Land Measurement Methods and the Use of Fractions During the Chola Period in South India*

V. Selvakumar

Department of Epigraphy and Archaeology Tamil University, Tanjavur, Tamil Nadu E-mail: selvakumarodi@gmail.com

Abstract

The medieval period saw advancement in land measurement methods, since land revenue was important for land administration. The inscriptions of the medieval period present various land measurement units and their symbols. The calculation of land area, taxable area and non-taxable area is mentioned in detail in these inscriptions. In the process various fraction units are mentioned in the inscriptions. This paper discusses some of the fractions and the methods of calculations in the medieval period in Tamil Nadu.

Keywords: Traditional Arithmetic, Chola Administration, Land Measurement, Fractions.

I. Introduction

Land measurement was an important aspect of medieval administration in India, since the states (governments) derived most of their income through land revenue. With the development of an organized administrative system, from the medieval period in South India, proper measures were adopted to accurately measure lands and to standardize the existing land measurement system, in order to collect land revenue. Questions such as 'How was land measured?' 'How was it assessed for tax?' and 'What was the ratio of tax in relation to the total productivity of land?' have been addressed by scholars (Subbaravalu, 2001a; 2001b; 2001c). However, more research in this area is necessary in order to understand the development and use of assessment methods, and especially the scientific concepts and operations related to measurement that were used during the medieval period. The land measurement system also has implications for understanding the development of history of science in India. This paper presents a few observations on the land measurement system under the Cholas.

In the medieval inscriptions, the measurement rods, measurement units of the total area of land, tax free area and amount of tax in coin

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or kind are frequently referred to. However, often only the area is mentioned without mentioning the size of the measurement rod used. The inscriptions mention the total area of a village and the areas that were under settlements, highways, canals, cremation grounds and tanks, which were exempted from tax, are mentioned, and after deducting the tax exempted lands, the taxable area is mentioned very accurately. Simple arithmetic operation to convey the taxable, nontaxable and total land area is found in many inscriptions. Comprehending this system of simple arithmetic calculation is not easy for many people, and only a few researchers of the modern times understand this system. The traditional measurement rods called kols or dandas, which were based on measurement units such as finger-breadth (angulam or viral), span (vitasti or cān in Tamil), foot (pāda or ati) and cubit (hasta or muzham), were used in the medieval times for measuring lands. The dimensions of these measurement rods are marked on the temple walls and on rocks in the remote areas in South India (Selvakumar, 2014; 2015). These measurement rods were named after their dimensions (e.g. 8 span rod, 16 span rod and 18 foot rod) or after the kings and their titles (e.g. ulagalantān kol and kaņdarakandan kol).

The land measurement system of the medieval times was very well developed across India and people in many parts of India were using locally defined measurement units for the calculation of an area. This knowledge still continues in the villages. The tradition of using body-part based measurement units for linear measurement perhaps goes back to the Harappan times, as evidence of Harappan scales have been found (Balasubramanian and Joshi, 2008). The Vedic texts mention about the various measurement units in the Sulba sutras (Sen and Bag, 1983) and the texts including the Artasāstra (Rangarajan, 1992) and those on architecture describe the various measurement units that were employed in ancient India. In Tamil Nadu, the late Medieval Tamil work of Kanakatikāram composed by Kāri Nāyanār describes the various measurement units (Kāri Nāyanār nd). These measurement units were taught in the schools of Madras Presidency during the colonial times (Babu, 2007). In Tamil Nadu, people belonging to the older generation, especially those who were educated in the first half of the twentieth century still remember the calculations based on fractions and the traditional units of measurements.

The inscriptions serve as a valuable source for understanding the measurement units and their diversity during the medieval period. The land areas were documented in the inscriptions, based on various units of fractions, and different varieties of measurement rods. References to these measurement rods are found in many parts of India. A few studies have been conducted on the land measurement methods of the Cholas, especially by P. Shanmugam (1987), Y. Subbarayalu (2001a; 2001b; 2001c), N. Karashima (2006), Kodumudi Shanmugan (2007) and Selvakumar (2014; 2015).

The use of very minute fractions as part of "Kīzh kaṇakku" system is found in the inscriptions and they were in use till the advent of the colonial times (Bhagavathy, 2003) and with the introduction of the modern education and mathematics, these traditional measurements and fractions lost their importance in education. Now there is an urgent need to reuse and teach these traditional measurements and fractions in order to improve the cognitive skills of the students.

