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Name: ……………………….

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2023

MST1C01 – Analytical Tools for Statistics – I

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

PART A

(Answer *any four* questions. Each carries 2 weightage)

1. If f is a bounded function and α is monotonic increasing on [a,b], then define the terms (i) Partition (ii) Refinement of partitions and (ii) Lower and Upper sums, of f with respect α .
2. Find $\int_1^{5.3}[x]\,dx$.
3. Define uniform convergence of (i) a sequence of functions and (ii) a series of functions.
4. State the Cauchy criterion for the convergence of a sequence of functions.
5. State the inversion theorem of a multivariable function.
6. Define (i) directional derivative and (ii) total derivative of a multivariable function.
7. Given L{ F(t)}= f(s) then find L{ e^{at} F(t)}.

(4*2=8 weightage)

PART B

(Answer *any four* questions. Each carries 3 weightage)

8. If $f \in R(\alpha)$ on [a,b] and $f \in R(\beta)$ show that $f \in R(c_1\alpha + c_2\beta)$ and $\int_a^b f\, d(c_1\alpha + c_2\beta) = c_1\int_a^b f\, d\alpha + c_2\int_a^b f\, d\beta$, where c_1, c_2 are constants.
9. Discuss on the convergence of the series $\sum_{n=1}^{\infty}\frac{n^2-1}{n^2+1}x^n, x > 0$.
10. Show that $\{f_n(x)\}$, where $f_n(x) = \frac{nx}{1+n^2x^2}, 0 \le x \le 1$, cannot be differentiated term by term at x=0.
11. Examine the function $x^2 + y^2 + x + y + xy$ for maximum and minimum.
12. If $f(x,y) = x^3 - y^2, x = e^t\cos t, y = \cos t + \sin t$, find $\frac{df(x,y)}{dt}$.
13. If the Laplace transform $\mathrm{L}\{\mathrm{F(t)}\} = \mathrm{f(s)}$, then show that $\mathrm{L}\{\mathrm{t^nF(t)}\} = (-1)^n\frac{d^n}{ds^n}\mathrm{f(s)}$.
14. Find inverse Laplace transform of (i) $\frac{1}{(s+a)(s+b)}$ and (ii) $\frac{1}{s^2(s+1)^2}$.

(4*3=12weightage)

PART C

(Answer *any two* questions. Each carries 5 weightage)

15. (a) If f is continuous on [a,b] and α is of bounded variation on [a,b], show that $f \in R(\alpha)$ on [a,b] .

(b) State and prove the first mean value theorem of Riemann Stieltjes integral.

16. (a) Prove that if a sequence $\{f_n\}$ converges to f uniformly on [a,b], and each function f_n is integrable, then f is integrable on [a,b] and the sequence $\int_a^b f_n dt$ converges uniformly to $\int_a^b f\,dt$ on [a,b].

(b) Test for the uniform convergence of $\{f_n\}$ where $f_n(x) = e^{-nx}, x \geq 0$.

17. (a) Check the continuity at (0,0) of $f(x,y) = \begin{cases} x\,y\,\frac{x^2-y^2}{x^2+y^2} & when\ (x,y) \neq (0,0) \\ 0 & when\ (x,y) = (0,0) \end{cases}$

(b) Show that the function $f(x,y,z) = 3\log(x^2+y^2+z^2) - 2(x^3+y^3+z^3)$ has only one extreme value , $\log\left(\frac{3}{e^2}\right)$.

18. (a) If the Laplace transform $L\{F(t)\} = f(s)$,then show that,

$L\{F^{(n)}(t)\} = s^n f(s) - s^{n-1}F(0) - \quad s^{n-2}F'(0) - \cdots - F^{n-1}(0)$ (in usual notations).

(b) Determine the Laplace transform of (a) $(1 + te^{-t})^3$ (b) $(t^2 + 1)^2$.

