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1B5N24205 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2024

BMT5B05– Abstract Algebra

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

PART - A
(All questions can be attended. Each questions carries 2 marks.)

1. State true or false :Usual addition + on the set **R** of real numbers induce a binary operation on the set $\mathbf{R}^*$ of non-zero real numbers.Justify your answer.
2. Give an example of a commutative and associative binary operation on $\mathbf{Z}^+$.
3. Is the set $\mathbf{Z}^+$ under addition a group ?Justify your claim.
4. Give an example of a finite group.
5. Find a subgroup of the group **Z**.
6. Define the symmetric group $S_n$on n letters.
7. Define orbits of a permutation on a set A.
8. Define alternating group A_n on n letters.
9. Define the left and right cosets of a subgroup H of the group G.
10. Find all the generators of $\mathbf{Z}_{10}$.
11. Define kernel of a homomorphism $\emptyset : R \to R'$ where R andR' are groups.
12. Give an example of a ring.
13. Define zero divisors in a ring.
14. Define integral domain.
15. State true or false : As a ring, **Z** is isomorphic to nZ for all n≥1.

(Ceiling : 25 Marks)

PART - B
(All questions can be attended. Each questions carries 5 marks.)

16. Show that the binary structures (**Q**, +) and (**Z**,+) under the usual addition are not isomorphic.
17. Define group and show that * defined on $\mathbf{Q}^+$ by $a*b = \frac{ab}{2}$ is a group.
18. Prove that every cyclic group is abelian.

19. If H is a subgroup of a finite group G, prove that the order of H is a divisor of the order of G.

20. If H is a subgroup of G and the relation $\sim_L$ defined on G by $a\sim_L b$ if and only if $a^{-1}b \in H$, prove that $\sim_L$ is an equivalence relation on G.

21. Define integral domain and prove that every finite integral domain is a field.

22. If H and K are subgroups of an abelian group G, show that HK = { hk : h∈ H, k ∈ K} is a subgroup of G.

23. Find all subgroups of Z_{18} and draw their subgroup diagram.

(Ceiling : 35 Marks)

PART - C

(Answer any two questions. Each questions carries 10 marks.)

24. (a) Prove that a subgroup of a cyclic group is cyclic.

    (b) Is every abelian group is cyclic ? Justify with example.

25. If R is a ring with additive identity 0, prove that for any a,b ∈ R

    (a) 0a = a0 = 0.

    (b) a(-b) = (-a)b = -(ab).

    (c) (-a)(-b) = ab.

26. (a) Prove that every permutation of a finite set is a product of disjoint cycles.

    (b) By an example show that the product of two cycles need not be a cycle.

27. If F is a field of quotients of D and L is any field containing D, prove that there exists map $\varphi ; F \to L$ that gives an isomorphism of Fwith a subfield of L such that $\varphi(a) = a$ for $a \in D$.

(2 x 10 = 20 Marks)

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1B5N24206 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2024

BMT5B06 – Basic Analysis

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A

All Questions can be attended. Each question carries 2 marks. Ceiling 25 marks

1. Define Denumerable set and give an example for Denumerable set.
2. If a and b in Rare such that a.b = 0, then either a = 0 or b = 0
3. Let a,b,c be any elements of R and if a > b the a + c > b + c.
4. Write the set of real numbers x satisfying $x^2 + x > 2$
5. Let a ∈ R. If x belongs to the neighborhood $V_\varepsilon(a)$ for every $\varepsilon > 0$, then prove that x = a
6. If $S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}$, find Inf S and Sup S
7. Define an unbounded set and give an example for unbounded set
8. An upper bound u of a non-empty set S in R is the supremum of S then prove that for every $\varepsilon > 0$ there exists an $S_\varepsilon \in S$ such that $u - S_\varepsilon < S$
9. Define subsequence of a sequence. Give an example of an unbounded sequence that has a convergent subsequence
10. Prove that convergent sequence of real numbers is a Cauchy sequence
11. Prove that the sequence $(1 + (-1)^n)$ is divergent.
12. Find the real part of $\frac{(4+5i)+2i^3}{(2+i)^2}$
13. Define bounded subset of a complex plane
14. Find the polar form of a complex number $z = -\sqrt{3} - i$
15. Find the real and imaginary part of the complex function $f(z) = z^2$

