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1M2A24085 (Pages : 2) Reg. No:..............................

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Degree Examination, April 2024**

**MMT2C06 – Algebra II**

(2022 Admission onwards)

Time: 3hours Max. Weightage : 30

## Part A
## Answer all questions. Each carries 1 weightage

1. Show that a commutative ring with unity is a field if and only if it has no proper non-trivial ideals.
2. Show that $\sqrt{1+\sqrt[3]{2}}$ is algebraic over Q.
3. Prove that squaring the circle is impossible.
4. State Isomorhism Extension theorem.
5. Does there exist a field of 3127 elements?. Justify your answer.
6. Find the spliting of $\{x^4 - 1\}$ over Q.
7. Is regular 60-gon is constructible? Justify your answer.
8. Define the $n^{th}$ cyclotomic polynomial over a field F.

**(8 × 1 = 8 weightage)**

## Part B
## Answer any two questions from each unit. Each carries 2 weightage

### Unit I

9. Let F is a field, Show that every ideal in F[x] is principle.
10. Prove that a finite extension E of a field F is an algebraic extension of F.
11. Show that a field F is algebraically closed iff every non-constant polynomial in F[x] factors in F[x] into linear factors.

### Unit II

12. Let $p$ be a prime and let $n \in Z^+$. If E and E$'$ are fields of order $p^n$, then prove that E $\simeq$ E$'$.

13. If E is a finite extension field of a field F then prove that E is seperable over F if and only if each $\alpha \in$E is separable over F.

14. Prove that every finite field is perfect.

## Unit III

15. Let K be a finite normal extension of a field F with Galois group G(K/F). For each intermediate field E with F $\leq$ E $\leq$ K. Let $\lambda(E)$ = G(K/E). Prove that

    (a) The fixed field of $\lambda(E)$ in K is E.

    (b) $\lambda$ is one to one.

16. Find all primitive $10^{th}$ roots of unity in $Z_{11}$.

17. Let F be a field of characteristic zero, and let $a \in$ F. Prove that if K is the splitting field of $x^n - a$ over F, then G(K/F) is a solvable group.

**(6 × 2 = 12 weightage)**

## Part C

**Answer any two questions. Each carries 5 weightage**

18. (a) Let E be a simple extension F($\alpha$) of a field F, and let $\alpha$ be algebraic over F. Let the degree of irr($\alpha$, F) be $n \geq 1$. Show that every element $\beta$ of E = F($\alpha$) can be uniquely expressed in the form $\beta = b_0 + b_1\alpha + \dots\dots\dots\dots + b_{n-1}\alpha^{n-1}$, where the $b_i$ are in F.

    (b) Let $\alpha$ be a zero of $x^2 + x + 1 \in Z_2[x]$. Show that there exist a field $\mathbb{Z}_2(\alpha)$ of four elements.

19. (a) State and prove the Conjugation Isomorphism theorem.

    (b) Prove that complex zeros of polynomials with real coefficients occur in conjugate pairs.

20. (a) Let E be finite extension of a field F. Let $\sigma$ be an isomorphism of F onto a field F$'$ and let $\overline{F'}$ be an algebraic closure of F$'$. Prove that the number of extensions of $\sigma$ to an isomorphism $\tau$ of E onto a subfield of $\overline{F'}$ is finite and independent of F$'$, $\overline{F'}$ and $\sigma$.

    (b) If F$\leq$E$\leq$K, where K is a finite extension field of a filed F, then prove that {K : F} = {K : E} {E : F}.

21. Let K be the splitting field of $x^4 + 1$ over Q.

    (a) Prove that

        i. Show that $[K : Q] = 4$

        ii. G(K/Q) is isomorhic to Kline 4-group.

    (b) Find an intermediate field E with $Q \leq E \leq K$ such that $[E : Q] = 2$.

**(2 × 5 = 10 weightage)**

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# FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Degree Examination, April 2024**

**MMT2C07 – Real Analysis II**

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

## Part A

*Answer ALL questions. Each question carries 1 weight.*

1. Give example of an algebra of sets that is not a $\sigma$-algebra.

2. If $A = \{1,2,3\}$, then prove that $m^*(A) = 0$.

3. Prove that a constant function is a measurable function.

4. Give example of a Lebesgue integrable function on $[0,1]$ that is not Riemann integrable.

5. Define $f^-$ of an extended real valued function $f$ on a set $E$.

   Find $f^-$ where $f(x) = \cos x$ on $[0,2\pi]$.

