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1M2A23113

(Pages : 2)

Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2023

MST2C06 – Probability Theory – II

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

Part A (Short answer type questions)

Answer any four questions. Each question carry weight two.

1. Define characteristic function of a random variable and show that it is uniformly continuous.
2. Prove that a characteristic function is real if and only if the associated random variable is symmetric about the origin.
3. State Kolmogorov inequality. How it is related to Chebyshev's inequality.
4. Explain how law of large numbers, central limit theorem and law of iterated logarithm differ. State Kolmogorov law of iterated logarithm.
5. State Lindberg-Feller central limit theorem. Deduce Linberg-Levy theorem from it
6. When do you say that a distribution is infinite divisible. What is its significance in central limit problem?
7. Define conditional expectation. Establish its linearity property.

Part B(Short essay type questions.)

Answer any four questions, each question has weight three.

8. State and prove inversion theorem of characteristic function.
9. Derive the characteristic function of a standard Cauchy distribution.
10. Establish Kolmogorov three series criterion.
11. For a sequence of IID random variables show that existence of expectation is a sufficient condition for applying law of large numbers.
12. State and prove Liapunove central limit theorem.
13. Establish Glivenko-Cantelli theorem.
14. If $\{x_n\}$ is a sequence of sub-martingale then for any $\alpha>0$, show that

    $P[\text{Max}(X_1,X_2,\ldots,X_n \geq \alpha)] \leq E[\,|X_n|\,]/\alpha$.

Part C (Long essay type questions)

Answer any two questions. Each question carries five weights.

15. State and prove Levy continuity theorem.

16. Establish Kolmogogorov strong law of large numbers for IID sequence of random variables.

17. A. Let $\{X_n\}$ be sequence of independent random variables such that $P[X_n = -n^{-\alpha}] = P[X_n = n^{\alpha}] = p$, $P[X_n = 0] = 1-2p$ $(0<p<1/2)$. For which value of α the sequence obeys the central limit theorem.

    B. If Lindberg-Feller condition is satisfied for a sequence $\{X_n\}$ of independent standard variables, prove that for every $\varepsilon>0$, as n tend to ∞

    $P[\mathrm{Max}_k |X_k| \geq \varepsilon\sqrt{n}]$ tend to zero.

18. State up-crossing inequality for martingales. Use it to prove martingale convergence theorem.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2023

MST2C07 – Applied Regression Analysis

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

PART A

Answer any four (2 weightages each)

1. Briefly explain the uses of regression.
2. Define a general Gauss Markov linear model.
3. How do you handle regression analysis when the errors are correlated?
4. Define residuals. What are the properties of residuals?
5. What is stepwise regression?
6. What is non-parametric regression and how does it differ from parametric regression methods?
7. Explain various link functions used in GLM.

(2 x 4=8 weightages)

PART B

Answer any four (3 weightages each)

8. For the simple linear regression model derive the properties of the least square estimators and the fitted regression model.
9. What do you meant by variable selection problem? Discuss the criteria for evaluating subset regression models.
10. Find the least square estimate of μ_1, μ_2 from the observational equations, $E(Y_1) = \mu_1$, $E(Y_2) = \mu_1 + \mu_2$, and $E(Y_3) = \mu_2$.
11. Explain various probability plots to examine the normality assumption in regression analysis.
12. What are orthogonal polynomials? Describe how it is used in regression Analysis.
13. Describe Poisson regression model. Discuss the inferential procedures of this model?
14. Derive the maximum likelihood estimator of generalized linear model.

(3x 4=12 weightages)

PART C

Answer any two (5 weightages each)

15. What do you meant by a multiple linear regression model? Derive the unbiased estimates of the model parameters along with their variances.
16. Discuss the state of affairs and consequences on account of possible departures from the underlying assumptions on a linear model.
17. Explain polynomial regression in one and several variables.
18. Discuss logistic regression models. How will you estimate the parameters in this model?

