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Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Statistics Degree Examination, November 2023

BST3B03 – Statistical Estimation

(2022 Admission onwards)

Time: 2½ hours

Max. Marks : 80

**Part A**

**Each question carries 2 marks**

1. Distinguish between estimator and estimate with an example.
2. For a Poisson distribution write the parameter and suggest an estimator for the parameter.
3. Briefly explain how sampling distribution is different from standard distributions.
4. Define Chi-square statistic.
5. What are the properties of student's t distribution?
6. List the desirable properties of a good estimator.
7. Define unbiasedness and give an example.
8. State Fisher – Neyman factorization theorem.
9. If X is standard normal variate and Y is a chi-square variate with n d.f. then comment on the distributions of t and  $t^2$  where  $t = \frac{X}{\sqrt{Y/n}}$ .
10. State the Cramer-Rao inequality.
11. Differentiate between point estimation and interval estimation.
12. What are the properties of moment estimators?
13. Define confidence coefficient.
14. Briefly explain Bayesian estimation method.
15. Write the confidence interval for the mean of a normal population when the standard deviation is unknown based on a large sample.

**(Maximum Mark = 25)**

**Part B**  
**Each question carries 5 marks**

16. Obtain the m.g.f of a chi-square distribution with  $n$  d.f.
17. If  $X$  has an  $F$  distribution with  $n_1$  and  $n_2$  d.f., find the distribution of  $1/X$  and give one use of this result.
18. If  $X$  and  $Y$  are independent chi-square random variables each with 1 d.f. then find  $\lambda$  such that  $P(X+Y > \lambda) = 1/2$ .
19. Explain the method of maximum likelihood estimation.
20. Define sufficiency. State the condition under which sample mean is sufficient for the population mean of a normal population and establish it.
21. Show that sample variance is a consistent estimator for the population variance though it is biased, in the case of normal population.
22. Let  $x_1, x_2, \dots, x_n$  be a random sample from the uniform distribution in the interval  $(0, \theta)$  and  $\theta > 0$ . Obtain the maximum likelihood estimator for  $\theta$ .
23. Obtain the 95% confidence interval for the variance of a normal population  $N(\mu, \sigma)$ .

**(Maximum Mark = 35)**

**Part C**  
**Each question carries 10 marks (Answer any TWO questions)**

24. If  $x_1, x_2, \dots, x_n$  are independent observations from standard normal distribution. Obtain the distributions of  $\bar{x}$  and  $U = x_1^2 + x_2^2 + \dots + x_n^2$ .
25. Derive the sampling distribution of sample variance  $S^2$  of a sample taken from normal population.
26. a) Examine sufficiency of the statistic  $\sum_{i=1}^n x_i^2$  for  $\sigma^2$  in the  $N(0, \sigma)$  distribution.  
b) For the rectangular distribution over the interval  $(\alpha, \beta)$ ,  $\alpha < \beta$ , find the maximum likelihood estimators of  $\alpha$  and  $\beta$ .
27. Obtain the 95% confidence interval for the difference of means of two normal populations  $N(\mu_1, \sigma_1^2)$  and  $N(\mu_2, \sigma_2^2)$  when (i)  $\sigma_1, \sigma_2$  known (ii)  $\sigma_1, \sigma_2$  unknown and from different populations.

**(2 x 10 = 20 Marks)**



## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Mathematics Degree Examination, November 2023

BST3C03 – Probability Distributions and Sampling Theory

(2022 Admission onwards)

Time: 2 hours

Max. Marks : 60

**SECTION-A**

Each question carries 2 Marks.

Maximum Marks that can be scored in this section is 20.

1. Define Cauchy distribution and write down its pdf.
2. What is convergence in distribution?
3. Give an example where cluster sampling can be used to select the sample.
4. Derive the mean of the geometric distribution.
5. If 5 and 6 are mode of a Binomial distribution. Write down the pmf.
6. State the conditions under which the Binomial distribution approaches the Poisson distribution.
7. State the lack of memory property of the exponential distribution.
8. What is a statistic? Give an example.
9. What is the relationship between Chi-Square and F variables?
10. If  $X_1$  and  $X_2$  are independent standard normal variables, what is the distribution of  $X_1^2 + X_2^2$  and  $\frac{x_1^2}{x_2^2}$
11. A r.v  $X$  has a uniform distribution over  $(-3,3)$ . Find  $P(|X| \leq 1)$ .
12. What do you mean by sampling distribution?

