1B2A24114 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Statistics Degree Examination, April 2024

BAS2C02- Life Contingencies

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

(Use of Scientific Calculator and Life tables are permitted)

PART-A (Short Answer)
Each question carries *two* marks. Maximum 20 Marks

1. What is life assurance contract?
2. Calculate $E\left(v^{K_x+1}\right)$ at an interest rate of 0%.
3. Find A_{40} (AM92 at 6%).
4. What does pure endowment mean?
5. What is life annuity?
6. A whole life annuity of 100 pa is payable annually in arrears to a life aged 65. Calculate the standard deviation of the benefits from this annuity, assuming AM92 Ultimate mortality and an annual effective rate of interest of 4%
7. What is a temporary life annuity?
8. Find $\ddot{a}_{30}$(AM92 at 4%) and $\ddot{a}_{75}$(PMA92C20 at 4%).
9. Define endowment assurance.
10. Find a_{65} (PFA92C20 at 4%).
11. Verify that $A_{65}= 1-d\ddot{a}_{65}$ using AM92 mortality and 4% pa interest.
12. Using AM92 mortality and 4% pa interest, calculate $a_{60}^{(12)}$.

Maximum Marks = 20

PART-B
Each question carries *five* marks. Maximum 30 marks

13. A male pension policyholder is currently aged 50 and he will retire at age 65, from which age a pension of £5,000 pa will be paid annually in advance. Before retirement he is assumed to experience mortality in line with AM92 Ultimate and after retirement in line with PMA92C20. Calculate the expected present value of the benefits assuming interest of 4% pa.

14. Calculate the expected present value of a payment of £2,000 made 6 months after the death of a life now aged exactly 60, assuming AM92 Select mortality and 6% pa interest.

15. A life insurance company issues a 3-year term assurance contract to a life aged exactly 42. The sum assured of 10,000 is payable at the end of the policy year of death. Calculate the expected present value of these benefits assuming AM92 Select mortality and an interest rate of 5% pa effective.

16. A whole life assurance contract, under which the sum assured of £40,000 is payable immediately on death, is issued to a life aged exactly 35. Using AM92 Select mortality and an interest rate of 6% pa effective, calculate:

(i) the expected present value of the benefits

(ii) the variance of the present value of the benefits.

17. In a certain population, the force of mortality equals 0.025 at all ages.

Calculate:

(i) the probability that a new-born baby will survive to age 5

(ii) the probability that a life aged exactly 10 will die before age 12

(iii) the probability that a life aged exactly 5 will die between ages 10 and 12.

18. Calculate the value of ${}_{0.5}p_{55.5}$ using ELT15 (Females) mortality and linear interpolation, based on the assumption of a uniform distribution of deaths between integer ages

19. Explain the process of constructing a life table.

Maximum Marks = 30

PART-C (Essay)

Answer any one question and carries 10 marks

20. Calculate the value of ${}_{1.75}p_{45.5}$ using AM92 Ultimate mortality and assuming that:

(i) deaths are uniformly distributed between integer ages.

(ii) the force of mortality is constant between integer ages

21. Calculate the expectation and standard deviation of the present value of the benefits from each of the following contracts issued to a life aged exactly 40, assuming that the annual effective interest rate is 4% and AM92 Ultimate mortality applies:

(i) a 20-year pure endowment, with a benefit of £10,000

(ii) a deferred whole life assurance with a deferred period of 20 years, under which the death benefit of £20,000 is paid at the end of the year of death, as long as this occurs after the deferred period has elapsed.

(1 x 10 = 10 Marks)

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1B2A24112 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Statistics Degree Examination, April 2024

BST2B02 - Bivariate Random Variables & Probability Distributions

(2022 Admission onwards)

Time: 2½ hours Max. Marks : 80

(Use of Scientific Calculator and Statistical table are permitted)

PART-A

[Each question carries 2 marks]

1. If the mean and variance of a binomial random variable X is 6 and 3. Find $P(X=1)$.
2. Two percentage of an item produced by a company is defective. Out of a packet of 100 items, use Poisson distribution to find the probability of getting 5 defective.
3. Verify whether $f(x,)=3x^2$, for $0<x<1$; $f(x)=0$, elsewhere, is a p.d.f. If so find $P(0.5<x<1.5)$.
4. Find the expected number of heads in 3 tosses of an unbiased coin.
5. The joint pmf of X and Y is $f(x,y)=\frac{x+y}{12}$, for $x=1,2; y=1,2$. Identify the probability distribution of Z, where $Z=X+Y$
6. If $\mu_r'=r!$ for a random variable X, find its m.g.f
7. What are the assumptions of central limit theorem
8. Define gamma distribution. Obtain its mean
9. Explain the term convergence in probability
10. Find the mean of a random variable X with first moment about 4 is given as 7.
11. Define conditional variance
12. If X and Y are two independent random variable with mean 10 and 20, and variances 2 and 3 respectively. Find the variance of $3X+4Y$.
13. Establish the relationship between geometric distribution and discrete uniform distribution
14. If X is a random variable following discrete uniform distribution over the numbers -1, 0 and 1: find the variance of X.
15. Define Lindeberg- Levy theorem.

