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2B3N24086 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2024

BMT3B03 – Theory of Equations and Number Theory

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section A
All questions can be attended
Each question carries 2 marks.

1. Show that $x^4 + 3x^2 + 3x + 2$ is divisible by $x + 2$.
2. Write the cubic equation with the roots 0, i, -i.
3. Find the quotient and remainder when $2x^4 - 6x^3 + 3x^2 - 5x + 1$ is divided by $x + 2$.
4. Find Δ of the equation $x^3 - 6x - 6 = 0$.
5. State Identity theorem.
6. How many real roots has the equation $x^6 - x^3 + 2x^2 - 3x - 1 = 0$.
7. Find the quotient and remainder when 78 is divided by 11.
8. State the Pigenhole Principle.
9. Express $(28, 12)$ as a linear combination of 28 and 12.
10. State Lamé's Theorem.
11. Find the canonical decomposition of 1863.
12. Determine whether the LDE $6x + 8y = 25$ is solvable.
13. Define Pseudoprime. Give an example.
14. Find $\sigma(28)$,where σ is a sigma function.
15. If $(a, b) = d$, prove that $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.

(Ceiling: 25 Marks)

Section B

All questions can be attended

Each question carries 5 marks.

16. Solve the cubic equation whose roots are a, b, c:
$2x^3 - x^2 - 18x + 9 = 0$ if $a + b = 0$.

17. Find an upper limit of the positive roots of the equation
$2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.

18. Prove that that f_m and f_n are relatively prime, where f_i is the ith Fermat number. m and n be distinct nonnegative integers.

19. Find the number of primes ≤ 100.

20. Prove that $[a, b] = \dfrac{ab}{(a, b)}$, where a and b be positive integers.

21. Find the remainder when 16^{53} is divided by 7.

22. If $ac \equiv bc (\mod m)$ and $(c, m) = 1$, prove that $a \equiv b (\mod m)$.

23. Prove that $\varphi(P^e) = p^e - p^{e-1}$, where p is a prime and e is any positive integer.

(Ceiling: 35 Marks)

Section C

Answer any two Question

Each question carries 10 marks.

24. Solve $x^3 - x^2 - 18x + 9 = 0$ by using cardan's formula.

25. (a) Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.
    (b) Solve the congruence $12x \equiv 48(mod 18)$.

26. State and prove Fermat's Little Theorem.

27. (a) If f is a multiplicative function, then prove that $F(n) = \sum_{d/n} f(d)$ is multiplicative.
    (b) Prove that the *tau* and *sigma* functions are multiplicative.

(2×10 = 20 Marks)

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2B3N24087 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Physics, Chemistry & Statistics Degree Examination, November 2024

BMT3C03 – Mathematics - 3

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks. Overall ceiling 20

1. Let $f(x) = x$ Evaluate $\int_0^4 \sin x \; dx$ by trapezoidal rule with $n = 4$.
2. Find the components and length of the vector $\vec{v}$ with given initial point $P: (0,0,1)$ and terminal point $Q: (1,0,1)$.
3. Let $\vec{a} = [1,1,1], \vec{b} = [-1,-1,-1]$and $\vec{c} = [3,3,3]$ =Verify whether $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$.
4. Find a parametric representation of the straight line through the origin in the direction of the vector $i - j$.
5. Find the directional derivative of $x + y$ at $(-1,1)$ in the direction of $i - j$.
6. State Stoke's theorem.
7. Write a parametric representation of the sphere $x^2 + y^2 + z^2 = 64$.
8. Evaluate the line integral $\int_C \boldsymbol{F(r)}.\boldsymbol{dr}$ where $\boldsymbol{F(r)} = \boldsymbol{xyi} + \boldsymbol{yzj} + \boldsymbol{zxk}$and C is the straight line segment $ti + tj + tk$, $0 \leq t \leq 1$.
9. Evaluate the integral $\int_C F.dr$ from $(0,1,2)$ to $(1,-1,7)$,where $\boldsymbol{F} = (3x^2dx + 2yz\,dy + y^2dz)$. Given that $\boldsymbol{F}$has potential $f(x,y,z) = x^3 + y^2z$.
10. Find the polar form of $5 - \sqrt{2}\,i$.
11. Find the value of the derivative $(z + i)/(z - i)$ at $-i$.
12. Define analytic function , Give an example.

Section B

All questions can be attended.

