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1M4A24205 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2024

MMT4C15 – Advanced Functional Analysis

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions. Each questions carries 1 weightage.

1. Define approximate eigen value of an operator $A \in BL(X)$. Give an example of an operator A and an approximate eigen value of A which is not an eigen value of A.
2. If A is a right shift operator on ℓ^p, then prove that $\sigma_e(A) = \emptyset$.
3. Let X be a Banach space over K and $k \in K$. If $k \in \sigma_a(A)$, then prove that

$$|k| \leq \inf_{n=1,2,3,...} \|A^n\|^{\frac{1}{n}} \leq \|A\|.$$

4. Show that ℓ^∞ is not reflexive.
5. Let X and Y be normed spaces and X be finite dimensional. Then prove that every linear map from X to Y is compact.
6. State Riesz representation theorem.
7. If A_n and B_n are sequences in $BL(X)$ and $A_n \to A$ and $B_n \to B$. Then prove that $A_n + B_n \to A + B$ and $A_nB_n \to AB$.
8. Show, by an example, that if $k \in \sigma_e(A)$ does not follow that $\bar{k} \in \sigma_e(A^*)$.

Part B

Answer two questions from each unit. Each questions carries 2 weightage.

Unit I

9. Let X be a Banach space and $A \in BL(X)$ and $\|A^p\| < 1$ for some positive integer p, Then prove that $I - A$ is invertible.
10. Let X and Y be normed spaces and $F, G \in BL(X, Y)$. Then prove that
    a). $(F + G)' = F' + G'$
    b). $\|F\| = \|F'\| = \|F''\|$.
11. Prove that dual of K^n with norm $\| \ \|_p$ is linearly isometric to K^n with norm $\| \ \|_q$.

Unit II

12. Let X be a normed space and $A \in CL(X)$. Then prove that 0 is the only possible limit point of the eigen spectrum of A.
13. Let X and Y be normed spaces. If $k \in K,\ F, G \in CL(X, Y)$, then prove that kF, $F + G \in CL(X, Y)$. Further prove that if F is compact bounded linear map and $H \in BL(Y, Z)$, then $HF \in CL(X, Z)$.
14. Let H be a Hilbert space, G be a subspace of H and g be a continuous linear functional on G. Then prove that there is a unique continuous linear functional f on H such that $f|_G = g$ and $\|f\| = \|g\|$

Unit III

15. Let H be a Hilbert space and $A \in BL(H)$. Then prove that there is a unique $B \in BL(H)$ such that $\langle A(x), y\rangle = \langle x, B(y)\rangle$ for all $x, y \in H$.
16. Let H be a Hilbert space and $A \in BL(H)$ be self adjoint .Then prove that A or $-A$ is positive if and only if $|\langle A(x), y\rangle|^2 = \langle A(x), x\rangle\langle A(y), y\rangle$ for all $x, y \in H$.
17. Let H be a Hilbert space and $A \in BL(H)$ be self adjoint. Then prove that

$$\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A, M_A].$$

Part C

Answer any two questions. Each questions carries 5 weightage.

18. Let X be a reflexive normed space. Then prove that
    a) X is Banach and it remains reflexive in any equivalent norm.
    b) X' is reflexive
    c) Every closed subspace of X is reflexive.
19. Let X be a nonzero Banach space over $\mathbb{C}$ and $A \in BL(X)$. Then prove that
    a). $\sigma(A)$ is nonempty.
    b). $r_\sigma(A) = \inf_{n=1,2,\cdots} \|A^n\|^{\frac{1}{n}} = \lim_{n\to\infty} \|A^n\|^{\frac{1}{n}}$.
20. Let H be a Hilbert space and $A \in BL(H)$. Then prove that
    a). $k \in \sigma(A)$ if and only if $\bar{k} \in \sigma(A^*)$.
    b). $\sigma_e(A) \subset \sigma_a(A)$ and $\sigma(A) = \sigma_a(A) \cup \{k : \bar{k} \in \sigma_e(A^*)\}$
21. Let $A \in BL(H)$. Then prove that
    a). If $R(A)$ is finite dimensional, then A is compact.
    b). If each A_n is a compact operator on H and $\|A_n - A\| \to 0$, then A is compact.
    c). If A is compact then so is A^*.

