1B2A23080

(Pages : 3)

Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, April 2023

BMT2B02 – Calculus – 2

(2022 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

All questions can be attended.
(Each question carries 2 Marks – Ceiling- 25 Marks.)

1. Find the area of the region enclosed by the curves $y = \sqrt{x}, x = 2$ and x – axis.
2. Find the volume of the solid obtained by revolving the region bounded by the graphs of $y = x^3$, $y = 8$ and $x = 0$ about the y axis.
3. Write an integral (no need to evaluate) giving the arc length of the graph of $y = \tan x$ from $P(0, 0)$ to $Q(\frac{\pi}{4}, 1)$.
4. Prove that $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$.
5. Find $\frac{dy}{dx}$ where $\ln(x + y) - \cos y - x^2 = 0$.
6. Find $\int \frac{\sqrt{\log x}}{x}\, dx$.
7. Prove the identity $\cosh^2 x - \sinh^2 x = 1$.
8. Evaluate $\lim_{x \to 0} \frac{e^x - 1}{x + \sin x}$.
9. Show that the sequence $\left(\frac{e^n}{n^2}\right)$ is divergent.
10. Give an example to show that sum of two divergent sequences may converge.
11. Does the series $\sum_{n=1}^{\infty} \tan^{-1} n$ converges?
12. Show that the series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 - 1}}$ diverges.
13. Test the convergence of $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$.

14. Give an example of an infinite series which is convergent, but not absolutely convergent.

15. Find the Taylor series of $f(x)=\dfrac{1}{1+x}$ at $x=2$.

Section B
All questions can be attended.
(Each question carries 5 Marks – Ceiling- 35 Marks.)

16. Find the number a such that the area of the region bounded by the graph of $x=(y-1)^2$ and the line $x=a$ is $\frac{9}{2}$.

17. Find the derivatives of (a) $y=\sin^{-1}\left(\dfrac{1}{x}\right)$ and (b) $y=(x+2)^{1/x}$.

18. Find $\int \frac{\sinh\sqrt{x}}{\sqrt{x}}\,dx$.

19. Show that $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x=e$.

20. Evaluate (a) $\lim\limits_{x\to 0^+}(x-\sin x)^{\sqrt{x}}$ and (b) $\lim\limits_{x\to \pi/2}(\tan x-\sec x)$.

21. Test the convergence of the series $\sum\limits_{n=1}^{\infty}\dfrac{\tan^{-1}n}{n^2+1}$.

22. Show that the infinite series $\sum\limits_{n=1}^{\infty}\dfrac{\sqrt{n}+\ln n}{n^2+1}$ converges.

23. Find the radius of convergence and the interval of convergence of $\sum\limits_{n=0}^{\infty}\dfrac{(-1)^n 2^n x^n}{\sqrt{n+1}}$.

Section C
Answer any TWO questions.
(Each question carries 10 Marks)

24. (a) Find the volume of the solid generated by revolving the region enclosed by the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ about the x axis.

(b) Find the area of the surface obtained by revolving the graph of $f(x)=\sqrt{x}$ on $[0,2]$ about the x axis.

25. (a) Evaluate $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$.

(b) State a comparison test for improper integrals. Use it to show that $\int_0^{\infty} e^{-x^2} dx$ is convergent.

26. (a) State squeeze theorem for sequences. Use it to prove that the sequence $\left(\frac{n!}{n^n}\right)$ converges to 0.

(b) Find the sum of the series $\sum_{n=1}^{\infty} \left[2(0.1)^n + 3(-1)^n (0.2)^n\right]$.

27. (a) Find a power series representation for $\tan^{-1} x$ by integrating a power series for $f(x) = 1/(1+x^2)$.

(b) Find the Maclaurin series of $\sin x$ and use it to find $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$.

(2 × 10 = 20 marks)

1B2A23081 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc (Physics, Chemistry & Statistics)Degree Examination, April 2023

BMT2C02 – Mathematics – 2

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended.

Each question carries 2 marks.

