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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Third Semester M.Sc Statistics Degree Examination, November 2024**

**MST3C10 – Time Series Analysis**

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

**PART A**

**Answer any four questions. Weightage 2 for each question.**

1. Define a time series and relate it with a discrete parameter stochastic process.
2. Distinguish between weak and strong stationary time series.
3. Define a Moving Average model of order  $q$  (MA( $q$ )) and find the autocorrelation of MA(1) model.
4. Stating the required conditions prove that an autoregressive process of order 1 (AR(1)) can be written as a MA process of infinite order.
5. Write down the general form of an ARMA( $p,q$ ) and ARIMA( $p,d,q$ ) model. Identify the constants ( $p,d,q$ ) of the model  $Y_t = 2Y_{t-1} - Y_{t-2} + \epsilon_t$ .
6. Describe the structure of correlogram of (i) Stationary series and (ii) Non stationary series.
7. Define ARCH(1,1) model. Give an example to show the application of ARCH models in financial time series analysis.

**(4x2=8 Weightage)**

**PART B**

**Answer any four questions. Weightage 3 for each question.**

8. Describe (i) Single exponential smoothing method and (ii) double exponential smoothing method.
9. Define auto-covariance function and auto-correlation function of a time series. Prove that the auto-covariance function of a stationary time series is positive definite.
10. Derive the autocorrelation of  $\{Y_t\}$ , where  $Y_t = \epsilon_t - 0.2\epsilon_{t-1} + 0.3\epsilon_{t-2}$  assuming  $\{\epsilon_t\}$  as a white noise process.

11. What do you mean by residual analysis of a time series? Explain any one test procedure for testing the presence of residual auto-correlations.
12. What do you mean by forecasting in time series? Explain the 1-step ahead forecasting procedure in an AR(p) process.
13. Find the spectrum of AR(1) and MA(1) models.
14. Define a GARCH(1,1) model and describe its properties.

(4x3=12 Weightage)

### PART C

Answer any 2 questions. Weightage 5 for each question.

15. (a) Explain Holt winters smoothing method for multiplicative seasonality.  
 (b) Let  $\{e_t\}$  be a zero mean white noise process. If  $Y_t = e_t + \theta e_{t-1}$ , find the autocorrelation function for  $\{Y_t\}$  both when  $\theta = 3$  and when  $\theta = \frac{1}{3}$ .
16. Derive the acf of an ARMA(p,q) process and obtain the invertibility conditions.
17. Explain the least square estimation method in AR(1) and MA(1) models.
18. State and prove Herglotz theorem.

(2x5=10 Weightage)



## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

## Third Semester M.Sc Statistics Degree Examination, November 2024

## MST3C11 – Design and Analysis of Experiments

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

**Part A (Answer any four questions, each carries 2 weightage)**

1. Write down the assumptions of ANOVA?
2. How do you deal with missing data in an experimental design?
3. Define main effect and interaction effect in an experimental design.
4. What are the basic Principle of design of experiments?
5. Give a example of  $2^2$  and  $2^3$  factorial experiment
6. Explain Youden square design?
7. Discuss the situation where intrablock analysis has been used?

(4x2 = 8 weightage)

**Part B (Answer any four questions, each carries 3 weightage)**

8. What do you mean by replication and local control? How do they increase the efficiency of an experiment?
9. Discuss the layout in the analysis of CRD?
10. For a BIBD prove that  $b \geq v$ , where  $b$  is the number of blocks and  $v$  denote the number of treatments.
11. Describe PBIBD with  $m$  associate classes. When does it reduces to BIBD?
12. Explain strip plot design with an example?
13. Differentiate the layout of LSD and GLSD?
14. Illustrate confounding of the interaction effect 'BC' with reference to  $2^3$  factorial experiments, having A, B, C as factors.

(4x3 =12 weightage)

**Part C (Answer any two questions, each carries 5 weightage)**

15. Explain in detail about the construction of BIBD with an example?
16. Explain the procedure of design of experiments using RBD.
17. Discuss the analysis of partially confounded  $2^3$  experiment.
18. State and prove Gauss Markov's theorem.

