1B2A23110 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Statistics Degree Examination, April 2023

BST2B02 – Bivariate Random Variables & Probability Distributions

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Part A

Each question carries 2 marks.

1. Define discrete continuous random variable and give one example.
2. Give the properties of probability density function.
3. Check whether $f(x) = 6x(1-x),\ 0 \leq x \leq 1$ is a probability density function?
4. If V(X)=2, then find V(3X – 4).
5. Define moment generating function. Give any two properties.
6. Define statistical independence of two random variables.
7. Define Poisson distribution and find its mean.
8. Obtain the moment generating function of binomial distribution?
9. Define hyper geometric distribution and find its mean.
10. Define the Beta distribution of first kind and second kind.
11. Define Cauchy distribution.
12. Define gamma distribution. Obtain its moment generating function.
13. Define Lindeberg-Levy central limit theorem for iid random variables.
14. Explain the concept of convergence in probability.
15. A random variable X is exponentially distributed with parameter 1. Use Chebychev's inequalities show that $P(-1 \leq X \leq 3) \geq \frac{3}{4}$.

Maximum Mark = 25

Part B

Each question carries 5 marks.

16. Find the distribution function F(x) of the random variable X with the following probability distribution

x :	0	1	4
f(x):	1/5	2/5	2/5

17. Given the pdf of a random variable X as

$$f(x) = \begin{cases} 5\,e^{-5x}; 0 < x < \infty \\ 0,\ \ otherwise \end{cases}$$

Show that $P(X \geq 7/X \geq 5) = P(X \geq 2)$.

18. If X is a random variable with pmf $p(x) = pq^x, x = 0,1,2, \ldots; o < p < 1, p + q = 1$.
Find the distribution of $Y = 2X$.

19. Define Chebychev's inequality. Use Chebychev's in equality to find how many times a fair coin must be tossed in order that the probability that the ratio of the number of heads to the number of tosses will lie between 0.45 and 0.55 will be atleast 0.95.

20. A random variable X is taking the values -2, 1 and 3 with probabilities
$P(X = -2) = 2\,P(X = 1) = P(X = 3)$. Obtain the variance of $3\,X^2 - 4$.

21. Let X and Y are two random variables with joint probability distribution

$$f(x,y) = \begin{cases} \frac{x + 2y}{27}; x, y = 0,1,2 \\ 0, \quad otherwise \end{cases}$$

Find the probability distribution of the random variable $Z = X + Y$.

22. If X and Y has joint probability distribution $f(x,y) = \frac{1}{4}$, for $(x,y) = (-3,-5), (-1,-1), (1,1), (3,5)$. Find $Cov(X,Y)$.

23. In a distribution exactly normal, 10.03% the items are under 25 Kg weight and 89.97% of the items are under 70 Kg weight. What are the mean and standard deviation of the distribution.

Maximum Mark = 35

Part C

Answer any two questions. Each question carries 10 marks.

24. Let X and Y are two random variables with joint probability mass function

$$f(x,y) = \begin{cases} \frac{x + 2y}{18}; x = 1,2;\ y = 1,2 \\ 0, \quad otherwise \end{cases}$$

Find (i) Correlation between X and Y, (ii) V(X/Y=2) and (iii) V(Y/X=1).

25. Given the joint pdf of two random variables X and Y,

$$f(x,y) = \begin{cases} 3(x + y); 0 < x < 1, 0 < y < 1, 0 < x + y < 1. \\ 0, \quad otherwise \end{cases}$$

Find (i) E(Y/X=x), and (ii) Cov(X,Y).

26. State and establish the Bernoulli's law of large numbers.

27. Derive the moment generating function of negative binomial distribution and hence show that *mean*<*variance*.

(2 x 10 = 20 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, April 2023

BST2C02 – Probability Theory

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

PART A

Each question carries 2 Marks.

Maximum Marks that can be scored in this part is 20

1. Define random experiment with example.
2. What do you meant by statistical regularity?
3. What are the axioms of probability?
4. Define mutual independence of 3 events.
5. Obtain the probability distribution of the number of heads when three coins are tossed together?
6. Find the Constant c of the probability density function of a random variable x $f(x) = cx^2, 0 \leq x \leq 1$.
7. Write any four properties of cumulative distribution function.
8. Define raw moment and central moment in terms of expectation.
9. For a random variable x show that $v(ax-b) = a^2 v(x)$, when they exist.
10. Define characteristic function of a random variable.
11. The Joint probability density function of two random variables X and Y is given by

    $f(x, y) = 24x(1-y), 0 < x < y < 1$

    $= 0$ otherwise

    Find marginal probability density function of Y?
12. Show that when X and Y are independent Cov (X, Y) = 0.

PART B

Each question carries 5 marks.

Maximum Marks that can be scored in this part is 30.

13. Define Sample point and sample space. Also specify the sample space of the random experiment "Three apples are distributed among three children".
14. State and prove Baye's theorem.

15. A continuous random variable X has the density function $f(x) = Kx(2-x)$, $0<x<1$. Find the constant K? Also obtain the distribution function?

