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1B6A24305 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2024

BMT6B10 - Real Analysis

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A
All questions can be attended
Each question carries 2 marks

1. Define Thomae's function.
2. State maximum minimum theorem for continuous functions.
3. Define non-uniform continuity criteria
4. State Weierstrass Approximation Theorem.
5. Let $f : [a, b] \to R$ be defined by $f(x) = k$. Show that f is Riemann integrable.
6. State first form of fundamental theorem of calculus
7. Evaluate $\int_1^4 \frac{\cos\sqrt{t}}{\sqrt{t}} dt$
8. Discuss the convergence of $g_n(x) = x^n$, $x \in R$, $n \in N$.
9. Define uniform norm on a set of bounded functions.
10. State the theorem on interchange of limit and integral.
11. State Cauchy's criterion for uniform convergence of series of functions.
12. Evaluate $\int_{-1}^{0} \frac{dx}{\sqrt[3]{x}}$.
13. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2}$.
14. Find Cauchy's Principal value of $\int_{-\infty}^{\infty} x dx$
15. Prove that $\Gamma n = (n-1)\Gamma(n-1)$.

Ceiling – 25 Marks

Section B
All questions can be attended
Each question carries 5 marks

16. State and prove Boundedness Theorem on Continuous functions.

17. If f : [a, b] $\to$R is continuous on [a, b], then prove that f$\in$R[a,b].

18. If $f\in$R[a, b] and if $|f(x)| \le M$ for all x$\in$[a, b], then prove that |f| is in R[a,b] and

$$\left|\int_a^b f(x)dx\right| \le M(b-a).$$

19. Let (f_n) be a sequence of continuous functions on A$\subset$R and suppose that (f_n) converges uniformly on A to a function f:A$\to$R. Then prove that f is continuous on A.

20. If f$\in$R[a, b] then prove that f is bounded on [a, b].

21. Prove that $\lim_{n\to\infty}\frac{\sin(nx+n)}{n} = 0$, for x$\in$R.

22. Show that even though the improper integral $\int_{-1}^{5}\frac{dx}{(x-1)^3}$ does not converge its Cauchy Principal value exists.

23. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Ceiling – 35 Marks

Section C
Answer any two questions
Each question carries 10 marks

24. (a) State and prove Uniform Continuity Theorem.

(b) If f:A$\to$R is uniformly continuous on a subset A of R and if (x_n) is a Cauchy sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in R

25. State and prove Cauchy's criterion for Riemann integrable functions.

26. (a) If f$\in$R[a, b], then prove that the value of the integral is uniquely determined.

(b) Prove that a sequence (f_n) of bounded functions on A $\subseteq$ R converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$ as n $\to\infty$.

27. (a) Prove that $B(m,n) = 2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\, d\theta$

(b) Prove that $B(m.n) = \frac{\Gamma m\Gamma n}{\Gamma(m+n)}$.

2×10 = 20 Marks

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2024

BMT6B11 - Complex Analysis

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

SECTION A

Answer the following questions. Each carries two marks

(Ceiling 25)

1. Show that $f(z) = e^x e^{-iy}$ is no where differentiable .
2. Evaluate $lim_{z \to 2+i} \frac{z^2-4z+5}{z^3-z-10i}$.
3. Define analytic function and singular point of a function with an example
4. If $f(z) = u + iv$ is analytic in a domain D then prove that its component functions u and v are harmonic in D.
5. Write $\mid exp(2z+i) \mid$ and $\mid exp(iz^2) \mid$ in terms of x and y.
6. Let C denote the curve defined by $y = 2x + 1, -1 \leq x \leq 0$. Then evaluate $\int_C (3x^2 + 6y^2) ds$.
7. If f is continous and $\mid f(z) \mid \leq M$ every where on a contour C and L is the length of C then prove that $\mid \int_C f(z)dz \mid \leq ML$
8. State Cauchy Goursat Theorem and solve $\int_C f(z)dz$ where $f(z) = \frac{z^2}{z^2+9}$ and $C : \mid z-1 \mid = 1$
9. Evaluate $\oint_C \frac{z^2-1}{z^2+1} dz, C : \mid z - i \mid = 1$
10. Prove that the series $\Sigma_{k=1}^{\infty} z_k$, where $z_k = i^k/k^2$ converges
11. Find the radius of convergence and circle of convergence of $\Sigma_{n=0}^{\infty} (\frac{6n+1}{2n+5})^n (z-2i)^n$
12. Obtain the Taylor series representation for $1/z^2$ about $z = 2$
13. Determin the zeros and their order of the function $f(z) = (z + 2 - i)^2$
14. Identify the type of singularity of (a) $\frac{z^2}{1+z}$ (b) $ze^{1/z}$
15. Find the residue at the singular point (a) $\frac{3e^z}{z^5}$ (b) e^{1/z^2}

