

## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Mathematics Degree Examination, November 2023

BMT1B01 – Basic Logic and Calculus – I

(2022 Admission onwards)

Time : 2 ½ hours

Max. Marks : 80

## Section A

All questions can be attended  
Each question carries 2 marks

- Write the sentence in *if-then* form: ' $x = 1$  is sufficient for  $x^2 = 1$ '
- Show that  $p \rightarrow q \equiv \sim q \rightarrow \sim p$
- Rewrite the implication in inferential form:  
 $[(p \rightarrow q) \wedge (\sim q)] \rightarrow \sim p$
- Find  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$
- Let  $f(x) = \frac{1}{x^2}$ . Evaluate  $\lim_{x \rightarrow 0^+} f(x)$ ,  $\lim_{x \rightarrow 0^-} f(x)$  and  $\lim_{x \rightarrow 0} f(x)$ , if they exist.
- Show that  $\lim_{x \rightarrow 2} [x]$  does not exist.
- Find the extrema of  $f(x) = x^2, -1 < x < 2$ .
- Verify that  $f(x) = x^2 + 1$  satisfies the hypothesis of Mean Value Theorem on  $[0, 2]$ .  
Find the value of  $c$ .
- Find the points of inflection of  $f(x) = x^4 - 4x^3 + 12$
- Find the horizontal and vertical asymptotes of  $g(x) = \frac{x}{x+1}$
- Find two numbers whose difference is 50 and whose product is minimum.
- Find all the anti-derivatives of  $f(x) = 1$  on  $(-\infty, \infty)$
- Evaluate  $\int \frac{\sin t}{\cos^2 t} dt$
- Evaluate the sum:  $\sum_{k=1}^n \frac{1}{n} \left(1 + \frac{k}{n}\right)^2$
- Divide the interval  $[2, 5]$  into  $n$  subintervals of equal length and let  $c_k$  be any point in  $[x_{k-1}, x_k]$ . Write  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{1 + c_k^2} \Delta x$  as an integral.

(ceiling 25 Marks)

**Section B**  
**All questions can be attended**  
**Each question carries 5 marks**

16. Let  $P(x, y): y^2 < x$  and  $x$  and  $y$  are real numbers. Determine the truth values of:  
 i)  $(\forall x)(\forall y)P(x, y)$     ii)  $(\exists y)(\forall x)P(x, y)$
17. Construct truth table and verify:  $\sim (p \rightarrow q) \equiv p \wedge \sim q$
18. Find  $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$ .
19. Using formal definition prove that  $\lim_{x \rightarrow 2} x^2 = 4$
20. Determine the intervals where the function  $f(x) = x + \frac{1}{x}$  is increasing and where it decreasing.
21. Prove that  $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$ .
22. Show that  $\int_a^b x \, dx = \frac{1}{2}(b^2 - a^2)$
23. Find the average value and  $c$  guaranteed by the Mean Value Theorem for Integrals for  $f(x) = 4 - 2x$  on  $[0, 2]$ .

(ceiling 35 Marks)

**Section C**  
**Answer any two questions**  
**Each question carries 10 marks**

24. a) Test the validity of the argument

$$p \vee q$$

$$q \vee r$$

$$\sim r$$

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$$\therefore p$$

- b) Prove by contradiction that there is no largest prime number

25. Let  $f(x) = \begin{cases} ax + b, & \text{if } x \leq 1 \\ 4, & \text{if } x = 1 \\ 2ax - b, & \text{if } x > 1 \end{cases}$ . Find the value of  $a$  and  $b$  that will make

$f$  continuous on  $(-\infty, \infty)$ .

26. Sketch the graph  $f(x) = x^3 - 3x^2 + 1$ .

27. a) Find  $\frac{dy}{dx}$  if  $y = \frac{1}{1+t^2} dt$

- b) Evaluate  $\int_{-2}^2 f(x) dx$  where,  $f(x) = \begin{cases} -x^2 + 1, & \text{if } x < 0 \\ x^3 + 1, & \text{if } x \geq 0 \end{cases}$

(2 x 10 = 20 Marks)



## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc (Chemistry, Physics &amp; Statistics) Degree Examination,

November 2023

BMT1C01 – Mathematics – I

(2022 Admission onwards)