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1M4A24207 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2024

MMT4E09 – Differential Geometry

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer All questions. Each question carries 1 weightage

1. Show that the graph of any function $f: \mathbb{R}^n \to \mathbb{R}$ is a level set for some function $F: \mathbb{R}^{n+1} \to \mathbb{R}$.
2. Find the integral curve through the point $p = (1, 1)$ of the vector field $\mathbb{X}(p) = (p, X(p))$, where $X(p) = (0,1)$ on $\mathbb{R}^2$.
3. Describe the spherical image of the 2-surface $x_2^2 + x_3^2 = 1$ oriented by $\frac{\nabla f}{\|\nabla f\|}$, where $f(x_1, x_2, x_3) = x_2^2 + x_3^2$.
4. Find the velocity, the acceleration and the speed of the curve $\alpha(t) = (cost,\ sint,\ t)$.
5. Let S be an n-surface in $\mathbb{R}^{n+1}$.let $\alpha: I \to S$ be a parametrized curve and let $\mathbb{X}$be vector field tangent to S along α. Verify that $(f\mathbb{X})' = f'\mathbb{X} + f\mathbb{X}'$.
6. Compute∇_vf where $f: \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x_1,\ x_2) = {x_1}^2 - {x_2}^2$, $v = (1, 1, \cos\theta, \sin\theta)$.
7. Show that local parameterizations of plane curves are unique up to reparametrization.
8. Define normal section of an n-surface.

(8x1= 8 weightage)

Part B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Let U be an open set in $\mathbb{R}^{n+1}$ and let $f: U \to \mathbb{R}$ be smooth. Let $p \in U$ be a regular point of f, and let $c = f(p)$. Then show that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$.
10. Let S be an n-surface in $\mathbb{R}^{n+1}$, $S = f^{-1}(c)$, where $f: U \to R$ is such that $\nabla f(q) \neq 0$ for all $q \in S$. Suppose $g: U \to R$ is a smooth function and $p \in S$ is an extreme point of g on S. Prove that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$.
11. Show that the two orientation on the n-sphere ${x_1}^2 + {x_2}^2 + \cdots. + {x_{n+1}}^2 = r^2$ of radius $r > 0$ are given by $\mathbb{N}_1(p) = (p, \frac{p}{r})$&$\mathbb{N}_2(p) = (p, \frac{-p}{r})$.

Unit II

12. Let S be an n-surface in $\mathbb{R}^{n+1}$. Let $p, q \in S$ and α be a piecewise smooth parametrized curve from p to q. Then show that parallel transport $P_\alpha: S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.
13. Show that the Weingarten map of an n-surface S at a point p in S is self adjoint.
14. Let $C = f^{-1}(r^2)$, where $f(x_1, x_2) = (x_1 - a)^2 + (x_2 - b)^2$, oriented by the outward normal $\frac{\nabla f}{\|\nabla f\|}$. Let $p = (a + r, b) \in C$. Find the local parameterization of C at p. Also compute the curvature of C at p.

Unit III

15. Find the Gaussian curvature $K: S \to \mathbb{R}$, where S is the cone $x_1^2 + x_2^2 - x_3^2 = 0, \quad x_3 > 0.$
16. Let $\varphi: U \to \mathbb{R}^3$ be given by $\varphi(\theta, \phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$ where $U = \{(\theta, \phi), \in \mathbb{R}^2 : 0 < \phi < \pi\}$ and $r > 0$.Then show that φ is a parametrized 2-surface.
17. State and prove the Inverse function theorem for n-surfaces.

(6x2= 12 weightage)

Part C

Answer any two questions. Each question carries 5weightages

18. (i) Let S be unit circle ${x_1}^2 + {x_2}^2 = 1$ and define $g: \mathbb{R}^2 \to \mathbb{R}$ by $g(x_1, x_2) = a{x_1}^2 + 2bx_1x_2 + c\,{x_2}^2$ where $a, b, c \in \mathbb{R}$. Show that the extreme point of g on S are the eigenvector of a matrix $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$.

    (ii) What do you mean by an oriented n-surface S in $\mathbb{R}^{n+1}$. Show that on a connected n-surface S in $\mathbb{R}^{n+1}$ there exists always exactly two orientations.
19. Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$ exhibited as a level set $f^{-1}(c)$ of a smooth function $f: \mathbb{R}^{n+1} \to \mathbb{R}$with $\nabla f(p) \neq 0$ for all $p \in S$. Then show that the Gauss map maps S onto the unit sphere S^n.
20. Let S be an n-surface in $\mathbb{R}^{n+1}$, let $p \in S$ and let $\mathbb{v} \in S_p$. Then prove there exists an open interval I containing 0 and a geodesic $\alpha: I \to S$ such that:

    (i) $\alpha(0) = p$ and $\dot{\alpha}(0) = \mathbb{v}$.