SECTION B

Answer the following questions. Each carries five marks
(Ceiling 35)

16. Show that for the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z}, when \; z \neq 0 \\ 0 \; when \; z = 0 \end{cases}$ even though partial derivative of component function exists and satisfy C-R equations at $z = 0$, $f(z)$ is not differentiable at $z = 0$.

17. Explain Cauchy Riemann Equation in polar co-ordinates.

18. Find the image of the annulus $2 \leq | z | \leq 4$ under the logarithmic mapping $w = Lnz$.

19. Integrate $\int_C \frac{tanz}{z^2-1} dz \;\; C : | z | = 3/2$ in clockwise direction

20. Let C be the arc of the circle $| z | = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Without evaluating integral show that $| \int_C \frac{dz}{z^2-1} | \leq \frac{\pi}{3}$

21. State Maximum Modulus Theorem and find the maximum of $sinz$ in the region R : $0 \leq x \leq \pi, \;\; 0 \leq y \leq 1$

22. Find all Maclaurin and Laurent series representation of the function $f(z) = \frac{-1}{(z-1)(z-2)}$ about $z = 0$.

23. Evaluate $\oint_C \frac{dz}{zsinz}$ where C is the unit circle in the positive direction.

SECTION C

Answer any two questions ($2 \times 10 = 20$ Marks)

24. (a)If f is a analytic in a simply connected domain D, then prove that f has an antiderivative in D, that is there exist a function F such that $F'(z) = f(z)$

    (b) Evaluate $\int_0^{1+i} e^{\pi z} dz$

25. (a) Let $f(z) = u(x, y) + iv(x, y)$ is differentiable at a point $z = x + iy$. Then prove that at z the first order partial derivatives of u and v exist and satisfy the Cauchy-Riemann equations.
    (b) If f is analytic in a simply connected domain D and let C be a simple closed contour lying entirely in D. Then prove that for any point z_0 in C
    $f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-z_0)} dz.$

26. State and Prove Taylor's Theorem

27. Show that $\int_{-\infty}^{\infty} \frac{2x^2-1}{x^4+5x^2+4} = \pi/4$

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2024

BMT6B12 - Calculus of Multivariable - 2

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A
All questions can be attended
Each question carries 2 marks

1. Define and write the equation of tangent plane and normal line.
2. Evaluate $\int_1^2 \int_0^1 3x^2y\, dx\, dy$.
3. Let $f(x,y) = x^2 + y^2 - 4x - 6y + 17$. Find the critical point of f and show that f has a relative minimum at that point.
4. Write the formula for finding the area of the surface.
5. Use a double integral to find the area enclosed by one loop of the three leaved rose $r = \sin 3\theta$.
6. Define Jacobian. State Change of variables in double integrals.
7. Find the gradient vector field of $f(x,y,z) = x^2 + xy + y^2z^3$
8. Find the divergence of $F(x,y,z) = xyz\, \boldsymbol{i} + x^3y^2z\, \boldsymbol{j} + xy^2\boldsymbol{k}$ at the point (1,-1,2).
9. Evaluate $\iiint_B (x^2y + yz^2)dV$, where $B = \{(x,y,z): -1 \le x \le 1, 0 \le y \le 3, 1 \le z \le 2\}$.
10. Evaluate $\int_c kz\, ds$, where k is a constant and C is the circular helix.
11. Determine whether the vector field $F(x,y) = (x^2 - 2xy + 1)i + (y^2 - x^2)j$ is conservative.
12. State Green's theorem.
13. Find a parametric representation of helicoid.
14. Identify the surface represented by $r(u,v) = 2\cos u i + 2 \sin u j + v k$ with parameter domain $D = \{(u,v) | 0 \le u \le 2\pi, 0 \le v \le 3\}$.
15. Define Surface integral of Scalar and Vector Fields.

