1B4A24202 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Mathematics Degree Examination, April 2024

BMT4B04 – Linear Algebra

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

All questions can be attended.
(Each question carries 2 Marks – Ceiling – 25 Marks.)

1. Write the augmented matrix for the linear system

$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 2x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

2. Describe the possibilities of the number of solutions (x,y) of a system of linear equation in the XY plane.
3. Prove that the product of symmetric matrices is symmetric if and only if the matrices commute.
4. Find the standard matrix for the operator defined by $T(x_1, x_2) = (2x_1 - x_2\,, x_1 + x_2)$.
5. Let $= \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$. Find the values of x for which the matrix A is invertible.
6. If A is a square matrix, then prove that $\det(A) = det(A^T)$.
7. If V is a vector space, then prove that $0\ \boldsymbol{u} = \mathbf{0}$ for any vector **u**.
8. Check whether the set of all pairs of the form $(x, y), x \geq 0$ is a subspace of R^2 with standard operations.
9. Check whether $(-1,2,4)$ and $(5, -10, -20)$ are linearly independent vectors in R^3.
10. A vector space can have more than one basis. Justify the statement.
11. Define dimension of a vector space. Give an example for an infinite dimensional vector space.
12. Find the image of $x = (1,1)$ under a rotation of $\frac{\pi}{6}$ radian about the origin.
13. If $T_1 : R^2 \to R^2$ is reflection about the y-axis and $T_2 : R^2 \to R^2$ is reflection about the x-axis, find $T_1 o\, T_2$.
14. Find the eigen values of the matrix $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$.
15. Prove that the similar matrices have the same determinant.

Section B

(Each question carries 5 Marks – Ceiling- 35 Marks)

16. Using elementary row operations solve the system of equations

$$x + y + z = 5$$
$$x + y - 4z = 10$$
$$-4x + y + z = 0$$

17. If A and B are $n \times n$ matrices, prove that $\text{trace}(A + B) = \text{trace}(A) + \text{trace}(B)$.

18. Prove that the set of all straight lines passing through the origin is a subspace of R^2.

19. Prove that the polynomials $1, x, x^2, \ldots, x^n$ form a linearly independent subset ofthe vector space of polynomials of degree $\leq n$.

20. If $S = \{v_1, v_2, \ldots, v_n\}$ is a basis for a vector space V, then prove that every vector v in V can be expressed in the form $v = c_1v_1 + c_2v_2 + \ldots + c_nv_n$ in exactly one way.

21. State and prove the dimension theorem for matrices.

22. Show that the operator $T: R^2 \to R^2$ defined by the equations

$$w_1 = 2x_1 + x_2$$

$w_2 = 3x_1 + 4x_2$ is one-one, and find $T^{-1}(w_1, w_2)$.

23. Prove that a square matrix A is invertible if and only if 0 is not an eigenvalue of A.

Section C

Answer any TWO questions

(Each question carries 10 Marks)

24. Solve the following linear system of equations by Gauss Elimination Method

$$x - y + 2z - w = -1$$
$$2x + y - 2z - 2w = -2$$
$$-x + 2y - 4z + w = 1$$
$$3x - 3w = -2$$

25. a) Determine whether the vectors $v_1 = (1,1,2)$, $v_2 = (1,0,1)$ and $v_3 = (2,1,3)$ spans the vector space R^3.

    b) Find the solution space of the homogeneous system given below.

$$\begin{bmatrix} 1 & -2 & 3 \\ -3 & 7 & 8 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

26. a)Let V be an n-dimensional vector space, and let S be a set in V with exactly n vectors. Then prove that

    a) S is a basis for V if and only if S spans Vor S is linearly independent.

    b) If W is a subspace of a finite dimensional vector space V ,then prove that $\dim(W) \leq \dim(V)$.

27. Find bases for the eigen spaces of the matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

(2 x 10 = 20 Marks)