Each question carries 5 marks. Overall ceiling 30

13. Sketch the graph and level curves at $c = 0,1,4$ of the function $f(x, y) = x^2 - y^2$.

14. Find the gradient of the function $F(x, y, z) = xy^2 + yx^2 - z^3$ at $(0, -1,1)$.

15. Find an equation for the tangent plane to the surface $z = xy^2$ at the point $(0,1,4)$.

16. Find the value of c if $u = (cxy - z^2)i + (x^2 + 2yz)j + (y^2 - cxz)k$ is irrotational.

17. If $f(x, y) = x^2y - 2xy$ and $R: 0 \le x \le 3, -2 \le y \le 0$, then evaluate $\iint_R f(x, y)\, dA$.

18. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x - y + 2z)\, dzdxdy$.

19. Integrate $\frac{z^4-3z^2+6}{(z+i)^3}$ in the counter clockwise sense around the circle $|z| = 1$.

Section C

Answer any one of the questions.

The question carries 10 marks.

20. Find the counterclockwise outward flux of the field $\boldsymbol{F} = (x - y)i + (y - x)j$ across the square bounded by $x = 0, x = 1, y = 0, y = 1$.

21. (a) Prove that $f(z) = x^2 + y^2$ is nowhere analytic.

    (b) Evaluate $\oint_C \frac{5z+7}{z^2+2z-3}\, dz$, where $C: |z - 2| = 2$.

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2B3N24088 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Computer Science Degree Examination, November 2024

BMT3C03(CS) – Mathematics

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

All questions can be attended.

Each question carries 2 marks. Overall ceiling 20

1. Evaluate $\int_0^2 |t\mathbf{i} + 2t^2\mathbf{j}|dt$.
2. Describe the curve represented by the vector equation $r(t) = (1-t)i + 3tj + 2tk$.
3. Find the intervals on which the vector valued function $\mathbf{r}(t) = e^{-t}\mathbf{i} + \cos\sqrt{4-t}\,\mathbf{j} + \frac{1}{t^2-1}\mathbf{k}$ is continuous.
4. Find an equation of the tangent plane to the graph of $z = \frac{1}{2}x^2 + \frac{1}{2}y^2 + 4$ at the point $(1, -1, 5)$.
5. Write the formula for three-dimensional Laplace Equation.
6. Find the level surface of $F(x, y, z) = x^2 + 3y^2 + 6z^2$.
7. Convert $(2, \pi/3, 1)$ in cylindrical coordinates to rectangular coordinates.
8. Integrate $f(z) = Re\ Z$ along the line segment from $z = 0$ to $z = 1 + i$.
9. Express $z = -2 + 2i\sqrt{3}$ in polar form.
10. Determine the principal value of the arguments of $(\sqrt{3} - i)^6$.
11. Find the domain of definition of the function $f(z) = \frac{1}{1-|z|^2}$.
12. Give the points at which the function $f(z) = \frac{z}{z-3i}$ will not be analytic.

Section B

All questions can be attended.
Each question carries 5 marks. Overall ceiling 30

13. Find the antiderivative $\boldsymbol{r}(t)$ of $\boldsymbol{r}'(t) = \cos t\,\mathbf{i} + e^{-t}\mathbf{j} + \sqrt{t}\mathbf{k}$ satisfying the initial condition r(0)=i+2j+3k.

14. Verify that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$, when $w = x^y + \sin(xy)$.

15. Find the directional derivative of $F(x, y, z) = \sqrt{x^2y + 2y^2z}$ at the point $(-2,2,1)$ in the direction of the negative z-axis.

16. Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.

17. Find the three cube roots of $-8i$.

18. Show that $f(z) = z^n (n \geq 1$ is an integer) is differentiable for all z.

19. If $z = 16x^2y^3 + 4x^3 + 7y^6 + 9$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

20. Evaluate $\int_C \bar{z}\, dz$, where C is given by $x = 3t, y = t^2, -1 \leq t \leq 4$.

Part C

Answer any one of the question.
The question carries 10 marks.

21. Prove that $u = x^2 - y^2, v = \frac{-y}{(x^2+y^2)}$ both are harmonic, but $u + iv$ is not an analytic function of z.

22. Show that if z_0 is any constant complex number interior to any simple closed contour C, $\oint_C \frac{1}{(z-z_0)^n} dz = \begin{cases} 2\pi i, & n = 1 \\ 0, & n \text{ an integer} \neq 1 \end{cases}$