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1B2A24082 (Pages : 2) Reg. No:...... ....

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, April 2024

BMT2B02 - Calculus 2

(2022 Admission onwards)

Time: 2 ½ hours Max. Marks : 80

Section - A

(All questions can be answered. Each question carries 2 marks.)

1. Write the formula for finding the volume of the solid generated by revolving about the y-axis the region between the y-axis and
   the graph of the continuous function $x = g(y)$, $c \leq y \leq d$.
2. If f is smooth on [a,b], write the formula for finding the length of the curve $y = f(x)$ from a to b.
3. Find the derivative of $f(x) = \log |\cos x|$.
4. Write the value of e correct to 3 decimal places.
5. If $y = 3^{\sin x}$, find $\frac{dy}{dx}$.
6. Find the range of the function $y = \cosh x$.
7. State the L- Hopital's rule for finding the limit of a function.
8. Determine whether the integral $\int_0^4 \frac{dx}{x-3}$ is proper or improper.
9. Write the inductive definition of the Fibonacci sequence.
10. Define bounded sequence.
11. Define the sequence of partial sums of an infinite series.
12. State the limit comparison test for the convergence of an infinite series.
13. Define the radius of convergence of a power series.
14. Write an example of a series which is conditionally convergent.
15. Write the Maclaurin's series expansion of sinx.

(Ceiling : 25 Marks)

Section - B

(All questions can be answered. Each question carries 5 m... ks.)

16. Find the area of the region bounded by the curve $y = x^2 - 1$, the x- axis, y-axis and the line $x = 3$.

17. Find the length of the asteroid $x^{2/3} + y^{2/3} = 1$.

18. Using logarithmic differentiation find the derivative of the function $y = (\tan x)^{\sqrt{2x+1}}$.

19. Show that $e = \lim_{h\to 0}(1+h)^{1/h}$

20. Prove that $\cosh^{-1}x = \log(x + \sqrt{x^2-1})$, $x \geq 1$.

21. Prove that the limit of a sequence if it exists, is unique.

22. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}(x-1)^n$.

23. Find the Taylor series of $f(x) = \sin x$ at $x = \frac{\pi}{2}$.

(Ceiling : 35 Marks)

Section- C

(Answer any two questions. Each question carries 10 marks.)

24. Prove that the geometric series $\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$ and its sum is $\frac{a}{1-r}$ and the series diverges if $r \geq 1$.

25. Find the Maclaurin series expansion for

    (a) $\cos x$.

    (b) $\cos^2 x$.

26. (a). Show that $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$.

    (b). For any two real numbers x and y, prove that $e^x . e^y = e^{x+y}$.

27. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x- axis.

(2 x 10 = 20 Marks)

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1B2A24083 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Chemistry, Physics & Statistics Degree Examination, April 2024

BMT2C02–Mathematics – 2

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended
Each question carries 2 marks

1. Evaluate $\frac{dy}{dx}$ where $y = tanh\sqrt{1+x^2}$
2. What is the polar equation corresponding to the Cartesian equation $x^2 + y^2 = 4$
3. Find the polar equation for the hyperbola with eccentricity $3/2$ and directrix $x = 2$.
4. Rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$ is
5. State Cayley- Hamilton theorem
6. Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$
7. State non decreasing sequence theorem.
8. Find $\lim_{n\to\infty} \frac{2^n}{5n}$
9. Sum the series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$
10. Is the series $\sum \frac{n^2}{2n}$ converges?
11. For what value of x, the series $\sum x^n$ converges?
12. Define Taylor series generated by a function f

(Ceiling: 20 Marks)

Section B
All questions can be attended
Each question carries 5 marks

13. Evaluate $\int_0^1 \frac{2dx}{\sqrt{3+4x^2}}$

14. Graph the curve $r = 1 - cos\theta$

15. Solve the system of equations using matrix.

$x_1 + 2x_2 + x_3 = 2,\ 3x_1 + x_2 - 2x_3 = 1, 4x_1 - 3x_2 - x_3 = 3, 2x_1 + 4x_2 + 2x_3 = 4$

16. Reduce the matrix $A = \begin{pmatrix} 1 & 2 & 0 -1 \\ 3 & 4 & 1\ 2 \\ -2 & 3 & 2\ 5 \end{pmatrix}$ into normal form

17. Find $\lim_{n\to\infty} a_n$ where $a_n = \left(\frac{n+1}{n-1}\right)^n$

18. Check the convergence of the series $\sum a_n$ where $a_n = \frac{(2n)!}{n!n!}$

19. Find the Taylor series generated by $f(x) = e^x$ at $x = 0$

(Ceiling: 30 Marks)

Section C
Answer any one question
Question carries 10 marks

20. (a) Write the equation of the region between the origin and the curve

$r = f(\theta), \alpha \leq \theta \leq \beta$

(b) Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + cos\theta)$

21. (a) Find the sum of the series $\sum \frac{(3^{n-1}-1)}{6^{n-1}}$

(b) Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

(1 x 10 = 10 Marks)

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1B2A24084 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Computer Science Degree Examination, April 2024

BMT2C02(CS) – Mathematics – 2

(2022 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A
All questions can be attended.
Each question carries 2 marks.

1. Define a sequence. Give example.
2. Evaluate the definite integral $\int_0^2 (3x + 5)dx$.
3. Find the length of the graph $\frac{x^3}{3} + \frac{1}{4x}$ on the interval $[1, 3]$.
4. Describe the curve represented by $x = 4\cos\theta$ and $= 3\sin\theta$; $0 \le \theta \le 2\pi$.
5. Find $\frac{d}{dx}[cosh^2(\ln 2x)]$.
6. Prove that $\frac{d}{dx}(sinhx) = coshx$.
7. List the terms of the sequence $\{\sin\frac{n\pi}{3}\}_{n=0}^{\infty}$.
8. Determine whether the sequence $\{(-1)^n\}$ converges or diverges.
9. Find $\lim_{n\to\infty}\frac{(-1)^n}{n}$.
10. Determine whether the series $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \cdots$ converges or diverges. If it converges find its sum.
11. Evaluate the area of the surface obtained by revolving the graph x^2 on [0, 1] about the x axis.
12. Find $\lim_{n\to\infty}\frac{1}{n^r}$ if $r > 0$.

(Ceiling: 20 Marks)

Section B

All questions can be attended

Each question carries 5 marks

13. Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$.
14. Find the area of the region bounded by the graphs of $y = 2 - x^2$ *and* $y = -x$.
15. Find the area of the surface obtained by revolving the graph of $f(x) = \sqrt{x}$ on the interval $[0, 2]$ about the x-axis.
16. Prove the identity $\cosh 2x = \cosh^2 x + \sinh^2 x$.
17. Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$ converges. If the series converges, find its sum.
18. Let $f(x) = e^x$. Find the Maclaurin series of f, and determine its radius of convergence.
19. Show that the series $\sum_{n=1}^{\infty} \frac{2n^2+1}{3n^2-1}$ is divergent.

(Ceiling: 30 Marks)

Section C

Answer any one question.

Question carries 10 marks

20. Find the area of the region S bounded by the graphs of $y = \cos x$ and $y = \left(\frac{2}{\pi}\right)x - 1$ and the vertical lines $x = 0$ and $x = \pi$.
21. a) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} n!\, x^n$.

    b) Find a power series representation for $\tan^{-1} x$ by integrating a power series representation of $f(x) = \frac{1}{(1+x^2)}$.