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1M2A24115 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2024

MST2C06 - Probability Theory - II

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

Answer any four questions. Each question carries 2 weightages.

1. Show that a characteristic function is uniformly continuous over the real line?
2. For any characteristic function φ, Show that,

   a) $Re(1-\varphi(t)) \geq \frac{1}{4} Re(1-\varphi(2t))$

   b) $|\varphi(t)-\varphi(t+h)|^2 \leq 2\{1- Re\,\varphi(h)\}$
3. If X_n' s are uniformly bounded and ΣX_n converges almost surely, show that $\Sigma V(X_n)$ and $\Sigma E(X_n)$ converge?
4. Let $X_1, X_2, \ldots\ldots$ be a sequence of iid random variables with common uniform distribution on [0,1]. Also let

   $Z_n = (\prod_{i=1}^{n} X_i)^{1/n}$. Show that $Z_n \xrightarrow{p} c$ where c is a constant. Find c?
5. Establish Lindberg – Levy central limit theorem?
6. Let x be an integrable random variable defined on $(\Omega, \mathbb{F}, P)$. If $\mathbb{F}_1 \subset \mathbb{F}_2$ are sub σ – fields of $\mathbb{F}$ show that $E[(X/\mathbb{F}_2)/\mathbb{F}_1] = E(X/\mathbb{F}_1)$.
7. Define a martingale, sub martingale and a super martingale?

(4 x 2 = 8 weightage)

PART B

Answer any four questions. Each question carries 3 weightages.

8. Derive the characteristic function of a standard Cauchy random variable?
9. If the series $\sum_r \frac{\mu_r'}{r!} t^r$ converges for some $t_o>0$, Show that the sequence of moments $\mu_r' = E(X^r)$ *determine the* distribution function uniquely?
10. State Lindberg -feller central limit theorem. Deduce Liaponove theorem using it?
11. State and establish Khintchine's weak law of large numbers.
12. State and prove Kolmogorov inequality?

13. Let $\{X_n\}$ be a sequence of iid random variables with finite mean $\mu = E(X_1)$, then show that $\frac{S_n}{n} \xrightarrow{P} \mu$ as $n \longrightarrow \infty$, where $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

14. If $\{X_n, \mathbb{F}_n\}$ is a martingale and ϕ is convex, and if $\phi(X_n)$ is integrable, show that $\{\phi(X_n), \mathbb{F}_n\}$ is a submartingale?

(4 x 3 = 12 weightage)

PART C

Answer any 2 questions. Each questions carries five weightages.

15. State and prove inversion theorem of characteristic function, use it to establish uniqueness theorem?

16. State and establish Kolmogorov strong law of large numbers for independent sequence of random variables?

17. Let $\{X_n\}$ be a sequence of independent random variables with the following distributions. In each case examine whether Lindberg condition hold,

    a) $P(X_n = \frac{1}{2^n}) = P(X_n = -\frac{1}{2^n}) = \frac{1}{2}$

    b) $P(X_n = \pm 1) = \frac{1-2^{-n}}{2}$

    $P(X_n = \frac{1}{2^n}) = P(X_n = -\frac{1}{2^n}) = \frac{1}{2^{n+1}}$

18. Write a note on the following,

    a) Doob-Meyer decomposition of a martingale.

    b) Stopping time.

(2 x 5 = 10 weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2024

MST2C07- Applied Regression Analysis

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

Answer any four (2 weightages each)

1. Define multiple linear regression model.
2. State Gauss Markov theorem.
3. Define multicollinearity. What are the consequences?
4. Explain how residual plot are used to check the assumption of normality of the errors in a linear model.
5. Describe how splines are used in polynomial regression.
6. Explain the use of dummy variables in regression analysis.
7. Explain the link function of a generalized linear model

(2 x 4=8 weightages)

PART B

Answer any four (3 weightages each)

8. Derive the interval estimation of the mean response of a simple linear regression model.
9. Let $y_i = \beta x_i + \varepsilon_i, i = 1,2$ where $\varepsilon_1 \sim N(o, \sigma^2), \varepsilon_2 \sim N(o, 2\sigma^2)$ and ε_1 and ε_2 are statistically independent. If $x_i = +1$ and $x_i = -1$, obtain the weighted least squares estimate of β and find the variance of your estimate.
10. Derive expression for Mallow's Cp Statistic.
11. Describe the method of detection of multicollinearity based on eigen values of matrix.
12. What is the form of Poisson regression model? Also give link functions used for a Poisson regression model?
13. Explain the nonlinear regression model. Discuss method of least squares of parameter estimation in this model.

(3x 4=12 weightages)

PART C

Answer any two (5 weightages each)

14. Obtain Maximum likelihood estimates of parameters of multiple linear regression model.
15. Define residuals and residual sum of squares. In a linear regression model $Y = X\beta + \varepsilon$, Show that the residual sum of squares $e'e = Y'Y - \hat{\beta}X'X\hat{\beta}$
16. (a) Describe stepwise regression procedure

    (b) Explain non parametric Regression
17. Explain in detail the analysis and inference methods involved in logistic regression model.