II. The Measurement Units of the Medieval Period

In the medieval inscriptions, the measurement of land is conveyed in terms of a whole unit, called *véli*, and then the smaller units are defined in relation to the *véli*, as fractions. The fractions used in the inscriptions are *mukkāl* (three fourth, 3/4), *arai* (half, 1/2), *kāl* (1/4 or quarter), *araikkāl* (1/8), *mā* (1/20), *kāņi* (1/64 or 1/80) and *muntiri* (1/320 or 1/256).

Kōl or *Danda*

Kol means a measurement rod in Tamil, and in Sanskrit it is known as *danda*. These terms refer to wooden measurement rods or scales or poles. The measurement rods were fundamental for the measurement of lands, and they were devised on the basis of smaller units such as angulam, piti, cān, ati and muzham. Their length varied from 8 piti to 22 cān. The length of these rods was marked on the temple walls and on rocks using symbols such as "+", "|" and tridents. The staff members of the village administration selected a wooden pole, probably bamboo or from other trees, and then cut the size of the pole according to the measurement marked on the temples or on the rocks in the remote areas of the villages. These measurement rods, which were used for land survey, were called according to their size or after the titles of the kings. Many villages had their own measurement rods, and there was no uniform measurement rod across medieval Tamil Country. An area covered by one unit of measurement rod in length and width formed a square, and it was called kuzhi in Tamil, i.e. one square rod. This kuzhi was the basic unit of land measurement used in the medieval times

Véli

Véli is a larger, whole unit of measurement of land, and this unit is still used by people in Tanjāvūr region, although for legal documents related to land ownership, they use metric measures. A véli is about 6.66 acre of land in Tanjāvūr region. Vėli means fence in Tamil; but, for area calculation a *véli* consists of 20 *mā* units of land. We do not know why this unit was named as *véli*; perhaps, it meant a unit of fenced or defined area. While quantifying lands, véli was used as a main unit and the smaller areas were conveyed as fractions of this main unit. A véli has 20 mā units, and 2000 kuzhi units (according to one type of calculation), means 20 by 100 or 200 by 10 or 40 by 50 kuzhi units, and it forms a rectangle shape. However, interestingly the term véli is not often directly mentioned in the inscriptions, only the number of whole véli units are mentioned, e.g. 93 3/4 means 93.75 véli. Although a véli refers to a specific area, it is not necessary that in all contexts véli refers to same extent of land. There are references to different véli units with their area varying from 2000 kuzhi to 10240 kuzhi units (Subbarayalu, 2001a: Table A p. 38), based on a 8 piti (bow-grip, which is the width of four fingers) rod or 12 span, 12 foot, 16 span, 16 foot and 18 span and 18 foot rods. Therefore, a véli in one region based on a 12 span rod would be different from a véli based on a 16 span rod.

Mā

 $M\bar{a}$ refers to the fraction of 1/20. In the context of area measurement, it refers to the 1/20 of a *véli*. In modern day convention in the Lower Kāveri valley, a $m\bar{a}$ consists of 100 *kuzhi* units, which means a square of 10 units x 10 units = 100 *kuzhi* units. Therefore, one type of $m\bar{a}$ unit forms a perfect square shape. Twenty $m\bar{a}$ units form one *véli*. The size of $m\bar{a}$ also depends upon the nature of the measurement rod and the number of *kuzhi* units. It should be remembered that 100 *kuzhi* units forming one $m\bar{a}$ was used only in certain regions. There are references to 128 *kuzhi* units forming one $m\bar{a}$. It appears that sometimes, a 16 foot rod or an 8 foot rod or any other rods were used to arrive at the unit numbers such as 128, 256, 512 and 1024. Hence the size of the $m\bar{a}$ unit was also not uniform on the ground. However, it appears that 100 *kuzhi* as one $m\bar{a}$ became an accepted standard at a later context.

Kāni

 $K\bar{a}ni$ refers to the fraction of 1/80. Four $k\bar{a}ni$ units make one $m\bar{a}$. Araikk $\bar{a}ni$ (half $k\bar{a}ni$) is 1/160 and eight araikk $\bar{a}ni$ units make one $m\bar{a}$. The term $k\bar{a}ni$ appears in the Sangam texts; in Nalatiyar, there is a reference to "muntiri mér k $\bar{a}ni$," which means that a $k\bar{a}ni$ is larger than a *muntiri*; however, no direct reference is present to specify the exact size of the fractions here. Interestingly, there seems to be another fraction associated with $k\bar{a}ni$; N. Subrahmanian (1966) in his *Pre-Pallavan Tamil Index* lists 1/64 as the meaning of $k\bar{a}ni$. Similarly, in the context of Karnataka, Jagadish and Rajaram Hegde (2012) list 1/64 as representing a $k\bar{a}ni$. In the contemporary usage, while $m\bar{a}$ and $v\acute{eli}$ are very frequently used, $k\bar{a}ni$ is rarely used in the Lower Kāveri valley, although it is also known to people. Like the other units, the size of $k\bar{a}ni$ depends upon the size of the measurement rod.