1. Using the definition, show that $\cosh^2 x - \sinh^2 x = 1$.
2. Find the Cartesian equation corresponding to the polar equation $r = 1 + 2\,r\,cos\theta$.
3. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$
4. Define elementary (raw) matrix.
5. What is the normal form of a matrix? Give an example.
6. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$
7. Show that $\lim_{n\to\infty} \frac{1}{n} = 0$
8. Find $\lim_{n\to\infty} \frac{2^n}{5n}$
9. Find $\sum_{n=1}^{\infty} \frac{4}{2^{n-1}}$
10. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
11. For what value of x, the power series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$, is convergent?
12. Define the Taylor series generated by the function $f(x)$ at a.

(Ceiling: 20 Marks)

Section B

All questions can be attended
Each question carries 5 marks

13. Evaluate $\int_0^1 \frac{2\,dx}{\sqrt{3+4x^2}}$

14. Graph the curve $r^2 = 4\cos\theta$

15. What are the basic elementary (raw) transformations? Explain with examples, one each.

16. Solve the following system of linear equations using Cramer's rule.

$$2x_1 + x_2 + 5x_3 + x_4 = 5, \quad x_1 + x_2 - 3x_3 - 4x_4 = -1,$$
$$3x_1 + 6x_2 - 2x_3 + x_4 = 8, \quad 2x_1 + 2x_2 + 2x_3 - 3x_4 = 2$$

17. Find $\lim_{n\to\infty} a_n$ where $a_n = \left(\frac{n+1}{n-1}\right)^n$

18. Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, -1 < x < 1$

19. State and prove n^{th} root test.

(Ceiling: 30 Marks)

Section C

Answer any one question.
Question carries 10 marks

20. Determine the characteristic roots and associated vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

21. (a) Find the length of the Cardioids $r = 1 - \cos\theta$

(b) Find the Taylor series and Taylor polynomial generated by $f(x) = e^x$ at $x = 0$.

(1x10 = 10 Marks)

1B2A23082 (Pages : 2) Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Computer Science Degree Examination, April 2023

BMT2C02(CS) – Mathematics – 2

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks.

1. Prove that $cosh^2x - sinh^2x = 1$.
2. Test the Convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{16}} \ldots$
3. Evaluate $\int sinh^2x\, dx$.
4. Show that $\lim_{n\to\infty} \frac{2n}{n^2+1} = 0$.
5. What is the polar coordinates of $(x, y) = (4, -2)$?
6. Write down the Maclaurien series for $cos\, x$.
7. Test the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n-1}$.
8. What is a p-series? Write the rule of its convergence.
9. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges.
10. The point $\left(4, \frac{\pi}{6}\right)$ is given in polar coordinates. Find its representation in rectangular coordinates.
11. Find the average value of $f(x) = 4 - x^2$ over the interval $[-1, 3]$.
12. Define a sequence. Give the first 5 terms of the sequence $\left\{\frac{\sqrt{n}}{2^{n-1}}\right\}$.

(Ceiling: 20 Marks)

Section B
All questions can be attended
Each question carries 5 marks

13. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}}$ - 1 where $0 \le x \le 1$.
14. Find the length of the perimeter of the cardioid $r = a(1 - cos\theta)$.
15. Using Maclaurin's series expand $\tan^{-1} x$. Hence deduce the Gregory series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots$

16. Determine whether the sequence $\left\{\frac{n}{n+1}\right\}$ converges or diverges

17. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on [0,2] about the x-axis.

18. State and Prove the Mean Value Theorem for Integrals.

19. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

(Ceiling: 30 Marks)

Section C

Answer any one question.
Question carries 10 marks

20. a) Evaluate $\int_1^{\infty} \frac{\ln x}{x^2} dx$, if it exists.

b) Find the area of the region shared by the cardioids $r = 2(1 + cos\theta)$ and $r = 2(1 - cos\theta)$.

21. Sketch the graphs of the polar equations, and reconcile your results by finding the corresponding rectangular equations.

a) $r = 2$

b) $\theta = \frac{2\pi}{3}$

(1x10 = 10 Marks)