1B4A23200 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Mathematics Degree Examination, April 2023

BMT4B04 – Linear Algebra

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

All questions can be attended.

(Each question carries 2 Marks – Ceiling- 25 Marks.)

1. Write the augmented matrix for the linear system $3x_1 - 2x_2 + x_3 = -5$, $x_1 - x_2 + 4x_3 = 0$, and $x_1 - 3x_3 = -2$.
2. Find all values of k for which the augmented matrix $\begin{bmatrix} 1 & k & -4 \\ 4 & 8 & 2 \end{bmatrix}$ corresponds to a consistent linear system.
3. If $A = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$, find trace of $(4B^T - A)$
4. If A is an invertible matrix, prove that A^T is invertible and $(A^T)^{-1} = (A^{-1})^T$.
5. Define subspace of a vector space. What are the subspaces of R^2?
6. Show that a set of two vectors is linearly independent if and only if neither vector is a scalar multiple of the other.
7. Prove that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $C^{\infty}(-\infty, \infty)$.
8. Find the coordinate vector of $v = (5, -1, 9)$ relative to the basis $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$, and $v_3 = (3, 3, 4)$.
9. State the dimension theorem for matrices.

10. Find the nullity of the matrix $\begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & -3 \\ 1 & 1 & -6 \end{bmatrix}$.

11. Use matrix multiplication to find the orthogonal projection of $(2,-5)$ onto the $y-$ axis.

12. Let $T_1(x_1,x_2,x_3)=(4x_1,-2x_1+x_2,-x_1-3x_2)$ and $T_2(x_1,x_2,x_3)=(x_1+2x_2,-x_3,4x_1-x_3)$.

Find an expression for $(T_2 \circ T_1)(x_1,x_2,x_3)$.

13. Find the characteristic equation of the matrix $A=\begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$..

14. Define an inner product space.

15. Find the cosine of the angle between $u=(-1,\ 5,\ 2)$ and $v=(2,\ 4,\ -9)$ with respect to the Euclidean inner product.

Section B

All questions can be attended.

(Each question carries 5 Marks – Ceiling- 35 Marks.)

16. Solve the linear system $2x_1+2x_2+2x_3=0,\ -2x_1+5x_2+2x_3=1,$ and $8x_1+x_2+4x_3=-1$ by Gauss – Jordan ellimination.

17. Express the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ as a product of elementary matrices.

18. Use row reduction to evaluate $\det(A)$ where $A=\begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$

19. Show that any subset $S=\{v_1,v_2,\ldots,v_r\}$ of R^n, where $r>n$, is linearly dependent.

20. Show that the vector space P_∞ of all polynomials with real coefficients is infinite dimensional.

21. Show that a set S of n vectors in an $n-$ dimensional vector space V is a basis for V if and only if S spans V or S is linearly independent.

22. Show that the operator $T: R^2 \to R^2$ defined by the equations

$w_1 = 2x_1 + x_2$, $w_2 = 3x_1 + 4x_2$ is one-to-one, and find $T^{-1}(w_1, w_2)$.

23. Find the eigen values and bases for the eigen spaces of the matrix $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$.

Section C

Answer any TWO questions.

(Each question carries 10 Marks)

24. (a) If $S = \{v_1, v_2, ..., v_n\}$ is a basis for a vector space V, then show that every vector v in V as a linear combination of vectors in S in exactly one way.

(b) Prove that any linearly independent set S of vectors in a finite dimensional vector space V is either a basis for V or can be enlarged to a basis for V.

25. Find basis for the row space and column space of the matrix

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}.$$

26. Consider the bases $B = \{u_1, u_2, u_3\}$ and $B' = \{u_1', u_2', u_3'\}$ for R^3 given by

$u_1 = [2 \ 1 \ 1]^T$, $u_2 = [2 \ -1 \ 1]^T$, $u_3 = [1 \ 2 \ 1]^T$, $u_1' = [3 \ 1 \ -5]^T$, $u_2' = [1 \ 1 \ -3]^T$,

and $u_3' = [-1 \ 0 \ 2]^T$.

(a) Find the transition matrix B to B'.

(b) Compute the coordinate vector $[w]_B$ and $[w]_{B'}$ of $w = [-5 \ 8 \ -5]^T$

27. (a) Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

(b) Define orthogonal matrices. Show that $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$. is orthogonal.

(2 × 10 = 20 marks)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, April 2023

BMT4C04 – Mathematics – 4

(2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

All questions can be attended. Each question carries 2 marks. Cieling 20 marks

1. Write an ordinary differential equation and specify its order.
2. Write example of a non-linear differential equation.
3. Verify that $y = \alpha e^{3x}$ satisfies the differential equation $y' = 3y$,
4. Verify that $e^{2x}\sin 2x$ is a solution of the differential equation $y'' - 4y' + 8y = 0$.
5. Give example of a second order linear non-homogeneous differential equation.
6. Test whether the functions e^{x} and e^{-x} are linear dependent or independent.
7. Evaluate $\mathscr{L}\{e^{2t}\}$.
8. Find $\mathscr{L}^{-1}\left\{\frac{s}{s^2+4}\right\}$.
9. State the first translation theorem of Laplace Transforms.
10. Prove that x^2 and x^3 are orthogonal functions on the interval $[-1,1]$.
11. Define Fourier series of an even function $f(x)$ in the interval $(-p,p)$.
12. Write the Heat Equation.

Section B

All questions can be attended. Each question carries 5 marks.
Cieling 30 marks

13. Solve the Initial Value Problem $\frac{dy}{dx} = \frac{2x}{3y}$, $y(0) = 1$.

14. Verify that the differential equation $(2x-1)dx + (3y+7)dy = 0$ is exact. Find the solution.

15. Solve the second order differential equation $y'' - 8y' + 12y = 0$.

16. Prove that $\{\sin 3x, \cos 3x\}$ is the fundamental set of solutions on any interval for the differential equation $y'' + 9y = 0$.

17. Prove that $\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}}$.

18. Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\}$.

19. Classify the Partial Differential Equation $3\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$.

Section C

Answer any One Question. Each question carries 10 marks.
10 marks from this section

20. Solve the initial value problem $x^2y'' - 5xy' + 8y = 0$; $y(2) = 32$, $y'(2) = 0$.

21. Using Laplace Transform, solve the initial value problem
$y'' + 5y' + 4y = 0$; $y(0) = 1$, $y'(0) = 0$.