6. State the Lebesgue Dominated Convergence Theorem.

7. Prove that a Lipschitz function is of bounded variation on $[a,b]$.

8. Define absolutely continuous function. Give one example.

**$8 \times 1 = 8$ Weights.**

## Part B

*Answer any TWO questions from each UNIT. Each question carries 2 weights.*

### Unit I

9. For two measurable sets $A$ and $B$, prove that $m^*(A \cup B) + m^*(A \cap B) = m^*(A) + m^*(B)$.

10. Let $\{A_k\}_{k=1}^{\infty}$ be an ascending sequence of measurable sets.

    Prove that $m\left(\cup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} m(A_k)$.

11. If $f$ and $g$ are measurable functions that are finite almost everywhere on a measurable set $E$, then prove that $f+g$ is measurable on $E$.

Unit II

12. State and prove monotone convergence theorem.

Will the theorem hold for a decreasing sequence of functions ? Give reason.

13. Let $f$ be integrable,,over $E$. If $\{E_n\}_{n=1}^{\infty}$ is a decreasing sequence of measurable subsets of $E$, then prove that $\int_{\bigcap_{n=1}^{\infty} E_n} f = \lim_{n\to\infty} \int_{E_n} f$.

14. Define the concept of converence in measure of a sequence of measurable fnctions to a measurable function on a set $E$. Give an example for this.

Unit III

15. Prove that a monotone fnction on the open inetrval $(a,b)$ is continuous except possibly at countable number of points of $(a,b)$.

16. Prove that a function $f$ is absolutely continuous on a closed and bounded interval $[a,b]$ if and only if it is an indefinite integral over $[a,b]$.

17. State and prove the Chordal Slope Lemma.

**$6\times2 = 12$ Weights.**

Part C

*Answer any TWO questions. Each question carries 5 weights.*

18. State and prove the Borel-Cantelli Lemma.

19. Prove that a non-negative extended real valued measurable function on a measurable set is the point-wise limit of an increasing sequence of simple functions.

20. State and prove the bounded convergence theorem.

21. State and prove the Vitali Covering Lemma.

**$2\times5 = 10$ Weights.**

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Degree Examination, April 2024**

**MMT2C08 – Topology**

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

## Part A

*Answer* **all** *questions. Each question carries* **1** *weightage.*

1. Let $(X, d)$ be a metric space. Show that the intersection of any finite number of open sets is open.
2. Define cofinite and co-countable topology on a set.
3. Let $A, B$ be subsets of a topological space $(X, \mathcal{T})$, then prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$.
4. Prove that a subset $A$ of a space $X$ is dense in $X$ iff for every nonempty open subset $B$ of $X$, $A \cap B \neq \emptyset$.
5. Define strong topology determined by a family of functions.
6. Let $X$ be a space and $A, B$ open subsets of $X$. If $A \cup B = X$, $A \cap B = \emptyset$ then show that $\overline{A} \cap \overline{B} = \emptyset$.
7. Show that every completely regular space is regular.
8. Define a cube, a Hilbert cube and a Cantor discontinuum.

**(8x1 = 8 weightage)**

## Part B

*Answer any* **two** *questions from each unit. Each question carries* **2** *weightages.*

### UNIT- I

9. Let $(X, \mathcal{T})$ be a topological space and $\mathcal{S}$ a family of subsets of $X$. Then prove that $\mathcal{S}$ is a sub-base for $\mathcal{T}$ if and only if $\mathcal{S}$ generates $\mathcal{T}$.
10. Prove that metrisability is a hereditary property.
11. Let $X$ be a space and $A \subset X$. Then prove that $int\ (A)$ is the union of all open sets contained in $A$.

## UNIT- II

12. Prove that every closed, surjective map is a quotient map.
13. Prove that every separable space satisfies the countable chain condition.
14. Show that every nonempty connected subset is contained in a unique component.

## UNIT- III

15. If $X$ is a regular topological space then prove that for any $x \in X$ and any open set $G$ containing $x$ there exists an open set $H$ containing $x$ such that $\overline{H} \subset G$.
16. Prove that a subset of $X$ is a box iff it is the intersection of a family of walls.
17. If the product is non-empty, then prove that each co-ordinate space is embeddable in it.

**(6x2 = 12 weightages)**

## Part C

*Answer any* **two** *questions. Each question carries* **5***weightages.*

18. (a) Let $(X, \mathcal{T})$ be a topological space and $\mathfrak{B} \subset \mathcal{T}$. Then prove that $\mathfrak{B}$ is a base for iff for any $x \in X$ and any open set $G$ containing $x$, there exist. $B \in \mathfrak{B}$ such that $x \in B$ and $B \subset G$.

    (b) Prove that, in a topological space two distinct topologies can never have the same family of subsets as a base for both of them.

19. (a) For a subset $A$ of a space $X$, prove that $\bar{A} = \{y \in X$: every nbd of $y$ meets $A$ non-vacously$\}$.