(2x5 = 10 weightage)

## FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

## Third Semester M.Sc Statistics Degree Examination, November 2024

## MST3C12 – Testing of Statistical Hypothesis

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

**Part A****Answer any four questions (2 weightages each)**

1. State generalized Neyman-Pearson lemma.
2. a) What is p- value ? How is it used in statistical test procedures?  
b) Differentiate simple and composite hypothesis with an example.
3. Explain the principle of invariance in testing of hypothesis.
4. Explain Bayesian tests.
5. Define Kendall's tau test. State its properties.
6. Explain run test and sign test.
7. How will you develop an SPRT for testing  $H_0: \mu < \mu_0$  against  $H_1: \mu > \mu_1$ , in the case of one parameter exponential distribution?

**(4 x 2=8 Weightage)****Part B****Answer any four questions (3 weightages each)**

8. Based on a sample of size n drawn from a distribution with density:

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the most powerful test of  $H_0: \theta = \theta_0$  vs  $H_1: \theta = \theta_1$ .

9. Show that  $\phi$  is invariant under G iff  $\phi$  is a function of T.
10. If  $l(x)$  is the likelihood ratio for testing  $H_0: \theta = \theta_0$  against  $H_1: \theta \neq \theta_0$ , where  $\theta$  is a scalar, then show that the asymptotic distribution of  $-2\log l(x)$  is  $\chi^2_{(1)}$ .
11. Define monotone likelihood ratio property. Examine whether the density:

$$f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}, -\infty < x < \infty$$

satisfies monotone likelihood ratio property.

12. a) Briefly explain median test.  
b) Explain Spearman rank correlation coefficient.
13. Show that the Kolmogorov test statistic is distribution free if the distribution function of the population is continuous.
14. Define A.S.N function. Let  $X \sim b(1, p)$ . Find the A.S.N of the SPRT  $H_0: p = p_0$  vs  $H_1: p = p_1$ .

**(4 x 3=12 weightage)**



**Part C**

**Answer any two questions (5 weightages each)**

15. a) Define unbiased test. If the power function of a test is continuous, then UMP  $\alpha$ -similar test reduces to the UMP unbiased test.
- b) Using a single observation taken from the pdf  $f(x)$ , obtain a most powerful size  $\alpha$  test for testing  $H_0: f(x) = \begin{cases} 4x, 0 < x < 1/2 \\ 4 - 4x, 1/2 \leq x < 1 \end{cases}$  against  $H_1: f(x) = 1, 0 < x < 1$ .
16. Show that the likelihood ratio test for testing  $H_0: \mu = \mu_0$  against  $H_1: \mu \neq \mu_0$  in sampling from  $N(\mu, \sigma^2)$  with  $\mu$  and  $\sigma^2$  unknown leads to t-test. What is the distribution of the test statistic under  $H_1$ ?
17. Describe the Mann Whitney U test for two independent samples. Derive the relationship between the Wilcoxon statistic  $W$  and  $U$ -statistic and the expression for the mean and variance of  $U$  under the null hypothesis.
18. a) Determine the expressions for the boundary values  $A$  and  $B$  of SPRT with strength  $(\alpha, \beta)$ .
- b) Obtain the OC function with respect to the SPRT for testing  $H_0: \lambda = \lambda_0$  against  $H_1: \lambda = \lambda_1$  based on observations from Poisson distribution at strength  $(\alpha, \beta)$ .

**(2 x 5 = 10 weightage)**

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Statistics Degree Examination, November 2024

MST3E01 – Operations Research – I

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

**Part A****(Answer any four questions. Weightage 2 for each question.)**

1. Define a convex set and extreme point of a convex set. Show that set of all feasible solution to a linear programming problem is a convex set.
2. Explain the use of slack, surplus, and artificial variables in solving a linear programming problem.
3. Differentiate between Big M method and Two-phase simplex method.
4. Write a note on economic interpretation of duality in a linear programming problem.
5. Define a transportation problem. Explain with the help of an example how an unbalanced transportation can be converted into a balanced one by an example.
6. Define what you mean by a loop in a transportation problem. What is its significance?
7. Discuss dominance property associated with a game, explain its role in solving a game.

