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1M4A23203 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2023

MMT4E11 – Graph Theory

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A
Answer all questions.
Each question carries 1weightage

1. Prove that every connected graph contains a spanning tree.
2. Define Hamiltonian and non Hamiltonian graphs with suitable examples.
3. If G is Hamiltonian and for every nonempty proper subset S of V, Prove that $\omega(G - S) \leq |S|$.
4. Define matching, perfect matching and maximum matching.
5. Define edge independent number and edge covering number of graph G.
6. Find the Ramsey number r(3,3).
7. Prove that every critical graph is a block.
8. State Euler's formula in planar graphs and verify for complete graph on four vertices.

Part B
Answer any two questions from each unit.
Each questions carries 2weightage.

Unit I

9. Show that graph G is forest if and only if every edge of G is cut edge.
10. Prove that closure of a graph is well defined.
11. If G is a non Hamiltonian simple graph $\nu \geq 3$, prove that G is degree majorised by some $C_{m,\nu}$.

Unit II

12. If a matching M in graph G is a maximum matching, Prove that G contains no M-augumenting path.
13. Prove that every 3-regular graph without cut edges has a perfect matching.
14. Prove that for a bipartite graph, $\chi' = \Delta$.

Unit III

15. If G is k-critical, Prove that $\delta \geq k-1$.

16. Prove that an inner bridge that avoids every outer bridge is transferable.

17. Prove that every planar graph is 5-vertex clourable.

Part C

Answer any two questions.

Each question carries 5weightage.

18. (a) Prove that $\tau(K_n) = n^{n-2}$.

(b) Show that a graph G with $\nu \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.

19. (a) If G is a simple graph with $\nu \geq 3$ and $\delta \geq \frac{\nu}{2}$, prove that G is Hamiltonian.

(b) Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

20. (a) Let G be a bipartite graph with bipartition (X,Y). Prove that G contains a matching that saturates every vertex in X if and only if $|N(S)| \geq |S| \; \forall \, S \subseteq X$.

(b) Prove that $\alpha + \beta = \nu$,where α is independent number and β is covering number of graph G.

21. (a) If G is connected simple graph and is neither an odd cycle nor a complete graph, prove that $\chi \leq \Delta$.

(b) Prove that a graph is planar if and only if it contains no subdivision of K_5 or $K_{3,3}$

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2023

MMT4C15 – Advanced Functional Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions. Each questions carries 1 weightage.

1. Let$X$ be a Banach space over K and $k \in K$. If$k \in \sigma(A)$, then prove that

$$|k| \leq \inf_{n=1,2,3,...} \|A^n\|^{\frac{1}{n}} \leq \|A\|.$$

2. Let X and Y be normed spaces and $F \in BL(X, Y)$. Then prove that $\|F\| = \|F'\| = \|F''\|$.
3. Is the sequence space ℓ^1 reflexive? Justify your answer.
4. Let H be a Hilbert space and F be nonempty closed subspace of H. Then prove that $F^{\perp\perp} = F$..
5. Let X be an inner product space, $\{u_1, u_2, \cdots\}$ be an orthonormal set in $X$and $f \in X'$. Then prove that $\sum_n |f(u_n)|^2 \leq \|f\|^2$.
6. Prove that Riesz representation theorem does not hold for an incomplete inner product space.
7. Let H be a Hilbert space and $A \in BL(H)$. Then prove that$\|AA^*\| = \|A\| = \|A^*A\|$.
8. Show, by an example, that if $k \in \sigma_e(A)$ does not follow that $\bar{k} \in \sigma_e(A^*)$.

Part B

Answer two questions from each unit. Each questions carries 2 weightage.

Unit I

9. Let X be a Banach space and$A \in BL(X)$ and $\|A^p\| < 1$for some positive integer p, Then prove that$I - A$ is invertible
10. Let $X = \ell^1$. Then prove that $x_n \xrightarrow{w} x$ in X if and only if $x_n \longrightarrow x$in X.
11. Prove that every closed subspace of a reflexive normed space is reflexive.