    (b) Let $(X, \mathcal{T}), (Y, \mathcal{U})$ be spaces and $f: X \to Y$ a function. Then show that $f$ is $\mathcal{T} - \mathcal{U}$ continuous if and only if for all $V \in \mathcal{U}, f^{-1}(V) \in \mathcal{T}$.

20. (a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

    (b) State and prove Lebesgue covering lemma.

21. (a) Define $T_1$ space and give an example. If $y$ is an accumulation point of a subset $A$ of a $T_1$space $X$. Then show that every neighborhood of $y$ contains infinitly many points of $A$.

    (b) Show that all metric spaces are $T_4$ .

**(2x5 = 10 weightages)**

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Degree Examination, April 2024**

**MMT2C09 – ODE and calculus of variations**

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

## Part A

**Answer all questions. Each question carries one weightage**

1. Locate and classify the singular points of the equation $x^2(x-1)y'' - 2xy' + xy = 0$ on the $x$ axis.
2. Find the indicial equation and roots of the equation $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$.
3. Show that $P_n(-x) = (-1)^n P_n(x)$, where $P_n(x)$ denote the Legendre polynomial of degree $n$.
4. Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$, where $J_n(x)$ is the Bessel function of the first kind of order n.
5. Describe the phase portrait of the system

$$\begin{cases} \frac{dx}{dt} = 1, \\ \frac{dy}{dt} = -1. \end{cases}$$

6. Explain the different types of critical points of an autonomous system.
7. Show that the zeros of the functions $a\sin x + b\cos x$ and $c\sin x + d\cos x$ are distinct and occur alternately whenever $ad - bc \neq 0$.
8. Define extremals and stationary curve of Euler's differential equation.

($8 \times 1 = 8$ weightage)

## Part B

**Answer any two questions from each unit. Each question carries 2 weightage**

### Unit I

9. Obtain the power series solution of the equation $y'' + xy = 0$.
10. Find the general solution of the equation $(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0$ near its singular point x = 3.
11. State and prove the orthogonality property of Legendre polynomials.

## Unit II

12. If $W(t)$ is the Wronskian of the two solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$$

then show that $W(t)$ is either identically zero or nowhere zero on $[a, b]$.

13. Find the general solution of the following system:

$$\begin{cases} \frac{dx}{dt} = 5x + 2y, \\ \frac{dy}{dt} = -x + y. \end{cases}$$

14. Determine the nature and stability properties of the critical point $(0, 0)$ for the linear autonomous system:

$$\begin{cases} \frac{dx}{dt} = -2x, \\ \frac{dy}{dt} = 3y. \end{cases}$$

## Unit III

15. State and prove Sturm Separation Theorem.

16. Check whether $f(x, y) = x^2|y|$ satisfies a Lipshitz condition on the rectangle $|x| \leq 1$ and $|y| \leq 1$. Also check if $\frac{\partial f}{\partial y}$ exist at all points on this rectangle.

17. Find the stationary function of

$$\int_0^4 [xy' - (y')^2] dx$$

which is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$.

($6 \times 2 = 12$ weightage)

## Part C

**Answer any two questions. Each question carries 5 weightage**

18. Find the general solution of Gauss's Hypergeometric equation.

19. Find two independent Frobenius series solution of the equation $xy'' + 2y' + xy = 0$.

20. Find the general solution of Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$ when $p$ is not an integer.

21. Explain Picard's method of successive approximations. Find the exact solution of the initial value problem $y' = x+y$, $y(0) = 1$. Apply Picard's method to find its approximate solution (staring with $y_0(x) = 1$) and compare with the exact solution.

($2 \times 5 = 10$ weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**Second Semester M.Sc Degree Examination, April 2024**

**MMT2C10 – Operations Research**

(2022 Admission onwards)

Time: 3 hours Max. Weightage : 30

SECTION A

**Answer all questions. Each carries 1 weightage**

1. Define a convex function. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semi definite, then prove that $f(X)$ is a convex function.

2. Write the general form of a linear programming problem

3. Solve graphically, Maximize $4x_1 + 5x_2$

   Subject to $x_1 - 2x_2 \leq 2$,

   $2x_1 + x_2 \leq 6$,

   $x_1 + 2x_2 \leq 5$, $x_1 \geq 0, x_2 \geq 0$.