(5x 2=10 weightages)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2024

MST2C08- Estimation Theory

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

(Use of Scientific Calculator and Life tables are permitted)

PART A

(Answer any 4 questions. Weightage 2 for each question)

1. Define a) Minimal sufficient statistic. b)ancillary statistic.
2. Give an example to prove that MLE's are not always unbiased.
3. Let $X_1, X_2, \dots, X_n$ be a sample from $U(-\frac{\theta}{2}, \frac{\theta}{2})$. Find a sufficient estimator of θ.
4. Define consistency. Let $X \sim U(0, \theta)$. Show that $X_{(n)} = \max(X_{(1)}, X_{(2)}, \dots, X_{(n)})$ is consistent.
5. Describe the method of construction of confidence intervals using pivots.
6. Explain the method of percentiles for estimation of parameters.
7. Define one parameter Cramer family of distributions.

(4*2=8 weightage)

PART B

(Answer any 4 questions. Weightage 3 for each question)

8. a) State and prove Neyman-Factorization theorem.

   b) Let $X_1, X_2, \dots, X_n$ be i.i.d Poisson with mean θ and let T=$\sum_{i=1}^{n} X_i$. Check whether T is sufficient for θ or not.
9. Define complete family of distributions. Explain using an example.
10. State and prove Cramer-Rao inequality.
11. a) Prove or disprove: " If T_n is a CAN estimator of θ then ${T_n}^k$ is a CAN estimator of θ^k, k is a known positive integer".

    b) Prove or disprove: MLE's are always consistent.
12. a) Let $X \sim B(1, \theta^2)$. Does there exist an unbiased estimator of θ? Justify your claim.

    b) Let $X \sim P(\lambda)$. Find a BLUE for λ

13. a) Explain the method of construction of confidence interval using maximum likelihood estimator. Illustrate with an example.

b)Distinguish between Bayesian and Fiducial interval.

14. Let $X_1, X_2, \ldots, X_n$ be a random sample of size drawn from a Poisson distribution with parameter λ. Assuming that the prior distribution of λ is $G(\alpha, \beta)$.

Find $100(1-\alpha)\%$ Bayesian confidence interval for λ. Compare it with classical shortest length confidence interval.

(4*3=12 weightage)

PART C

(Answer any two questions. Weightage 5 for each question)

15. State and prove Cramer-Huzurbazar theorem.

16. Apply method of moment estimation to estimate the parameter $\theta = (\mu, \sigma)$ of the following distribution with pdf.

$$f(x;\theta) = \frac{1}{\sigma} e^{\frac{-(x-\mu)}{\sigma}}, x > \mu, \sigma > 0.$$

Show that $(X_{(1)}, \sum_{j=1}^{n}(X_j - X_{(1)}))$ is complete sufficient statistic for $\theta = (\mu, \sigma)$.

17. a) State and Prove Lehmann-Scheffe theorem.

b) Explain One parameter Exponential family of distributions.

18. State and Prove Rao-Blackwell theorem

(2*5=10 weightage)

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1M2A24118 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2024

MST2C09 - Stochastic Processes

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

(Use of Scientific Calculator is permitted)

PART A
Answer any four (2 weightage each)

1. Define Stochastic Process. Explain its classification with examples.
2. Let Sn be the waiting time for the occurrence of n^{th} renewal and m(t) be the renewal function of renewal process. Show that $E\{S_N(t)+1\} = E(X1)\ \{1 + m(t)\}$.
3. State and prove Chapman -Kolmogorov equations for Markov chains.
4. Compute the density function Tx, the time until Brownian motion hits x.
5. Derive the relationship between Poisson Process and Geometric distribution.
6. Define Non homogeneous Poisson Process. Obtain its Probability generating function
7. Explain open queueing networks and closed queueing networks.

(4 x 2 =8weightage)

Part B
Answer any four (3 weightage each)

8. Explain basic characteristic of a queueing system.
9. State and prove Ergodic theorem and converse of the Ergodic theorem.
10. (a) Define recurrence and obtain a condition for a chain to be recurrent.

    (b) Check whether Poisson process is stationary or not
11. Explain Yule-Furry process. Obtain its probability distribution
12. If the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $1/\lambda$ then prove that event E form a Poisson Process with mean λt.
13. Explain Stationary and weakly Stationary Processes with the help of examples.
14. (a)Show that the renewal function $m(t) = \sum_{n=1}^{\infty} F_n(t), \forall t$, where $F_n(t) = P(S_n \leq t), n \geq 0$.

    (b) Explain Stopping Time

(4x3=12weightage)

PART C
Answer any two (5 weightage each)

15. Define period of the state of a Markov chain. Show that periodicity is a class property. Given an example.
16. Derive the p.m.f of Poisson Process.
17. (a)Explain renewal reward process and regenerative process.

    (b) Explain Semi Markov process

    (c)What do you mean by a queue? Briefly explain Kendall's notation
18. Derive the steady state probabilities of M/M/s model

(2x5=10weightage)