Muntiri

Muntiri is the smallest unit in the series of traditional fractions, and it refers to 1/320 of a *véli*. Four *Muntiri* units form one *kāni*, 16 *muntiri* units form one *mā* and 320 *muntiri* units form one *véli*. The fractions below *muntiri* are called *kīzh* and the fractions that are smaller than *muntiri* are conveyed by using *kīzh* (*Kīzh*= i.e. those units which are below *muntiri*; *kīzh* means below) along with units such as *mā*, *mukkāl*, *arai*, *kāl*, and *araikkāl*. The unit of *muntiri* is not commonly used by people, like the three above-mentioned categories. Sometimes, *muntiri* is given a meaning of 1/256 (Subrahmanian, 1966). The size of *muntiri* depends upon the size of the *véli* and the system of calculation.

Kuzhi, the basic unit

Kuzhi in Tamil means a pit. *Kuzhi* is a square unit and it is the Tamil equivalent for *caturam* in Sanskrit; it covers an area of one rod by one rod in size. A *véli* consists of 2000 *kuzhi* units, according to traditional accounts. It is not clear as to why 2000 *kuzhi* units were considered to have formed one larger unit of *véli*. Interestingly, the units of 2000 *kuzhi* do not yield a perfect square root, and it can be framed by a rectangle of 20 $k\bar{o}l \ge 10 k\bar{o}l$ units. The actual size of a *kuzhi* depends upon the size of the measurement rod.

The nature of Fractions

In the context of land measurement, the fractions are reduced in the series of 1/20, 1/40, 1/80, 1/160 and 1/320, and they appear as multiples of 20, 40, 80, 160 and 320. A *véli* is divided into 320 *muntiri* units. Why 320? Why 1/320 has to be a unit? The answer perhaps lies in the dimension of the measurement rods. The measurement rods of 16 span/foot were frequently used during the Chola times and hence, the multiples of 16 span rod and decimal based numbers of 10 and 20 resulted in 320 square units. However, we are not sure, if the 16 feet rod was used in the Lower Kaveri valley and it might have been used at least in some contexts. It appears that multiples of 8 as 16, 32, 64, 128 and 256 were also used as units or sub-units of area measurement. The area unit of $m\bar{a}$ units is defined as consisting of 100 or 128 or 256 or 512 units.

Kīzh Fractions below muntiri, i.e. below 1/320

The $K\bar{i}zh$ fractions are very interesting in the inscriptions and Karashima argues that this system was introduced by Rajaraja I to standardize the assessment of land for taxation (2006), and he has established a connection between *matakku* and $k\bar{i}zh$. After the smallest unit of fraction *Muntiri*, $K\bar{i}zh$ is used in the inscriptions. $K\bar{i}zh$ fractions are the fractions below, i.e. smaller than 1/320. $K\bar{i}zh$ arai, $K\bar{i}zh$ araikkal and $K\bar{i}zh$ mukkal are some of the smaller fractions. If $K\bar{i}zh$ appears after $K\bar{i}zh$ it means 1/320 x 1/320. In the Medieval inscriptions, sometimes, $K\bar{i}zh$ is repeated thrice and in a rare case it appears four times.

 $K\bar{i}zh = K\bar{i}zh$ refers to the fractions below 1/320

Kīzh Muntiri refers to the fraction of = $1/320 \ge 1/102,400$

Kīzh Kīzh Muntiri = 1/320 x 1/320 x 1/320 = 1/3,27,68,000

 $K\bar{i}zh K\bar{i}zh K\bar{i}zh Muntiri = 1/320 x 1/320 x 1/320 x 1/320 = 1/10,48,57,60,000$

Very minute fractions are mentioned in the Tanjāvūr temple inscriptions. A unit of the last series $K\bar{i}zh K\bar{i}zh K\bar{i}zh Muntiri \ge 1/2$ is used in the Tanjāvūr temple inscriptions and it is considered to be the smallest unit found in the inscriptions. It refers to the unit of 1/5242,8800000 of a *véli*, according to Venkayya (1913).