4. Write the dual of the following LPP:

   Maximize $f = 2x_1 + x_2 - x_3$,

   Subject to $2x_1 - 5x_2 + 3x_3 \leq 4$,

   $3x_1 + 6x_2 - x_3 \geq 2$,

   $x_1,\ x_3 \geq 0, x_2$ unrestricted

5. Explain the unbalanced transportation problem with an example and convert it as balanced transportation problem

6. Explain Caterer problem

7. Explain saddle point of a function

8. Define a game. Give the characteristics through which we can classify various types of games.

$8 \times 1 = 8$ weightage

## SECTION B

**Answer any two questions from each unit. Each carries 2 weightage**

### Unit 1

9. Define a feasible solution. Prove that a vertex of $S_F$ is a basic feasible solution.

10. Let $f(X)$ be a convex differentiable function defined in a convex domain $K \subseteq E_n$. Then $f(X_0), X_0 \in K$, is a global minimum if and only if $(X - X_0)'\nabla f(X_0) \geq 0$ for all $X$ in $K$.

11. Solve: Maximize $5x_1 + 3x_2 + x_3$

    Subject to $2x_1 + x_2 + x_3 = 3$,

    $-x_1 + 2x_3 = 4$,

    $x_1, x_2, x_3 \geq 0$.

### Unit 2

12. Prove that if the primal problem is feasible, then it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa.

13. Solve the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the following table

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| $O_1$ | 3 | 2 | 5 | 4 | 25 |
| $O_2$ | 4 | 1 | 7 | 6 | 35 |
| $O_3$ | 7 | 8 | 3 | 5 | 30 |
| | 10 | 18 | 20 | 42 | |

14. The manager of an agricultural farm of 80 hectares learns that for effective protection against insects, he should spray atleast 15 units of chemical $A$ and 20 units of chemical $B$ per hectare. Three brands of insecticides are available in the market which contain these chemicals. One brand contains 4 units of $A$ and 8 units of $B$ per kg and costs Rs 5 per kg, the second brand contains 12 and 8 units respectively and costs Rs 8 per kg, and the third contains 8 and 4 units respectively and costs Rs 6 per kg. It is also learnt that more than 2.5 kg per hectare of insecticides will be harmful to the crops. Determine the quantity of each insecticide he should buy to minimize the total cost for the whole farm.

### Unit 3

15. Find the minimum path from $v_1$ to $v_8$ in the graph with arcs and arc lengths.

| Arc | (1,2) | (1,3) | (1,4) | (2,3) | (2,6) | (2,5) | (3,5) | (3,4) | (4,7) |
|---|---|---|---|---|---|---|---|---|---|
| Length | 1 | 4 | 11 | 2 | 8 | 7 | .3 | 7 | 3 |
| Arc | (5,6) | (5,8) | (6,3) | (6,4) | (6,7) | (6,8) | (7,3) | (7,8) | |
| Length | 1 | 12 | 4 | 2 | 6 | 10 | 2 | 2 | |

16. Solve by branch and bound method

$Max \;\; 13x_1 + 3x_2 + 3x_3$

$Subj \;\; 7x_1 + 6x_2 - 3x_3 \leq 8, \;\; 7x_1 - 3x_2 + 6x_3 \leq 8, \;\; x_1, x_2, x_3$ non negative integers

17. Give the algorithm for the maximum flow problem. Prove that this algorithm solves the problem of maximum flow and that thr maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.

$6 \times 2 = 12$ weightage

## SECTION C

**Answer any two questions. Each carries 5 weightage**

(Ceiling 35)

18. Solve using Two Phase method:

Minimize $f(X) = 4x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \geq 1$

$x_1 + 4x_2 \geq 2$

$x_1, \; x_2 \geq 0$

19. (*a*) Prove tha dual of a dual is primal. (*b*) Solve using Dual simplex method:

Minimize $2x_1 + 3x_2$

Subject to $2x_1 + 3x_2 \leq 30$,

$x_1 + 2x_2 \geq 10$,

$x_1 \geq 0, x_2 \geq 0$.

20. A Project consists of activities $A, B, C, ...M$. In the following data $X - Y = c$ means $Y$ can start after $c$ days of the work on$X$. $A, B, C$ start simultaneously and $K$and$M$ are the last activities and take 14 and 13 days respectively.

$A - D = 4, B - F = 6, B - E = 3, C - E = 4, D - H = 5D - F = 3, E - F = 10, F - G = 4, G - I = 12H - I = 3H - J = 3, J - K = 8, I - K = 7, I - L = 7, L - M = 9$. Find the least time of completion of the project.

21. (*a*) State and prove the Minimax theorem.

(*b*) Define a saddle point of a function. Let $f(X, Y)$ be such that both $\max_X \min_Y f(X, Y)$ and $\min_Y \max_X f(X, Y)$ exist. Then prove that the necessary and sufficient condition for the existence of a saddle point $(X_0, Y_0)$ of $f(X, Y)$ is that $f(X_0, Y_0) = \max_X \min_Y f(X, Y) = \min_Y \max_X f(X, Y)$.

$2 \times 5 = 10$ weightage