16. If X is a random variable with probability density function $f(x) = 6x(1-x)$, $0<x<1$. Find the probability density function of $Y = 2X$.

17. Define moment generating function of a random variable x. Also state and prove any two properties of it.

18. Find mean and variance of a random variable X with probability density function

$P(X=x) = q^{x-1}p$, $x=1, 2, \ldots\ldots$

$p+q=1$

19. Joint probability density function of 2 random variables X and Y is

$f(x, y) = 3(x+y)$; $0<x<1$, $0<y<1$

Find Cov (X, Y)?

PART C

Answer any one question and carries 10 Marks

20. a) Define joint probability and marginal probability with example.

b) A bowl contains 2 Roses and &4 Jasmine flowers and a second bowl contains 4 Roses and 3 Jasmine flowers. If a flower is selected at random from one of the bowls, what is the probability that it is a rose flower?

21. For two random variables state and prove:

a) Addition theorem on expectation

b) Multiplication theorem on expectation

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Statistics Degree Examination, April 2023

BAS2C02 – Life Contingencies

(2022 Admission onwards)

Time: 2 hours | Max. Marks : 60

PART-A (Short Answer)

Each question carries *two* marks. Maximum 20 Marks

1. Write down the expressions for ${}_tp_x$ and ${}_tq_x$ in terms of the function l_x.
2. Define force of mortality.
3. Define term assurance.
4. Using an interest rate of 6% *pa* effective and AM 92 Ultimate mortality, calculate $A_{50:\overline{15}|}^{1}$
5. Define whole life level annuity.
6. Write down the formula for variance of the present value under whole life immediate annuity in arrears
7. Find

   i) $\ddot{a}_{30}$(AM92at4%)

   ii) $\ddot{a}_{75}$(PMA92C20 at 4%)
8. A level annuity of £1,000 *pa* is to be paid continuously to a 40- year-old male for the rest of his life. On the basis of 4% *pa* interest and AM92 Ultimate mortality, calculate the expected present value of this annuity.
9. Calculate the value of $A^{1}_{40:\overline{25}|}$ using AM 92 mortality and 4% *pa* interest
10. Calculate the expected present value of a payment of £2,000 made 6 months after the death of a life now aged exactly 60, assuming AM 92 Select mortality and 6% *pa* interest.
11. Evaluate the value of $\ddot{a}_{60:\overline{10}|}$ using AM92 mortality and 6% per annum interest.
12. Write down the expression for the expected present value of a life annuity of 1 *pa*, payable in arrears *m* times a year to a life age dx

Maximum Marks=20

Each question carries *five* marks. Maximum 30 marks

13. Calculate the following probabilities assuming AM 92 mortality applies:

i) $_{10}p_{90}$

ii) $_{5}p_{[79]+1}$

iii) $_{2}q_{[70]}$

iv) $_{5|}q_{[60]}$

v) $_{10|15}q_{50}$

14. In a certain population, the force of mortality equals 0.025 at all ages.

Calculate:

a) the probability that a new-born baby will survive to age 5

b) the probability that a life aged exactly 10 will die before age 12

c) the probability that a life aged exactly 5 will die between ages 10 and 12.

15. Explain endowment assurance contracts. Derive its mean and variance.

16. An annuity is payable continuously throughout the lifetime of a person now aged exactly 60, but for at most 10 years. The rate of payment at all times t during the first 5 years is £10,000 *pa*, and there after it is £12,000*pa*.

The force of mortality of this life is 0.03 *pa* between the ages of 60 and 65, and 0.04 *pa* between the ages of 65 and 70.

Calculate the expected present value of this annuity assuming a force of interest of 0.05 *pa*.

17. Explain deferred annuities .Derive it's mean .

18. A life office has just sold a 25-year term assurance policy to a life aged 40. The sum assured is £50,000 and is payable at the end of the year of death.

Calculate the variance of the present value of this benefit, assuming AM 92 Ultimate mortality and 4% *pa* interest.

19. A man aged exactly 40 buys a special 25-year endowment assurance policy that pays £30,000 on maturity. The man pays a premium of £670 at the start of each year throughout the 25 years, or until death if that happens first. Should that happen, all premiums paid so far are returned without interest at the end of the year of death

Calculate the expected present value of the benefits payable under this policy assuming AM 92 Select mortality and 4% *pa* interest.

Maximum Marks=30

PART-C (Essay)

Each question carries *ten* marks. Maximum 10 marks

20. Calculate the value of $_{1.75}p_{45.5}$ using AM92 Ultimate mortality and assuming that:

   a) Deaths are uniformly distributed between integer ages.

   b) The force of mortality is constant between integer ages.

21. Calculate the expectation and standard deviation of the present value of the benefits from each of the following contracts issued to a life aged exactly 40, assuming that the annual effective interest rate is 4% and AM92 Ultimate mortality applies:

   i) A 20-year pure endowment, with a benefit of £10,000

   ii) A deferred whole life assurance with a deferred period of 20 years, under which the death benefit of £20,000 is paid at the end of the year of death, as long as this occurs after the deferred period has elapsed.

Maximum Marks=10