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1B3N23086 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Mathematics Degree Examination, November 2023

BMT3B03 – Theory of Equations and Number Theory

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A

All questions can be attended
Each question carries 2 marks.

1. Show that $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $x - 3$.
2. Write the cubic equation with the roots 0, 1, 2.
3. Find the quotient and remainder when $3x^4 - 2x^3 + 2x^2 - 5x + 1$ is divided by $x - 2$.
4. Find Δ of the equation $x^3 + 6x^2 + 9x + 8 = 0$.
5. State Fundamental theorem of Algebra.
6. How many real roots has the equation $x^6 + x^4 - x^3 - 2x - 1 = 0$.
7. State well ordering principle.
8. Find the quotient and remainder when -325 is divided by 13.
9. Express $(18, 28)$ as a linear combination of 18 and 28.
10. State Lamé's Theorem.
11. Find the canonical decomposition of 1661.
12. Determine whether the LDE $12x + 18y = 30$ is solvable.
13. Define Pseudoprime. Give an example.
14. Find $\varphi(81)$, where φ Euler's Phi function.
15. Prove that the product of any two integers of the form $4n + 1$ is also of the same form.

(Ceiling: 25 Marks)

Section B

All questions can be attended

Each question carries 5 marks.

16. Solve the equation
$3x^3 - 16x^2 + 23x - 6 = 0$, if the product of the root is 1.

17. Find an upper limit of the positive roots of the equation
$2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.

18. Factorise into real linear and quadratic factors : $x^4 + x^3 + x^2 + x + 1$.

19. Prove that there are infinitely many primes.

20. Find six consecutive integers that are composites.

21. Prove that if p/ab, then p/a or p/b, where p is a prime.

22. Find the remainder when 3^{247} is divided by 17.

23. Prove that $a^{\phi(m)} \equiv 1(\mod m)$,where m is a positive integer and a any integer with $(a, m) = 1$.

(Ceiling: 35 Marks)

Section C

Answer any two Question

Each question carries 10 marks.

24. Examine whether the equation
$x^6 + 3x^5 - 36x^4 - 45x^3 + 93x^2 + 132x + 140 = 0$ has integral roots or not.

25. (a) Find the number of positive integers ≤ 3000 and divisible by 3, 5 or 7.
(b) Solve the congruence $12x \equiv 48(mod 18)$.

26. State and prove Wilson's Theorem.

27. (a) If f is a multiplicative function, then prove that $F(n) = \sum_{d/n} f(d)$ is multiplicative.
(b) Prove that the *tau* and *sigma* functions are multiplicative.

(2×10 = 20 Marks)

1B3N23087 (Pages : 2) Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Physics, Chemistry & Statistics Degree Examination,

November 2023

BMT3C03 – Mathematics – 3

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks. Overall ceiling 20

1. Let $f(x) = \sin x$. Evaluate $\int_0^{\pi} \sin x \ dx$ by trapezoidal rule with $n = 2$.
2. Find the components and length of the vector $\vec{v}$ with given initial point $P: (3,9,1)$ and terminal point $Q: (3-9,1)$.
3. Let $\vec{a} = [1,3,2], \vec{b} = [2,0,-5]$. Find $\vec{a}.(\vec{a}-\vec{b})$ and $\vec{a}.(\vec{b}-\vec{a})$
4. Find a parametric representation of the straight line through the point $(4,2,0)$ in the direction of the vector $i + j$.
5. Find the directional derivative of $x^2 + y^2$ at $(1,1)$ in the direction of $2i - 4j$.
6. State Green's theorem in the plane.
7. Write a parametric representation of the sphere $x^2 + y^2 + z^2 = 16$.
8. Evaluate the line integral $\int_C \boldsymbol{F}(\boldsymbol{r}).\boldsymbol{dr}$ where $\boldsymbol{F}(\boldsymbol{r}) = \boldsymbol{5zi + xyj + x^2zk}$ and C is the straight line segment $ti + tj + tk, 0 \le t \le 1$.
9. Evaluate the integral $\int_C \boldsymbol{F}.\boldsymbol{dr}$ from $(0,1,2)$ to $(1,-1,7)$, where $\boldsymbol{F} = (3x^2dx + 2yz\,dy + y^2dz)$. Given that F has potential $f(x,y,z) = x^3 + y^2z$.
10. Find the polar form of $3 + 3\sqrt{3}\,i$.
11. Find the value of the derivative $(z-i)/(z+i)$ at i.
12. Find an upper bound for the absolute value of the integral.