(Ceiling 25 Marks)

Section B
All questions can be attended
Each question carries 5 marks

16. Show that the point (0,0) is a critical point of $f(x, y) = y^2 - x^2$ but that it does not give rise to a relative extremum of f.

17. Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$

18. Evaluate $\int\int\int_T z\, dV$ where T is the solid in the first octant bounded by the graphs of $z = 1 - x^2$ and $y = x$.

19. Evaluate $\int\int_R \cos\left(\frac{x-y}{x+y}\right) dA$, where R is the trapezoidal region with vertices (1,0), (2,0), (0,2) and (0,1).

20. Show that the divergence of the electric field $\boldsymbol{E}(x, y, z) = \frac{kQ}{|\boldsymbol{r}|^3}\boldsymbol{r}$, where $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, is zero.

21. Suppose that $\boldsymbol{F}$ is a continuous vector field in a region $\boldsymbol{R}$. Then $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ is independent of path iff $\int_C \boldsymbol{F} \cdot d\boldsymbol{r} = 0$ for every closed path C in $\boldsymbol{R}$.

22. Evaluate $\int\int_S x\, dS$, where S is the part of the plane $2x + 3y + z = 6$ that lies in the first octant.

23. Find the flux of the vector field $\boldsymbol{F}(x, y, z) = y\boldsymbol{i} + x\boldsymbol{j} + 2z\boldsymbol{k}$ across the unit sphere $x^2 + y^2 + z^2 = 1$

(Ceiling 35 Marks)

Section C
Answer any two questions
Each question carries 10 marks

24. Find the relative extrema of $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$

25. Evaluate $\int\int\int_T \sqrt{x^2 + z^2}\, dV$, where T is the region bounded by the cylinder $x^2 + z^2 = 1$ and the planes $y + z = 2$ and $y = 0$.

26. Let $\boldsymbol{F}(x, y, z) = 2xyz^2\boldsymbol{i} + x^2z^2\boldsymbol{j} + 2x^2yz\,\boldsymbol{k}$.
   (a) Show that $\boldsymbol{F}$ is conservative, and find a function f such that $\boldsymbol{F} = \nabla f$
   (b) If $\boldsymbol{F}$ is a force field, find the work done by $\boldsymbol{F}$ in moving a particle along any path from (0,1,0) to (1,2,-1).

27. Evaluate $\oint (y^2 + \tan x)\, dx + (x^3 + 2xy + \sqrt{y})dy$, where C is the circle $x^2 + y^2 = 4$ and is oriented in a positive direction.

(2x10 = 20 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2024

BMT6B13 - Differential Equations

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A

All questions can be attended. Each question carries 2marks.

1. Find an integrating factor for the equation $\frac{dy}{dt} + 4y = e^{-t}$.
2. Show that the equation $\frac{dy}{dx} = \frac{x^2}{1-y^2}$ is separable.
3. Solve $\frac{dy}{dx} = \frac{y}{x}$.
4. State the existence and uniqueness theorem for first-order linear equations.
5. Solve $y'' + 5y' + 6y = 0$.
6. Find the Wronskian of $y_1 = e^t \sin t, y_2 = e^t \cos t$.
7. State Abel's theorem.
8. Determine the longest interval in which the solution of the initial value problem $ty'' + 3y = t,\ \ y(1) = 1,\ y'(1) = 2,$ is certain to exist.
9. Find the general solution of $y'' + 9y = 0$.
10. Find a particular solution of $y'' - 3y' - 4y = 3e^{2t}$.
11. Find $\mathcal{L}^{-1}\left[\frac{1-e^{2s}}{s^2}\right]$.
12. Find the Laplace transform of $\{4\,t \cos 2t - 5\,e^{2t}\}$.
13. Write the explicit form of $g(t) = u_1(t) + 2u_3(t) - 6u_4(t)$.
14. What is the primitive period of $\sin 3x$.
15. Write the formulae for computing the Fourier coefficients in the Fourier series expansion of a periodic function $f(x)$ of period $2L$.

(Ceiling 25 Marks)

PARTB

All the questions can be attended. Each question carries 5 marks.