Unit II

12. Let X be a normed space and Y be a Banach space. If $F_n \in CL(X,Y), F \in BL(X,Y)$ and $\|F_n - F\| \to 0$, then prove that $F \in CL(X,Y)$.
13. Let X and Y be normed spaces and $F \in BL(X,Y)$.If$F \in CL(X,Y)$, then prove that $F' \in CL(Y',X')$. Further prove that the converse holds if Y is a Banach space.
14. State and prove Riesz representation theorem.

Unit III

15. Let H be a Hilbert space and $A \in BL(H)$. Then prove that there is a unique $B \in BL(H)$ such that $\langle A(x), y\rangle = \langle x, B(y)\rangle$for all $x, y \in H$.
16. Let H be a Hilbert space and $A \in BL(H)$ be self adjoint .Then prove that A or $-A$ is positive if and only if $|\langle A(x), y\rangle|^2 = \langle A(x), x\rangle\langle A(y), y\rangle$for all $x, y \in H$.
17. Let H be a Hilbert space and $A \in BL(H)$be self adjoint. Then prove that

$$\{m_A, M_A\} \in \sigma_a(A) = \sigma(A) \subset [m_A, M_A].$$

Part C

Answer any two questions. Each questions carries 5 weightage.

18. Let $1 \leq p < \infty$and $\frac{1}{p} + \frac{1}{q} = 1$. Then prove that the dual of $\ell^p$is ℓ^q.
19. Let X be a normed space and $A \in BL(X)$ be of finite rank
    a).Prove that$\sigma_e(A) = \sigma_a(A) = \sigma(A)$.
    b).Show that $\sigma(A) \not\subset \sigma_a(A)$, in general.
20. Let X be a normed space and$A \in CL(X)$. Then prove that
    a). Every nonzero spectral value of A is eigen value of A.
    b). If X is infinite diamensional, then $0 \in \sigma_a(A)$.
21. Let $A \in BL(H)$. Then prove that
    a). If $R(A)$ is finite dimensional, then A is compact.
    b). If each A_n is a compact operator on $H$and $\|A_n - A\| \to 0$, then A is compact.
    c). If A is compact then so is A^*.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2023

MMT4E09 – Differential Geometry

(2019 Admission onwards)

Time: 3 hours

Max. Weightage : 30

Part A

Answer all questions. Each question carries 1 weightage

1. Sketch the graph of the function $f(x_1, x_2) = x_1$.
2. Let $f: U \to R$ be a smooth function, where $U \subset R^{n+1}$. Prove that the gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.
3. For what values of c is the level set $f^{-1}(c)$ an n-surface,where $f(x_1, x_2) = {x_1}^2 + {x_2}^2$.
4. Define spherical image of an oriented n-surface.
5. Find the velocity and acceleration of the parametrized curve $\alpha(t) = (\cos 3t, \sin 3t)$.
6. Let **X** and **Y** be smooth vector fields along the parametrized curve $\alpha: I \to R^{n+1}$.Verify that $(X \dot{+} Y) = \dot{X} + \dot{Y}$.
7. Compute $\nabla_v f$, where $f(x_1, x_2) = {x_1}^2 - {x_2}^2, v = (1,1, \cos\theta, \sin\theta)$.
8. Let $\phi: U_1 \to U_2$ and $\varphi: U_2 \to R^k$ be smooth, where $U_1 \subset R^n$ and $U_2 \subset R^m$.Verify the chain rule $d(\varphi o \phi) = (d\varphi\ o\ d\phi)$.

(8×1=8 weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. State and prove the Lagrange multiplier theorem.
10. Prove that a connected n-surface S in R^{n+1} has exactly two orientations.
11. Let S be a compact connected oriented n-surface in R^{n+1} exhibited as a level set $f^{-1}(c)$ of a smooth function $f: R^{n+1} \to R$ with $\nabla f(p) \neq 0$ for all $p \in S$. Prove that the Gauss map maps S onto the unit sphere S^n.