**Part B****(Answer any four questions. Weightage 3 for each question.)**

8. Show that extreme point of the set of all feasible solution of an LPP always corresponds to a basic feasible solution.
9. Write a note on the degeneracy and cycling in an LPP.
10. Explain the steps involved in solving a linear programming problem by a dual simplex method.
11. Explain the steps involved in finding an initial basic feasible solution to a transportation problem by Vogel's approximation method.
12. Explain the term sensitivity analysis in an LPP. Discuss how dual simplex method is useful in sensitivity analysis.
13. Discuss the steps involved in solving an assignment problem.
14. Explain game problem as a linear programming problem.

### Part C

(Answer any two questions. Weightage 5 for each question)

15

a) Solve graphically

$$\text{Maximize } z = x_1 + x_2$$

Subject to

$$-2x_1 + x_2 \leq 1$$

$$x_1 + x_2 \leq 3$$

$$x_1 \leq 2, x_1 \geq 0, x_2 \geq 0$$

b) Use simplex method to solve

$$\text{Maximize } z = 3x_1 + x_2 + 7x_3$$

Subject to

$$x_1 + x_2 + x_3 \leq 20$$

$$2x_1 + x_2 + 3x_3 \leq 50$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

16

a) What is zero-one programming? Illustrate it with an example.

b) Describe Gomory's method of solving an all-integer programming problem

17

a) Solve the transportation problem

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
O <sub>1</sub>	11	17	14	30	8
O <sub>2</sub>	20	12	13	10	9
O <sub>3</sub>	22	15	18	20	13
	12	6	8	4	

b) Solve the assignment problem

	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>	J <sub>4</sub>
A	8	11	16	15
B	12	18	19	20
C	22	19	16	23
D	17	15	18	20

18

a) State and prove fundamental theorem of game

b) Solve the game graphically

	Player B
Player A	$\begin{bmatrix} 3 & 10 & 4 & 8 \\ 7 & 6 & 6 & 9 \end{bmatrix}$



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Third Semester M.Sc Statistics Degree Examination, November 2024

MST3E04 – Life Time Data Analysis

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

**PART A****Answer any four (2 weightages each)**

1. Define threshold parameters.
2. Compare and contrast continuous and discrete lifetime distributions
3. How does censoring affect the analysis of lifetime data?
4. What methods are available for dealing with truncated data?
5. Describe type-2 censored test plans and their role in lifetime data analysis.
6. Explain the concept of mixture models in the context of lifetime data analysis.
7. What are continuous multiplicative hazard models?

**(2 x 4=8 weightages)****PART B****Answer any four (3 weightages each)**

8. How are likelihood-based methods applied for inference in log-location scale distributions?
9. Provide examples of descriptive and diagnostic plots commonly used in the analysis of lifetime data.
10. Discuss the properties and applications of the Weibull distribution in survival analysis.
11. How is semi-parametric maximum likelihood estimation carried out for continuous observations in survival analysis?
12. How is inference carried out under the exponential model in lifetime data analysis? Discuss the large sample theory.
13. Explain the rank test for comparing distributions say the log-rank test.
14. What are the properties of the Nelson-Aalen estimator, and how is it used to estimate cumulative hazard functions?

**(3x 4=12 weightage)**



**PART C**

**Answer any two (5 weightages each)**

15. Define censoring in the context of lifetime data analysis. What are the statistical methods used to account for censoring?
16. For the data on remission times (in days) given below, Obtain the Kaplan-Meier estimator of survival function  $S(t)$  at  $t=1, 10, 29$  and  $60$ .  
1, 1, 2, 4, 4, 6, 6, 6, 7, 8, 9, 9, 10, 12, 13, 14, 18, 19, 24\*, 26, 29, 31\*, 42, 45\*, 50\*, 57, 60, 71\*, 83\*, 91. (Here \* denote the censored observations).
17. Explain the concept of life tables
18. Describe the Accelerated Failure Time model and its applications.

**(5x2=10 weightage)**