    (ii) If $\beta: \hat{I} \to S$ is any other geodesic in S with $\beta(0) = p$ and $\dot{\beta}(0) = \mathbb{v}$then $\hat{I} \subset I$ and $\beta(t) = \alpha(t)$ for all $t \in \hat{I}$.
21. Show for a parameterized n-surface $\varphi: U \to \mathbb{R}^{n+1}$ in $\mathbb{R}^{n+1}$ and for $p \in U$, there exists an open set $U_1 \subset U$ about p such that $\varphi(U_1)$ is an n-surface in $\mathbb{R}^{n+1}$.

(2x5= 10 weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2024

MMT4E11 – Graph Theory

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A
Answer all questions.
Each question carries 1weightage

1. Prove that an edge e is cut edge of G if e is not contained in the cycle of G.
2. Prove that in a nontrivial loopless connected graph G has at least two vertices that are not cut vertices.
3. If G is 2-connected, prove that any two vertices of G lie on a common cycle.
4. Define matching, perfect matching and maximum matching.
5. Prove that a set $S \subseteq V$ is an independent set of G if and only if V\S is a covering set of G.
6. Find the Ramsey number r(3,5).
7. Prove that in a critical graph, no vertex cut is a clique.
8. Prove that a graph is embeddable in plane if it is embeddable in thesphere..

Part B
Answer any two questions from each unit.
Each questions carries 2weightage.

Unit I

9. Show that graph G is forest if and only if every edge of G is cut edge.
10. Prove that $k \leq \kappa' \leq \delta$.
11. If G is a simple graph with $\nu \geq 3$ and $\delta \geq \frac{\nu}{2}$, prove that G is Hamiltonian.

Unit II

12. Let G be a bipartite graph with bipartition (X,Y). Prove that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S| \; \forall \; S \subseteq X$.
13. If a matching M in graph G is a maximum matching, Prove that G contains no M-augumenting path.
14. Prove that every 3-regular graph without cut edges has a perfect matching.

Unit III

15. Let G be a k-critical graph with 2-vertex cut {u,v }, Prove that $d(u) + d(v) \geq 3k - 5$.
16. If G is connected simple graph and is neither an odd cycle nor a complete graph, prove that $\chi \leq \Delta$.
17. Prove that every planar graph is 5-vertex clourable.

Part C

Answer any two questions.

Each question carries 5weightage.

18. (a)If e is a linkof G, prove that $\tau(G) = \tau(G-e)+\tau(G.e)$.

    (b) Show that a graph G with $\nu \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.

19. (a)Prove that closure of a graph is well defined.

    (b)Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

20 (a) Prove that every k-regular bipartite graph has perfect matching.

    (b)For any two integers $k \geq 2$ and $\ell \geq 2$, prove that

$$r(k, \ell) \leq r(k, \ell - 1) + r(k - 1, \ell).$$

21. (a) Prove that in a simple graph G $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge e of G

    (b)Prove that a digraph D contains a directed Path of length $\chi - 1$.

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2024

MMT4E14 – Computer Oriented Numerical Analysis

(2022 Admission onwards)

Time: 1 ½ hours Max. Weightage : 15

Part A (Short Answer Questions)
(Answer all questions. Each question has weightage 1)

1. Write the output of 7//3, 5/3 and 7%3.
2. Explain *break* statement with examples.
3. Write a short note on the data structures List and Tuple.
4. Explain functions in python with example.

(4x 1=4 weightage)

Part B
(Answer any three from the following five questions. Each question has weightage 2)

5. Write a python program to find the root of the equation $x^2 - 5x + 6 = 0$ using Newton Raphson method.
6. Write a python program to find the $\int_a^b f(x)dx$ using trapezoidal rule.
7. Write a python program programme to find the value of function using Lagranges interpolation.
8. Write a python program to solve the initial value problem by using Runge Kutta method of order 4.
9. Write a python programme to solve a system of equations having n equations and n unknowns.

(3x2=6 weightage)

Part C
Answer any one from the following two questions. Each question has weightage 5

10. Write a python programme to find the root of the given continuous function f(x) on [a,b] by using bisection method.
11. Write a python programme to find the integral using tabulated values by the method of Simpson rule.

(1x5=5 weightage)