(Maximum Mark= 25)

PART-B
[Each question carries 5 marks]

16. Show that $Cov(X,-Y) = -Cov(X,Y)$, where X and Y are two random variables.

17. Derive the m.g.f of a normal distribution with mean μ and variance σ^2.

18. Obtain the probability distribution of X, if the distribution function of X is

$$F(x) = \begin{cases} 0, \text{ if } x < -2 \\ \frac{1}{8}, \text{ if } -2 \le x < 0 \\ \frac{1}{2}, \text{ if } 0 \le x < 3 \\ \frac{7}{8}, \text{ if } 3 \le x < 5 \\ 1, \text{ if } x \ge 5 \end{cases}$$

Also find $P(X > -2 / X < 4)$

19. If X follows the distribution $f(x) = e^{-x}$, $x > 0$. Obtain $P\{|X-1| > 2\}$ by Tchbycheve's inequality and compare it with actual probability.

20. Examine the effect of the shifting of the origin and change of scale on the m.g.f of a random variable.

21. Obtain the mean and variance of exponential distribution.

22. If $X \rightarrow N(12,4)$. Obtain (i) $P(X \le 20)$ (ii) $P(0 \le X \le 24)$ and (iii) $P\{|X-12| \ge 8\}$.

23. State and prove Bernoulli's weak law of large numbers

(Maximum Mark= 35)

PART-C
[Each question carries 10 marks. Answer any Two questions]

24. (a) If X and Y are two random variables, prove that $E(X) = E[E(X/_{Y=y})]$

    (b) Let X and Y have joint density is given by $f(x,y) = 2-x-y$, $0 < x < 1$, $0 < y < 1$.
    Find $Cov(X,Y)$.

25. Discuss the properties and applications of normal distributions. Derive expressions for its r^{th} central moments.

26. State and prove weak law of large numbers. Examine whether WLLN hold good for the sequence of random variables $\{X_i\}$, $i = 1,2,.....$ where $P\{X_i = \pm\sqrt{2i-1}\} = 0.5$

27. Let X and Y be two random variables with joint p.d.f $f(x,y) = 8xy$, $0 \le x \le y \le 1$.
    Obtain

    (i) The marginal pdf of X and Y

    (ii) $P(X < 1/4 \,/\, \frac{1}{2} < y < 1)$

(2x10= 20 Marks)

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1B2A24113 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, April 2024

BST2C02- Probability Theory

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

(Use of Scientific Calculator is permitted)

PART A
Each question carries 2 Marks.
Maximum Marks that can be scored in this part is 20

1. Define sample point and sample space.
2. Write any two limitations of classical definition of probability.
3. Define random experiment with example.
4. Define conditional probability.
5. Distinguish between discrete and continuous random variables.
6. Define distribution function of a random variable.
7. The probability density function of a random variable X is $f(x) = k\ x^2, x=1,2,3,4$. Find the value of k?
8. Define expectation of a random variable.
9. If $V(X) = 16$ Find $V((2x+4)/6)$?
10. Define characteristic function of a random variable.
11. Define conditional expectation.
12. Show that when X and Y are independent Cov (X, Y) = 0.

PART B
Each question carries 5 marks.
Maximum Marks that can be scored in this part is 30.

13. If A and B are two events such that $P(A) = 1/3$, $P(B) = 1/4$ and $P(A \cap B) = 1/8$ Find $P(A/B)$ and $P(A/B^c)$
14. State and prove Baye's theorem.
15. Define probability mass function and probability density function.Also state any two properties of them.
16. A continuous random variable X has probability density function $f(x) = Ax^2$, $0 < x < 10$ Determine (a) A (b) $P(2 < X < 5)$

17. Define row moments and central moments. Also derive the relation between them (using expectation)

18. Find mean deviation about median of a random variable X with probability density function $f(x) = 2x, 0 < x < 1$

19. Joint probability density function of two random variables X and Y is $f(x, y) = 2; 0 < x < y < 1$ Find variance of X given Y=y?

PART C

Answer any one question and carries 10 Marks

20. State and prove addition and multiplication theorem on probability.

21. The joint probability density function of (X, Y) is given below

X Y	0	1
-1	1/8	2/8
1	3/8	2/8

Find (a) V(x) (b) V(y) (c) covariance (X, Y)

(1 x 10 = 10 Marks)