Section B

All Questions can be attended. Each question carries 5 marks. Ceiling 35 marks

16. Let S be a nonempty bounded set in R. Let a < 0 and let aS = {as : s ∈ S}. Prove that Sup (a S) = a Inf S

17. State and Prove Cantor's theorem

18. State and Prove Bernoulli's Inequality

19. Show that every convergent sequence is bounded

20. Let (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then the sequence $(\sqrt{x_n})$ of positive square roots converges and Lim $(\sqrt{x_n}) = \sqrt{x}$

21. Prove that every Contractive sequence is a Cauchy sequence and therefore is convergent

22. Find the three cube roots of $z = i$

23. Find an upper bound for $\left| \frac{-1}{z^4 - 5z + 1} \right|$ if $|z| = 2$

SECTION C

Answer any Two Questions. Each question carries 10 Marks.

24. Let a >0. Construct a sequence (s_n) of real numbers that converges to a by using Monotone Convergence theorem.

25. State and prove the existence and uniqueness of Nested Intervals Property

26. If c >0, then prove that lim $(c^{\frac{1}{n}}) = 1$

27. (a) Prove that $| |z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$ and interpret the result geometrically

    (b) Evaluate Im $(\bar{z}^{\,2} + z^2)$

(2 x 10 = 20 Marks)

1B5N24207 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2024

BMT5B07 – Numerical Analysis

(2022 Admission onwards)

Time: 2hours Max. Marks : 60

Section A

All questions can be attended. Each question carries 2marks.

1. Use Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x = 0$,o n $[0, 1]$.
2. Write sufficient conditions for the existence and uniqueness of a fixed point.
3. Use method of False Position to find p_2 for $f(x) = x^2 - 6$ with $p_0 = 1$.
4. Determine the coefficient polynomials $L_0(x), L_1(x)$ and $L_2(x)$ through the nodes $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$.
5. Construct a forward difference table for $f(x) = x^3 + 2x + 1$ for $x = 1, 2, 3, 4, 5$.
6. Compute $f'(0.4)$ by using the data $f(0.0) = 0.0000, f(0.2) = 0.74140, f(0.4) = 1.3718$.
7. Use three-point endpoint formula to determine $f'(8.1)$ by using the data $f(8.1) = 16.94410, f(8.3) = 17.56492, f(8.5) = 18.19056$.
8. Approximate $\int_1^{1.6} \frac{2x}{x^2-4} dx$ using Simpson's rule.
9. The Trapezoidal rule applied to $\int_0^2 f(x)\, dx$ gives the value 4 and Simpson's rule gives the value 2. What is $f(1)$?
10. Use the Composite Simpson's rule to approximate the integral $\int_{-2}^{2} x^3 e^x\, dx,\ n = 4$.
11. Show that the IVP $y' = \frac{4t^3 y}{1+t^4}$ $0 \le t \le 1,\ y(0) = 1$ has a unique solution.
12. Write the conditions for the well-posedness of an IVP $y' = f(t, y), a \le t \le b, y(a) = \alpha$.

(*Ceiling ... 20 Marks*)

Section B

All questions can be attended. Each question carries 5marks.

13. Use Newton's Method to find the root of $x^4 - 5x^3 + 9x + 3 = 0$ accurate to six decimal places in the interval [4,6]. Use $p_0 = 5$.

14. Determine Lagrange interpolating polynomial of degree at most two to approximate $f(0.45)$. for the function $f(x) = \sqrt{1+x}$, using the points $x_0 = 0$, $x_1 = 0.6$ and $x_2 = 0.9$. Also, find the absolute error of the approximation.

15. Let $f(x) = xe^x$. Use second derivative midpoint formula to approximate $f''(2.0)$ by using the following table with $h = 0.2$. Compare the result to the exact value.

x	1.8	1.9	2.0	2.1	2.2
$f(x)$	10.889365	12.703199	14.778112	17.148957	19.855030

16. Use open Newton-Cotes formula for $n = 3$ to approximate the integral $\int_{0.5}^{1} 5xe^{3x^2}\,dx$.

17. Use the Composite Trapezoidal rule to approximate the integral $\int_0^{\pi} x^2 \cos x\; dx,\; n = 6$.

18. Use Euler's method to approximate the solutions for the initial value problem

    $y' = \frac{2}{t}y + t^2e^t,\; 1 \le t \le 2,\; y(1) = 0$, with $h = 0.1$.