Time : 2 hours

Max. Marks : 60

## Section A

All questions can be attended  
Each question carries 2 marks

1. Find  $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$ .
2. Show that  $y = \sin \frac{1}{x}$  has no limit as  $x$  approaches zero from either side.
3. For what value of  $a$ ,  $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$  is continuous at every  $x$ ?
4. Define  $g(4)$  in a way that extends  $g(x) = \frac{(x^2-16)}{(x^2-3x-4)}$  to be continuous at  $x = 4$ .
5. Suppose that  $f(-1) = 3$  and  $f'(x) = 0$ . Then find  $f(x)$ .
6. Find  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$ .
7. Find the horizontal tangents of the function  $f(x) = \frac{x^3}{3} - \frac{x^2}{2} + 1$ .
8. State first derivative theorem for local extreme values.
9. Define point of inflection of a graph of a function. Give an example.
10. Show that the value of  $\int_0^1 \sin x^2 dx$  cannot possibly be 2.
11. Find all points in the interval  $[0, 1]$  at which the function  $f(x) = -3x^2 - 1$  assumes its average value.
12. If  $y = \int_1^{\sin x} 3t^2 dt$  find  $\frac{dy}{dx}$  by differentiating integral directly.

(Ceiling: 20 Marks)

**Section B**  
**All questions can be attended**  
**Each question carries 5 marks**

13. Find the derivative of  $g(t) = \tan(5 - \sin 2t)$ .
14. Show that the equation  $x^3 - 15x + 1 = 0$  has three solutions in the interval  $[-4, 4]$ .
15. Find an equation of the tangent line to the curve  $y = \frac{8}{\sqrt{x-2}}$  at  $(6, 4)$ .
16. Find the tangent and normal to the curve  $x^2 + xy - y^2 = 1$  at the point  $(2, 3)$ .
17. A hot balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is  $\frac{\pi}{4}$ , the angle is increasing at the rate of 0.14 rad/mm. How fast is the balloon rising at the moment?
18. Find the absolute extrema of  $h(x) = x^{\frac{2}{3}}$  on  $[-2, 3]$ .
19. Find the total area of the region between x-axis and the graph of the function  $f(x) = -x^2 - 2x, -3 \leq x \leq 2$ .

**(Ceiling: 30 Marks)**

**Section C**  
**Answer any one question**  
**Question carries 10 marks**

20. Graph the function  $y = \frac{x^3+1}{x}$ . Include the equations of asymptotes and dominant terms.
21. (i) Find the intervals on which  $f(x) = -x^3 + 12x + 5, -3 \leq x \leq 3$  is increasing and decreasing. Where does the function assume extreme values and what are these values?  
  
(ii) Find the asymptotes of the graph of  $y = -\frac{x^2-4}{x+1}$ .

**(1 × 10 = 10 Marks)**



## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Computer Science Degree Examination, November 2023

BMT1C01(CS) – Mathematics – I

(2022 Admission onwards)

Time : 2 hours

Max. Marks : 60

## Section A

All questions can be attended  
Each question carries 2 marks

1. The power set of the set consists of the letters of the word MATHS is.....
2. Let A be the set of all straight lines. A relation on R is defined by  $xRy$  iff x is perpendicular to y, for all  $x, y \in A$ . Is this a transitive relation? Justify.
3. Dual of  $(A \cap B^c)^c \cup B$  is.....
4. Negate the statement  $\neg(p \wedge \neg q \vee r)$ .
5. Define Proposition. Give an example.
6. Write a short note on universal and existential quantifiers.
7. Position of a particle at a time t is given by the function  $p(t) = 3t^2 + 2\sqrt{t}$ . Find the velocity and acceleration of the particle at  $t = 2$ .
8. Find  $\lim_{x \rightarrow \frac{\pi}{4}} (2x^2 - \cot x)$ .
9. Find  $\frac{d}{dx} \left( \frac{x}{x^2+1} \right)$ .
10. Define an increasing function. Give an example.
11. Find all the asymptotes of  $f(x) = \frac{x^2-4}{x-1}$ .
12. If  $f'(x) = 2x$ , for all  $x \neq -2$  and  $f(-2) = 3$ . Find  $f(2)$

(Ceiling: 20 Marks)

## Section B

All questions can be attended  
Each question carries 5 marks

13. In an examination 40% students passed in Maths only, 30% students passed in Physics only and 10% students failed in both subjects. If 400 students passed in Physics, find the total number of students by drawing Venn diagram.
14. Show that  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ .
15. If P and Q are two relations on a set A, Show that  $P \cap Q$  is a relation on A.