Unit 2

12. Show that a parametrized curve α in the unit n-sphere is a geodesic if and only if α is of the form $\alpha(t) = \cos at\, e_1 + \sin at\, e_2$ where $e_1$and e_2 are orthonormal vectors in R^{n+1}.
13. Let S be an n-surface in R^{n+1}, let $p, q \in S$ and let α be a piecewise smooth parametrized curve from p to q. Then prove that the parallel transport $p_\alpha: S_p \to S_q$ along α is a vector space isomorphism.
14. Show that the Weingarten map of the n-sphere $x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = r^2$ of radius r is multiplication by $\frac{1}{r}$.

Unit 3

15. Let V be a finite dimensional vector space and let $L: V \to V$ a self-adjoint linear transformation on V. Prove that there exists an orthonormal basis for V consisting of eigen vectors of L.

16. Find the Gaussian curvature of the parametrized torus φ in R^3 represented by $\varphi(\theta, \phi) = ((a + b\cos\phi)\cos\theta, (a + b\cos\phi)\sin\theta, b\sin\phi)$.

17. Let S be a compact, connected oriented n-surface in R^{n+1} whose Gauss Kronecker curvature is nowhere zero. Prove that the Gauss map $N: S \to S^n$ is a diffeomorphism.

(6×2=12 weightage)

Part C

Answer any two questions. Each carries 5 weightage.

18. (a)Let X be a smooth vector field on an open set $U \subset R^{n+1}$ and let $p \in U$. Prove that there exists a maximal integral curve of X passing through the point p.

 (b) Find the integral curve through p = (1,0) of the vector field **X** on R^2 given by $X(x_1, x_2) = (x_1, x_2, -x_2, x_1)$.

19. Prove that the Weingarten map at each point p of an oriented n-surface in R^{n+1} is self-adjoint; that is $L_p(v).w = L_p(w).v$

 (b) Let C be the circle $f^{-1}(r^2)$, where $f(x_1, x_2)=(x_1 - a)^2 + (x_1 - b)^2$, oriented by the inward normal $\frac{-\nabla f}{\|\nabla f\|}$. Prove that the curvature is $\frac{1}{r}$ at each point.

20. Let C be a connected oriented plane curve and let $\beta: I \to C$ be a unit speed global parametrization of C. Prove that β is either one to one or periodic. Also show that β is periodic if and only if C is compact.

21. a) Let S be an n-surface in R^{n+1} and let $f: S \to R^k$. Then prove that f is smooth if and only if $f o \varphi: U \to R^k$ is smooth for each local parametrization $\varphi: U \to R^k$.

 b) State and prove the inverse function theorem for n-surfaces.

(2 × 5 = 10 weightage)

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Name: ....

. FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Mathematics Degree Examination, April 2023

MMT4E14 – Computer Oriented Numerical Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 15

PART A (1 Weightage each)

(Answer all questions)

1. Describe various data types in Python.
2. Which are the statements that controls the flow of the program in Python.
3. How will you create, read and write files using Python?
4. Explain try-except and try-finally statements in Python.

PART B (2 Weightage each)

(Answer any Three)

5. Write a Python program to find the root of an equation using Newton Raphson method.
6. Write a Python program for differentiation when the function is given as a set of n points
7. Write a Python program to find $\int_0^1 \frac{1}{1+x^2}dx$ using Trapezoidal rule.
8. Write a Python program to solve $10x+y+z=12$, $x+10y+z=12$, $x+y+10z=12$, using Gauss-Seidel Method
9. Write a Python program to fit a polynomial using Newton Interpolation.

PART C (5 Weightage each)

(Answer any One)

10. Write a Python program to solve the initial value problem $f(x,y)=\frac{x-y}{x+y}$, $y(0)=1$ using Runge-Kutta Method of order 4.
11. Write a Python program for Triangular factorization.