19. Use the midpoint method to approximate the solution to the initial value problem

    $y' = 1 + (t-y)^2,\; 2 \le t \le 3, y(2) = 1$, with $h = 0.5$.

(Ceiling ... 30 Marks)

Section C

Answer any ONE question.

20. (a) Let $f(x) = e^x - 3x$. With $p_0 = 0$ and $p_1 = 1$, find p_4 using Secant Method.

    (b) Use Stirling's formula to approximate $f(0.65)$ using the following data:

t	0.5	0.6	0.7	0.8	0.9
$f(t)$	1.64872	1.82212	2.01375	2.22554	2.46227

21. (a) Use Taylor's method of order two to approximate the solution for the initial-value problem

    $y' = 1 + \frac{y}{t},\; 1 \le t \le 2,\; y(1) = 2$, with $h = 0.25$.

    (b) Given $\frac{dy}{dt} = \frac{2ty+e^t}{t^2+te^t}$ where $y(1) = 0$. Find $y(1.4)$ using fourth order Runge-Kutta method taking $h = 0.2$.

(1 X 10 = 10 Marks)

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1B5N24208 (Pages : 4) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2024

BMT5B08 – Linear Programming

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Session A

All questions can be attended. Each question carries 2 marks.

1) Define a canonical maximization linear programming problem.
2) Sketch the constraint set and find the extreme points of constraint set of the following LPP

$$\textbf{Maximize } P = 2x + 3y$$
$$\textbf{Subject to } 3x + 2y \le 6$$
$$x + 2y \le 8$$
$$x, y \ge 0$$

3) Show that the linear programming problem

$$P = 3x + 2y$$
$$subject\ to\ \ 2x - y \le -1$$
$$x - 2 \ge 0$$
$$x, y \ge 0$$

is infeasible.

4) Find the basic solution of the following tableau.

x_1	−1/2	3/2	15/2
x_2	−1	1	5
t_P	1/2	1/2	25/2
−1	−100	50	−2500
	$= x_3$	$= t_F$	$= C$

5) Prove that in a maximum basic feasible tableau, the basic solution is a feasible solution.
6) State the dual canonical maximization problem of the following problem

Minimize $g(x, y, z) = 2x + 3y + 2z$

Subject to $2x + 3y \le 2,\ x - y + z \ge 1,\ z \ge 2,\ x, y, z \ge 0$

7) Write the canonical maximum and minimum problem represented by

	x_1	x_2	-1	
y_1	1	-1	-1	$= -t_1$
y_2	-1	-1	-1	$= -t_2$
-1	1	-2	0	$= f$
	$= s_1$	$= s_2$	$= g$	

8) State the duality equation.

9) Define complementary slackness.

10) Apply minimum entry method to obtain an initial basic feasible solution of the transportation problem

6	5	4	**10**
3	7	2	**16**
5	10	8	**10**
4	6	3	**12**
15	**17**	**16**	

11) Give the mathematical form of a general balanced assignment problem.

12) A basic feasible solution of a transportation problem.

6	1	9	3	**70**
11	5	2	8	**55**
10	12	4	7	**90**
85	**35**	**50**	**45**	

is $x_{11} = 25, x_{14} = 45,\ x_{21} = 20,\ x_{22} = 35,\ x_{31} = 40,\ x_{33} = 50.$

Check the optimality of this solution.

Session B

All questions can be attended. Each question carries 5 marks.

13) Find all the extreme points of the constraint set of the following LPP.

Maximize $f(x, y, z) = x - 2y - z$

Subject to $10x + 5y + 2z \leq 1000$

$$2y + 4z \leq 800$$

$$x, y, z \geq 0$$

14) Formulate the following problem mathematically and solve it graphically.

Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram. Food Y contains 8 units of Vitamin A per gram and 12 units of Vitamin B per gram. Food X costs 12 rupees per gram and Y costs 20 rupees per gram. The daily minimum requirements of Vitamin A and Vitamin B are 100 units and 120 units respectively. Find the minimum cost of the product mix.

15) Solve the following canonical LPP

$$\textit{Minimize } C = x_1 - 3x_2 + 2x_3$$

$$\textit{Subject to } 3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq -12$$

$$4x_1 - 3x_2 - 8x_3 \geq -10$$

$$x_1, x_2, x_3 \geq 0$$

16) Solve the non-canonical LPP

$$\textit{Maximize } f(x, y) = x + y$$

$$\textit{Subject to } 2x + y = 5$$

$$x - y = -2$$

$$x + 3y = 6$$

$$x, y \geq 0$$

17) Prove that a pair of feasible solution of dual canonical linear programming problem exhibit complementary slackness if and only if they are optimal solution.

18) By applying VAM, find a basic feasible solution of the following transportation problem

Warehouses

Factories					
	4	8	7	5	30
	6	2	9	6	50
	5	4	6	3	80
	20	60	55	40	

19) Solve the assignment problem

	P_1	P_2	P_3	P_4
J_1	1	4	6	3
J_2	9	7	10	9
J_3	4	5	11	7
J_4	8	7	8	5

Session C

Answer any one. Question carries 10 marks

20) Prove that the following problem has infinitely many solutions and find all the solutions.

$$\text{Maximize} \quad f(x,y,z) = x - y + z$$

$$\text{Subject to} \quad x + y \geq 2$$

$$z - y \geq 3$$

$$2x + z \leq 8$$

21) Write the transportation algorithm and using this solve the transportation problem.

5	9	10	6	4
10	7	5	5	5
4	5	5	4	4
6	5	7	5	3
3	4	4	3	

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1B5N24209 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2024

BMT5B09 – Calculus of Multivariable - 1

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended

Each question carries 2 marks

1. Find the polar coordinates of the point $(1, -1)$ taking $r > 0$ and $0 \leq \theta \leq 2\pi$.
2. Sketch the curve represented by $x = \sqrt{t}$ and $y = t$.
3. Convert the polar equation $r^2 = 4r\cos\theta$ to a rectangular equation.
4. Find parametric equations for the line passing through the points $(2, 1, 4)$and $(1, 3, 7)$.
5. Express the point $(-\sqrt{2}, \sqrt{2}, 2)$ in rectangular co-ordinates in terms of cylindrical co-ordinates.
6. Find an equation in spherical coordinates for the paraboloid with rectangular equation $4z = x^2 + y^2$.
7. (a) Find $\lim_{t\to 0} \langle e^{-t}, \frac{\sin t}{t}, \cos t \rangle$.

   (b) let $\mathbf{r}(s) = 2\cos 2s\mathbf{i} + 2\sin 2s\mathbf{j} + 4s\mathbf{k}$ where $s = t^2$.Find $\frac{d\mathbf{r}}{dt}$.
8. Determine the velocity vector, speed and acceleration vector of an object that moves along the plane curve described by the position vector $\mathbf{r}(t) = 2\cos t\mathbf{i} + \sin t\mathbf{j}$.
9. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^2 + xy - x^2 z + yz^2 = 0$
10. Let $z = 2x^2 - xy$.Find Δz.
11. Sketch the curve defined by the vector function $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ $0 \leq t \leq 2\pi$.

12. Show that $\lim_{(x,y)\to(0,0)} \frac{3xy}{3x^2+y^2}$ does not exist.

(Ceiling 20 Marks)

Section B

All questions can be attended

Each question carries 5 marks

13. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x = e^{-t}$ and $y = e^{2t}$.

14. Determine the slope of the tangent line to the cardioid $r = 1 + \cos\theta$ at the point where $\theta = \frac{\pi}{6}$.

15. Find the length of the cardioid $r = 1 + \cos\theta$.

16. Determine the points of intersection of $r = \cos\theta$ and $r = \cos 2\theta$.

17. Identify and sketch the surface $4x - 3y^2 - 12z^2 = 0$.

18. Find parametric equations for the line of intersection of the planes defined by $2x - 3y + 4z = 3$ and $x + 4y - 2z = 7$.