**SECTION-B**

Each question carries 5 Marks.

Maximum Marks that can be scored in this section is 30.

13. Show that for the Poisson distribution, the coefficient of variation is the reciprocal of the standard deviation.
14. State and prove weak law of large numbers.
15. A manufacturer claims that at most 10% of his product is defective. To test this claim, 18 units are inspected and his claim is accepted if among these 18 units, at most 2 are defective. Find the probability that the manufacturer's claim will be accepted if the actual probability that a unit will be defective is 0.05.

16. Establish the lack of memory property of the geometric distribution.
17. Define an F variate and give its pdf. Give an example of a statistic that follows F distribution.
18. Let  $X_i$  assume the values  $i$  and  $-i$  with equal probability. Show that the law of large numbers cannot be applied to the sequence of independent variables  $X_1, X_2, \dots$ .
19. Derive the mean and variance of the rectangular distribution.

### SECTION-C

(Answer any one Question and carries 10 marks)

20. a. State and prove Tchebycheff inequality.  
b. A r.v has mean value 5 and variance 3. What is the least value of  $(|X - 5| < 3)$  ?
21. Derive the mean and variance of the Normal distribution

(1 x 10 = 10 Marks).



FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE  
Third Semester B.Sc Statistics Degree Examination, November 2023

**BAS3C03 – Life Contingencies and Principles of Insurance**

(2022 Admission onwards)

Time: 2 hours

Max. Marks : 60

**PART-A (Short Answer)**

**Each question carries two marks. Maximum 20 Marks**

1. Define Principle of equivalence.
2. Define Retrospective Reserve.
3. What do you mean by Gross premium reserves?
4. What do you mean by Whole life insurance?
5. Define Gross premium.
6. What is the principle of insurance?
7. Define Agriculture Insurance.
8. Define Fidelity Guarantee.
9. What do you mean by Business Interruption cover?
10. Who is a risk averse person?
11. What do you mean by optimal insurance?
12. Define utility function.

**Maximum Marks = 20**

**PART-B (Paragraph)**

**Each question carries five marks. Maximum 30 Marks**

13. Calculate  $P(5 < K_{50:60} < 10)$  assuming that the two lives are both independently subject to AM92 mortality.
14. Explain the different types of life insurance.
15. Discuss the different premium payment structures.
16. Briefly explain different types of miscellaneous insurance.
17. Hubert, aged 60, is applying to buy a whole life immediate annuity from an insurance company, with his life savings of £200,000. Calculate the largest amount of level annuity, payable annually in arrears, that the insurer could pay if it requires a probability of loss from the contract of no more than 10%. Assume PMA92C20 mortality, interest of 5% *pa*, and expenses of 1% of each annuity payment.

18. A life aged exactly 50 buys a 15-year endowment assurance policy with a sum assured of £50,000 payable on maturity or at the end of the year of earlier death. Level premiums are payable monthly in advance. Calculate the monthly premium assuming AM92 Ultimate mortality and 4% *pa* interest. Ignore expenses.

19. Calculate:

(i)  ${}^P\overline{62:65}$

(ii)  ${}^3\overline{50:50}$

assuming that the two lives are both independently subject to AM92 Ultimate mortality.

**Maximum Marks = 30**

### **PART-C**

**(Answer any one Question and each carries 10 marks)**

20. A 25-year endowment assurance policy provides a payment of £75,000 on maturity or at the end of the year of earlier death. Calculate the annual premium payable for a policyholder who effects this insurance at exact age 45.

Expenses are 75% of the first premium and 5% of each subsequent premium, plus an initial expense of £250.

Assume AM92 Select mortality and 4% *pa* interest.

21. A 10-year term assurance with a sum assured of £500,000 payable at the end of the year of death, is issued to a male aged 30 for a level annual premium of £330.05. Calculate the prospective and retrospective reserves at the end of the fifth policy year, *i.e.* just before the sixth premium has been paid.

Assume AM92 Ultimate mortality and 4% *pa* interest. Ignore expenses.

**(1 x 10 = 10 Marks)**