16. Test the continuity of the function  $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

17. Find the equations of all the lines having slope -1 that are tangent to the curve

$$y = \frac{1}{(x-1)^2}$$

18. Find the points of inflection of the function  $f(x) = 3x^4 - 4x^3 + 1$ .

19. Find the local and absolute extreme values of the function  $\frac{1}{3}x^3 - 2x^2 + 4x$ ,  $0 \leq x < \infty$ .

(Ceiling: 30 Marks)

### Section C

Answer any one question  
Question carries 10 marks

20. i) a) Evaluate  $\lim_{x \rightarrow \infty} \frac{x + \sin x}{2x + 7 - 5 \sin x}$       b) Find  $y'$  at  $(-1, 1)$  if  $x^2 + 3xy + y^2 = -1$ .

ii) Define contingency. Show that  $(p \vee q) \wedge (\neg q)$  is a contingency.

21. Graph the function  $f(x) = \frac{1}{6}(x^3 - 6x^2 + 9x + 6)$ .

## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

## First Semester BVOC AUTOMOBILE Degree Examination, November 2023

## SDC1MT01 – Mathematics

(2022 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

## PART A

Answer all questions.

Each question carries Two marks.

1. Find  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$
2. Find  $\lim_{x \rightarrow \infty} \frac{x - 5}{3x - 4}$
3. State Rolle's theorem for derivative.
4. Differentiate  $f(x) = 6x^3 + 2x^2 + x + 4$ .
5. Find the slope of the tangent line to the graph of  $f(x) = 3\sqrt{x}$  at  $x = 9$ .
6. A sphere of radius  $r$  millimetres has volume  $V = \frac{4}{3}\pi r^3$  cubic millimeters. Find the rate of increase of volume with respect to radius at  $r = 3$ .
7. Find the second derivative of  $f(x) = 2x^3 - 5x^2 + 4x + 8$
8. Evaluate  $\int_0^3 (x^2 + x - 1) dx$
9. If  $y = f(x)$  and  $2x^2 + 5y^2 = 1$ , express  $dy/dx$  in terms of  $x$  and  $y$ .
10. Find the sum of the first 100 integers.
11. Find the inflection points of  $f(x) = x^3 - x^2$ .
12. Analyze the concavity of  $f(x) = 10x^3$  at the points  $x = -1$ ,  $x = 0$ , and  $x = 1$ .
13. Find the volume of a ball of radius  $r$ .
14. Is the function  $f(x) = x^4 + 3x^2 + 12$  odd or even?
15. A bus travels  $2x^2$  meters in  $x$  seconds. Find the instantaneous velocity at the time  $x = 4$ .

[ Ceiling = 25 Marks]



## PART B

Answer *all* questions.

Each question carries *Five* marks.

16. Differentiate  $f(x) = (2x^3 - 5)(4x + 8)$
17. Find the equation of the tangent line to the parametric curve  $x = t^3$  and  $y = t^5$  at  $t = 2$ .
18. Find the critical points of the function  $f(x) = x^3 - x$ . Are they local maximum or minimum points?
19. Find  $\sum_{j=3}^{102} (j - 2)$ .
20. A car has position  $2t^4 + 5t - 6$  at time  $t$ . Find the velocity and acceleration of the car at  $t = 2$ .
21. Find  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$
22. Draw the graph of the step function  $g$  on  $[-1, 2]$  defined by

$$g(x) = \begin{cases} -4 & \text{if } -1 \leq x < 0 \\ 2 & \text{if } 0 \leq x \leq 1 \\ 3 & \text{if } 1 < x \leq 2 \end{cases}$$

Compute the signed area of the region between its graph and the  $x$  axis.

23. Find the area between the graphs  $y = x^2$  and  $y = x + 3$  on  $[-1, 1]$ .

[ Ceiling = 35 Marks]

## PART C

Answer any *two* questions

Each question carries *Ten* marks.

24. Sketch the graph of  $f(x) = 2x^3 + 8x + 1$ .
25. Calculate an approximate value for  $\frac{1}{(2.01)^2 + (2.01)^3}$ .
26. (a) Find the average value of  $f(x) = x^2$  on  $[0, 2]$ .
- (b) The region under the graph of  $y = x^2$  on  $[0, 1]$  is revolved about the  $x$  axis. Sketch the resulting solid and find its volume.
27. (a) Show that if  $f'(x_0)$  exists, then  $f$  is continuous at  $x_0$ .
- (b) Does  $\lim_{x \rightarrow \infty} \frac{|x|}{x}$  exist?

[2 x 10 = 20 Marks]