(2 x 5=10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2023

MST1C02 – Analytical Tools for Statistics – II

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any four questions. Weightage 2 for each question)

1. Define vector space and give one example.
2. Define basis and dimension of a vector space.
3. Define unitary matrix. Show that determinant of a unitary matrix has modulus 1.
4. Define Row and column space of a matrix.
5. Define null space and state rank-nullity theorem.
6. Briefly describe Spectral representation of a real symmetric matrix.
7. Define the rank, signature and index of a real quadratic form. State the inter-relationship between them, if any.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Define linearly independent and linearly dependent vectors. Check whether the vectors $(2,3,-1,-1), (1,-1,-2,-4), (3,1,3,-2), (6,3,0,-7)$ are linearly dependent.
9. Describe Gram-Schmidt orthogonalization process.
10. Describe the method of finding inverse of a matrix by partition.
11. Show that the sum of the characteristic roots of the matrix A is the trace of A and the product of the characteristic roots of A is the determinant of A.
12. Define geometric and algebraic multiplicities of a matrix. Prove that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.
13. Define g-inverse of matrix A. Find g-inverse of $A = \begin{bmatrix} 4 & 2 & 1 \\ -1 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.
14. Briefly describe definiteness of a quadratic form. Classify the following quadratic form $3{x_1}^2 + {x_2}^2 + 5{x_3}^2 - 3x_1x_2 + 8x_1x_3 + 10x_2x_3.$

(4 x 3= 12 weightage)

Part C
Long Essay Type questions
(Answer any two questions. Weightage 5 for each question)

15. (a) Define vector subspace. If $\mathcal{W}_1$ and $\mathcal{W}_2$ are two subspaces of a vector space $\mathcal{V}$, then prove that $\mathcal{W}_1 \cap \mathcal{W}_2$ is a subspace of $\mathcal{V}$.

    (b) Show that $\mathcal{W}_1 \cup \mathcal{W}_2$ is a subspace if and only if one is subset of the other.

16. Define inner product. State and Prove Cauchy-Schwartz's inequality.

17. (a) Define rank of a matrix. If r(A) is rank of A then show that

$$r(A) + r(B) - n \leq r(AB) \leq \min(r(A), r(B)).$$

    (b) Also show that $r(A + B) \leq r(A) + r(B)$.

18. (a) State and prove the necessary and sufficient condition that a real quadratic form is positive definite.

    (b) Define Moore-Penrose inverse of a matrix and show that it is unique.

(2 x 5= 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2023

MST1C03 – Probability Theory – I

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A: Short Answer Type Questions

(Answer any four questions. Weightage 2 for each question.

1. Differentiate between discrete and continuous random variables. Give an example for each .Examine whether the following is a distribution function.

   $F(x) = 0, \quad x \leq 0$

   $\quad\quad = 1, \quad x > 0.$

2. Define a sigma field. By an example show that union of two sigma field need not be a sigma field.
3. Explain sigma field generated by a class of sets. Illustrate it by a simple example.
4. Discuss the concept of Lebesgue measure and Lebesgue –Stieltjes measure.
5. Describe the concept of product measure
6. State and prove Jensen's inequality.
7. State convergence in probability and almost sure convergence. By an example show that convergence in probability need not imply almost sure convergence.

Part B: Short essay/problem solving.
(Answer any four questions, weightage three for each question.

8. Show that a distribution function is right continuous. Also show that a distribution function can have at the most a countable number of discontinuity points.
9. State and prove correspondence theorem associated with a distribution function.
10. Obtain the moment generating function of a multinomial random vector. Use it to obtain its first and second order moments.

11. Define convergence in probability and convergence in distribution. How they are related? Establish.
12. State and prove basic in equality.
13. If A_n is sequence of independent events show that $P(\limsup A_n)$ is either equal to zero or one.
14. State and prove Helly's theorem on a sequence of distribution function.

Part C: long essay

Answer any two questions. Weightage five for each question.