16. Solve the IVP $y' = \frac{3x^2+4x+2}{2(y-1)}, y(0) = -1$ and determine the interval in which the solution exists.
17. Check the exactness and solve the differential equation $(2x+4y)+(2x-2y)y' = 0$.
18. Use the method of reduction of order to find a second solution of the differential equation $t^2y'' + 2ty' - 2y = 0,\ t > 0,\ y_1(t) = t$.
19. Use the method of variation of parameters to find the general solution of the differential equation $y'' - 5y' + 6y = e^{2t}$.
20. Using the definition of Laplace transform find $L[\cos at]$.
21. Using convolution theorem, find $\mathcal{L}^{-1}\left[\frac{1}{s^2(s-1))}\right]$.
22. Obtain the half range cosine series of $f(x) = x$, when $0 < x < 2$.
23. Replace the PDE $xu_{xx} + u_t = 0$ by a pair of ordinary differential equations using the method of separation of variables.

(Ceiling 35 Marks)

PART C

Answer any *two* questions. Each question carries 10 marks.

24. (a) Solve the differential equation $xdy - ydx = 0$.
    (b)Solve the IVP $y' = y,\ y(0) = 1$, by Picard's iteration method (do three steps). Also find the exact solution.
25. Find the general solution of the differential equation $y'' - 4y' + 4y = t^3e^{2t} + te^{2t}$.
26. Using Laplace transformation solve $y'' + 3y' + 2y = \delta(t-5) + u_{10}(t),\ y(0) = 0,$ $y'(0) = 1/4$.
27. Find the Fourier series for the function $f(x) = \begin{cases} -k, & when -\pi < x < 0 \\ k, & when \quad 0 < x < \pi \end{cases}$ and $f(x+2\pi) = f(x)$and hence deduce the Madhav-Gregory series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots$.

(2 x 10 = 20 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Mathematics Degree Examination, April 2024

BMT6B14(E03) - Mathematical Programming with Python and Latex

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended. Each question carries 2 marks.

1. Modify the expression *print 5**2+5*2* to get a result 40.
2. What is the difference between a[6:] and a[:6] where a= 'mathematics'.
3. What is **List** in Python?
4. Explain **while** statement in Python?
5. Write a python program that uses *if...else* statement.
6. Write the effect of break statement in Python.
7. What is meant by slicing of a string ?, Give example.
8. What is the output of the following Python program?

```
From numpy import *
   a = arrange(20)
   b = reshape(a, [4,5])
print b
```

9. Write a Python code for plotting the points (0,0) , (1,-1) and (4,5) on the plane.
10. Write a short note on 2D plots using colours using Python.
11. Write a Latex statement to create the matrix $\begin{bmatrix} 1.1 & -2.4 & 4.3 \\ 6.7 & -5.6 & 3.4 \end{bmatrix}$
12. Name two environments in Latex and explain the purpose.

(Ceiling... 20 marks)

Section B
All questions can be attended.
Each question carries 5 marks.

13. Write a short note on operators and their precedence in python.

14. What is the Scope of variables in python?

15. Write a Python program to find the inverse of the matrix $\begin{bmatrix} 4 & 2 & -2 \\ 2 & -3 & 3 \\ -6 & -2 & 1 \end{bmatrix}$

16. Write a python program to draw a pie chart for the following data:

Item:	Frogs	Hogs	Dogs	Logs
Percentage:	25	25	30	20

17. Write a short note on the trapezoidal rule to find $\int_a^b f(x)\,dx$. Write a Python program to evaluate $\int_0^{\pi} sinx\,dx$, using 100 subintervals of $[0, \pi]$.

18. Write a Python program to evaluate $\cos(x) = 1 - \frac{x^2}{1!} + \frac{x^4}{4!} + \cdots$

19. Write a Latex command to for

(i) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (ii) $\sum_{n=0}^{\infty} a_n\,(x - x_0)^n$

(Ceiling... 30 marks)

Section C
Answer any One Question.

20. (a) Write a Python Program to input an integer to check whether it is a perfect number.
(b) Implement the bisection method in Python to find the root of a function with a given tolerance level.

21. (a) Prepare a sample index using Latex.
(b) Typeset a Python program to generate the multiplication table of 5, using verbatim.

($1\times 10 = 10$ marks)