(Ceiling 20 Marks)

Section B

All questions can be attended.
Each question carries 5 marks. Overall ceiling 30

13. Sketch the graph and level curves at $c = 0,1,2$ of the function $f(x, y) = x^2 + y^2$.
14. Find the gradient of the function $F(x, y, z) = xy^2 + 3x^2 - z^3$ at $(4, -2, 8)$.
15. Find an equation for the tangent plane to the surface $z = x^2y$ at the point $(2,1,4)$.
16. Find the value of a if $u = (axy - z^2)i + (x^2 + 2yz)j + (y^2 - axz)k$ is irrotational.
17. If $f(x, y) = x^2y - 2xy$ and $R: 0 \leq x \leq 3, -2 \leq y \leq 0,$ then evaluate $\iint_R f(x, y)\ dA$.
18. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z)\ dzdxdy$.
19. Integrate $\frac{z^4 - 3z^2 + 6}{(z+i)^3}$ in the counter clockwise sense around the circle $|z| = 1.5$.

(Ceiling 30 Marks)

Section C

Answer any one of the questions.
The question carries 10 marks.

20. Using divergence theorem evaluate the net outward flux of the field $F = (x^3\, i + x^2\, y\, j + x^2 z\, k)$ across the closed surface S consisting of the cylinder $x^2 + y^2 = 4, 0 \leq z \leq 1$ and the circular disks $z = 0$ and $z = 1$; $(x^2 + y^2 \leq 4)$.
21. (a) Prove that $f(z) = x^2 + y^2$ is nowhere analytic.

    (b) Evaluate $\oint_C \frac{5z+7}{z^2+2z-3}\, dz$, where $C: |z - 2| = 2$.

(1 x 10 =10 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Computer Science Degree Examination, November 2023

BMT3C03(CS) – Mathematics

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks. Overall ceiling 20

1. Show that the function $f(z) = z^2 + 2z + 1$ is analytic for all z.
2. Express the complex number $\frac{2}{i}$ in the form $a + ib$.
3. Evaluate $\int_0^1 \mathbf{r}(t)dt$, where $\mathbf{r}(t) = t^2\mathbf{i} + 5t\mathbf{j} + 6t^3\mathbf{k}$.
4. The characteristic equation of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ is______________.
5. The Rank of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is______________.
6. Convert $(1, 0, -\sqrt{3})$ given in rectangular coordinates to spherical coordinates.
7. State Cayley Hamilton Theorem.
8. Find the Polar form of the complex number $z = -\sqrt{3} + i$.
9. Find the derivative of $f(z) = (4z + 3)(Z^2 - 8z + 4i)$.
10. If $z = 3x^2y^2 + 9x^3 + y^6 + 5$, find $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$.
11. The vector function $r(t) = a \cos t\, i + a \sin t\, j, 0 \le t \le 2\pi$ represents
12. Show that $\frac{z+\bar{z}}{2} = Re(z)$.

Section B

All questions can be attended.
Each question carries 5 marks. Overall ceiling 30

13. Show that $f(z) = \bar{z}$ is no where differentiable.

14. Find the parametric equation of the tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at $P: (1, \sqrt{2}, 0)$.

15. Find the gradient of the function $f(x, y) = e^x y + \sin(xy)$ at (2,0).

16. Find all solutions of the equation $z^4 + 1 = 0$.

17. Find all Eigen values of the matrix $A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

18. If **F** is a vector field having continuous second partial derivatives, then prove that div(curl **F**)=0.

19. Compute $(2 - 2i)^5$.

20. Let C denote the line segment $y = 2x + 1, -1 \leq x \leq 0$ and $G(x, y) = 6x^2 + 3y^2$. Evaluate the line integrals $\int_C G(x, y) dy$.

Part C

Answer any one of the question.
The question carries 10 marks.

21. State Cauchy's Integral formula. Evaluate $\oint_C \frac{z^2 - (\frac{1}{3})}{z^3 - z} dz$, where C is the circle $|z - 1/2| = 1$, oriented in the counter clockwise direction.

22. Let $u(x, y) = \log_e(x^2 + y^2)$. Verify that u is harmonic. Find the harmonic conjugate function of u and form the corresponding analytic function $f(z) = u + iv$.