15. What you mean by sigma field induced by a random variable and a sigma field generated by a sequence of random variables. Use it to obtain tail-sigma field. Also establish Kolmogorov 0-1 law.
16. State and prove Holder's inequality .Use it to obtain Liapunov inequality.
17. Show that every distribution function can be written as the convex combination of a step function and a continuous distribution function. Use It to obtain the decomposition of

$$
\begin{aligned}
F(x) &= 0, && x<0 \\
&= e^{-\mu}, && 0\leq x<1 \\
&= e^{-\mu} + (1-e^{\mu})(1-e^{-(x-1)}), && x\geq 1
\end{aligned}
$$

18. State and prove Slutstkey's theorem. Also show that convergence in probability implies convergence in law.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2023

MST1C04 – Distribution Theory

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Answer any four (2 weightages for each)

1. Determine the MGF of the Negative binomial distribution. Check that it has the additive property.
2. Define power series distributions. Obtain its mean.
3. Derive beta type II distribution from Gamma variates.
4. Define Lognormal distribution. Find the rth order raw moment of if
5. What do you mean by Mixture distributions? Write an example.
6. Derive pmf of rth order statistic of Geometric variate.
7. Define noncentral F statistic **(2 x 4=8 weightages)**

Part B

Answer any four (3 weightages each)

8. Demonstrate that the Binomial distribution tends to the Poisson distribution under specific conditions (to be described).
9. State and prove renovsky formula
10. Let X and Y be independent random variates. Then X + Y follows Normal if and only if both X and Y follows Normal.
11. Let X and Y are independent Gamma variates, then show that both $\frac{X}{Y}$ and $\frac{X}{X+Y}$ are independent to $X + Y$
12. Let $f(x)$ and $F(X)$ are PDF and CDF of random variable X. Then derive the distribution of range. Also, find the distribution of range when X follows an exponential distribution.
13. Derive non-central t distribution.
14. Derive the MGF of chi-square random variate. Check that is holds additive property

(3 x 4=12 weightages)

Part C

Answer any two (5 weightages each)

15. **A)** A box contains N identical balls numbered 1 through N. Of these balls, n are drawn at a time. Let $X_1, X_2, \dots, X_n$ denote the numbers on the n balls drawn. Let $S_n = \sum X_i$. Find $Var(S_n)$.

    B) Show that geometric distribution possesses Lack of memory property

16. Let $X \sim N(0,1)$, derive the rth order central moments

17. **A)** State and prove Cauchy-Schwarz inequality.

    B) Justify that for any two random variables X and Y, X^2 and Y^2 are independent implies X and Y are independent.

18. Let $X \sim N(0,1)$, Then find the distribution of X^2 and $\sum(X_i - \bar{X})^2$.

(5 x 2=10 weightages)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2023

MST1C05 – Sampling Theory

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Short Answer Type Questions

Answer any four questions. (Weightage 2 for each question)

1. Write about Sampling frame. Explain various defects associated with it.
2. Write about systematic sampling.
3. Write about Multistage Sampling.
4. Explain Murthy's unordered estimator
5. Define ratio estimator. Derive its bias.
6. If the regression on Y on X is perfectly linear, the variance of the regression estimator becomes zero. Is it true? Prove.
7. Write a note on proportional allocation.

(4 x 2 = 8 weightage)

Part B

Short Essay Type / Problem solving type questions

Answer any four questions. (Weightage 3 for each question)

8. Obtain an unbiased estimate of population mean in simple random sampling without replacement. Find the variance of the estimate.
9. Obtain the mean and its variance in equal cluster sampling
10. Show that for a population with linear trend $V_{st} : V_{sy} : V_{ran} \approx 1/n : 1 : n$
11. Derive Hartley - Ross unbiased ratio type estimator.
12. Derive Neyman allocation.
13. (a) Write about probability sampling

    (b) What are non -sampling errors? Explain its sources
14. Write about π **ps** sampling.

(4 x3=12 weightage)

Part C

Long Essay Type questions

Answer any two questions. (Weightage 5 for each question)

15. (a) Prove that $V(ran) \geq V(prop) \geq V(opt)$.

    (b) Explain the principles of stratification.

16. (a) Differentiate between Cumulative Total Method and Lahiri's method. Explain them with the help of an example.

    (b) Explain the general selection procedure in PPS sampling.

    (c) Compute the gain due to PPS sampling with replacement compared to simple random sampling.

17. (a) Derive the sampling variance of regression estimator.

    (b) Differentiate between Hansen & Hurwitz method and Politz- Simmon's technique.

18. (a) Show that in SRSWOR Sample mean $\bar{y}$ is the BLUE of $\bar{Y}$.

    (b) Give any three estimators of population mean in cluster sampling where clusters are of unequal size and discuss their properties.

(2 x5=10 weightage)