(5x2=10 weightages)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2023

MST2C08 – Estimation Theory

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

PART A(Short Answer type)

(Answer any 4 questions. Weightage 2 for each question)

1. Define sufficiency. Obtain a sufficient statistic for θ in $U(0, \theta)$ based on a random sample of size n.
2. Let $X_1, X_2, ..., X_n$ be a random sample from $Bernoulli(p)$. Obtain an unbiased estimator of p^2.
3. Define Fisher information matrix. Obtain the information matrix for $N(\theta, \sigma^2)$.
4. Obtain the MLE of θ based on a random sample of size 'n', from the population with p.d.f. $f(x) = \frac{1}{2}e^{-|x-\theta|}$, $-\infty < x < \infty$.
5. State Cramer-Huzurbazar Theorem.
6. Define informative and non-informative priors. Give examples.
7. Distinguish between UMA and UMAU confidence intervals.

($4 \times 2 = 8$ weightage)

PART B(Short Essay type questions)

(Answer any 4 questions. Weightage 3 for each question)

8. Prove that "every one to one function of a sufficient statistic is also sufficient". Give an example to show that in general a continuous function of a sufficient statistic need not be sufficient.
9. State Cramer-Rao inequality. Find the Cramer-Rao lower bound for the variance of unbiased estimators of θ in exponential distribution with mean θ.
10. Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$. Show that $(\prod_{i=1}^{n} X_i)^{\frac{1}{n}}$ is a consistent estimator of $\frac{\theta}{e}$.
11. Define CAN estimator. Find the CAN estimator of $e^{-\lambda}$ when $X \sim P(\lambda)$.
12. Explain the method of moment estimation. Obtain moment estimators of the parameters in $Gamma(\alpha, \beta)$ based on a random sample of size n.

13. Define Bayes estimator. show that under squared error loss Bayes estimator is the mean of the posterior distribution.

14. Explain the concept of shortest length confidence interval. Obtain the shortest confidence interval for θ from $U(0, \theta)$ in terms of the order statistic $X_{(n)}$.

($4 \times 3 = 12$ weightage)

PART C (Long Essay type questions)
(Answer any 2 questions. Weightage 5 for each question)

15. Define complete sufficiency and minimal sufficiency. Prove that a complete sufficient statistics is minimal sufficient. Is the converse true? justify your claim.

16. State and prove sufficient conditions for an estimator to be consistent. Obtain a consistent estimator of μ^2 in $N(\mu, \sigma^2)$ based on a random sample of size n.

17. By stating the necessary regularity conditions prove the asymptotic normality of MLE.

18. Define unbiased confidence intervals. Consider a random sample of size n from exponential distribution with mean λ. Obtain an unbiased confidence interval of λ.

($2 \times 5 = 10$ weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, April 2023

MST2C09 – Stochastic Process

(2022 Admission onwards)

Time: 3 hours

Max. Weightage : 30

PART A

Answer any four (2weightage each)

1. Define a Stochastic process. Explain the classification with the help of an example.
2. Show that states of a one-dimensional symmetric random walk are recurrent.
3. Explain open system
4. Explain Compound Poisson Process
5. Show that renewal function satisfies the renewal equation.
6. Derive the backward differential equation satisfied by a birth and death process
7. Explain the significance of Little's formula in queuing theory

(4x2=8weightage)

Part B

Answer any four (3 weightage each)

8. Derive Poisson Process
9. Define periodicity. Show that it is a class property.
10. Explain how Poisson process is related to binomial and uniform distribution.
11. State and prove Chapman-Kolmogorov equation.
12. State and prove elementary renewal theorem
13. (a) Derive Wald's equation

    (b) Show that the renewal function $m(t) = \sum_{n=1}^{\infty} F_n(t), \forall t$, where $F_n(t) = P(S_n \leq t), n \geq 1, \forall t$
14. Describe M/G/I queueing system.

(4x3=12weightage)

PART C

Answer any two (5 weightage each)

15. (a) State and Prove ergodic theorem of Markov Chain.

    (b) Explain Gamblers ruin problem. Derive Ultimate winning of the Gambler.
16. Explain Yule–Furry Process. Find its probability distribution. Hence or otherwise find its mean and variance.
17. Find the steady state probability distribution for M/M/S queue.
18. (a) State and prove central limit Theorem on Renewal process.

    (b)Explain Renewal reward process

(2x5=10 weightage)