19. Find the curvature of a circle of radius a.

(Ceiling 30 Marks)

Section C

Answer any one question

20. (a) A moving object has an initial position and an initial velocity given by the vectors $\mathbf{r}(0) = \mathbf{j} + \mathbf{k}$ and $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$.Its acceleration at time t is $\mathbf{a}(t) = \mathbf{i} - t\mathbf{j} + (1+t)\mathbf{k}$. Find its velocity and position at time t.

    (b) A particle moves along a curve described by the vector function $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$.Find the tangential scalar and normal scalar components of acceleration of the particle at any time t.

21. (a) Prove that $\lim_{(x,y)\to(a,b)} x = a$

    (b) Suppose a point charge Q(in coulombs) is located at the origin of a three dimensional coordinate system.This charge produces an electric potential V(in volts) given by $V(x,y,z) = \frac{kQ}{\sqrt{x^2+y^2+z^2}}$ where k is a positive constant and x, y and z are measured in meters

    i. Find the rate of change of the potential at the point $P(1,2,3)$ in the direction of the vector $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

    ii. In which direction does the potential increase most rapidly at P and what is the rate of increase.

$(1 \times 10 = 10)$

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1B5N24210 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2024

(Open Course)

BMT5D03– Linear Mathematical Models

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended

Each Question carries 2 marks

1. Find the slope of the line joining the points (2, -1) and (3,2).
2. Let $f(x) = 2x + 3$. Find the value of x such that $f(x)=5$.
3. Write a short note on the objective function in a linear programming problem.
4. Define the corner point of the feasible region.
5. Write the augmented matrix for the system of equations $2x + 3y - z = 1$, $3x + 5y + z = 3$.
6. Let $A= \begin{bmatrix} 2 & -1 \\ 5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Find AB.
7. Graph the inequality $3x + y < 4$.
8. What is the feasible region for solving a system of inequalities?
9. Find $A^T + B$, where $A= \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 3 & 5 \end{bmatrix}$.
10. Explain the role of the slack variable in an optimization problem.
11. Write the standard form of a maximization problem.
12. If B is the inverse of the matrix $A = \begin{bmatrix} -1 & 0.5 \\ 0.7 & 3 \end{bmatrix}$. Then B^{-1} is ....

(Ceiling 20 mark)

Section B

All questions can be attended

Each questions carries 5 marks

13. Find the least square line for the set of points (1,1),(1,2) (3,1) (4,2).
14. Find the equation of the line passing through (3,7) and perpendicular to the line $3x - 4y = 11$.
15. Graph the feasible region for the following system of inequalities

    $x + 3y \leq 6, \quad 2x + 4y \leq 7$

16. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$

17. Formulate a Linear programming problem (LPP).

Two products 'A' and 'B' are to be manufactured. Single unit of 'A' requires 2.4 minutes of punch press time and 5 minutes of assembly time, while single unit of 'B' requires 3 minutes of punch press time and 2.5 minutes of welding time. The capacity of punch press department, assembly department, and welding department are 1200 min/week, 800 min/week and 600 min/week respectively. The profit from 'A' is Rs.60 and from 'B' is Rs.70 per unit. Formulate LPP such that, profit is maximized.

18. Write the dual of the LPP

Maximize $Z = 3x + 5y$

Subject to $x + y \leq 10$

$2x + y \leq 8$

$x, y \geq 0$

19. Use graphical method to solve

Maximize $Z = 3x + 4y$

$2x + y \leq 4$

$-x + 2y \leq$

$x, y \geq 0$

(Ceiling 30 Marks)

Section C

Answer any One question

20. Solve the following linear programming problem using simplex method.

Maximize $Z = 2x_1 + 5x_2 + x_3$

Subject to $x_1 - 5x_2 + 2x_3 \leq 30$

$4x_1 - 3x_2 + 6x_3 \leq 72$

$x_1, x_2, x_3 \geq 0$

21. Use the Gauss –Jordan method to solve the following system of equations,

$x + 2y - 7z = -2$

$-2x - 5y + 2z = 1$

$3x + 5y + 4z = -9$

(1x10 = 10 marks)