Nature of Minute Fractions

Noboru Karashima points out that such small fractions were the results of conversion of old units into standard units as part of standardization (2006). These fractions are so minute that one wonders why they had to measure such micro units. We do not know if these minute units had any significance in land assessment. But, before going into this aspect let us understand the actual dimension of these micro units.

Conversion of Fractions

In order to understand these fractions, we have to convert them to metric measures. There were different measurement rods and we are not sure which rod was used for the determination of a *kuzhi* in an area unless there is a specific reference. According to the traditional account, in the Lower Kāveri valley, a *kuzhi* consists of 12 feet by 12 feet = 144 sq. ft. In some cases, it appears that 16 feet rod was used leading to 16 feet x 16 feet = 256 sq ft.

Land Measurement Methods and Use of Fractions

a) Calculation based on 16 span rod and 100 kuzhi = 1 $m\bar{a}$

Calculation up to kīzh muntiri

1 *kuzhi* by 16 span rod = 16 x 16 square $c\bar{a}n$ = 256 square $c\bar{a}n$

If $1 \ m\bar{a} = 100 \ kuzhi$, then 256 x 100 = 25,600 square $c\bar{a}n$

20 $m\bar{a} = 1 \ v\acute{e}li = 512,000 \ square \ c\bar{a}n$

Therefore, one *muntiri* of a *véli* is 512,000/320 = 1600 square $c\bar{a}n$, which is 40 x 40 square $c\bar{a}n$.

There are references to fractions further below *muntiri*, i.e. $k\bar{i}zh$ *muntiri*, which is below 1600 square $c\bar{a}n$.

Kīzh muntiri equals to $1/320 \ge 1/320 = 1/320$ of 1600 square $c\bar{a}n = 5$ square $c\bar{a}n$.

From the above account, measuring an area up to 5 square $c\bar{a}n$ is very much possible and practicable, and hence the unit of $k\bar{i}zh$ muntiri of a véli appears very relevant.

If we have to go further below $K\bar{i}zh \ muntiri$, for the purpose of clarity it is better go to the sub-units of a $c\bar{a}n$. We have to divide a $c\bar{a}n$ into *angulam* units, as illustrated below.

Calculation beyond kizh muntiri

As mentioned earlier, if we need to understand the measurements beyond $k\bar{i}zh$ muntiri, we have to split a $c\bar{a}n$, into its sub-unit angulam. According to convention,

1 span/ *cān* is equal to 12 *angulam*,

1 square $c\bar{a}n = 12 \times 12 = 144$ square *angulams*

Therefore, $K\bar{i}zh$ Muntiri (1/320 x 1/320) of a véli is = 144 x 5 square $c\bar{a}n$ (as illustrated above) = 720 square angulams.

Therefore, $K\bar{i}zh \ K\bar{i}zh \ Muntiri \ (1/320 \ x \ 1/320 \ x \ 1/320) = 1/320$ x 720 = 2.25 square *angulams*, which is a square of 1.5 x 1.5 square *angulams*.

Therefore, measuring such a small area is also very much feasible, and hence the use of such an unit should also be considered relevant.

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Calculation beyond kīzh kīzh muntiri
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To go beyond *kīzh kīzh muntiri*, we need to further reduce an *angulam* unit.

One *angulam* is equal to 8 *tōrai* or the width of a paddy or rice grain, according to tradition.

Then 1 square *angulam* is equal to $8 \ge 8 \ t \ \overline{o}rai = 64$ square $t \ \overline{o}rai$ units.

From the above account we know that $K\bar{i}zh K\bar{i}zh Muntiri$ (1/320 x 1/320 x 1/320) =2.25 square *angulams* x 64 = 144 square *torai* units.

Therefore, $K\bar{i}zh \ K\bar{i}zh \ K\bar{i}zh \ Muntiri$ (1/320 x 1/320 x 1/320 x 1/320 x 1/320)= 1/320 x 144 square $t\bar{o}rai$ units.

= 0.45 square *tōrai*.

Which means 1/320 X 1/320 x 1/320 X 1/320 = 1/1048,57,60,000 of a *véli* =0.45 square *tōrai*.

If an *angulam* is 1.76 cm in an average, then 1 *tōrai* is 1.76/8 =about 2.2 mm

Therefore, 1 square $t\bar{o}rai$ is = 4.84 sq mm.

Then, 0.45 square $t\bar{o}rai$ is = 2.18 square mm, which is 1.476 mm x 1.476 mm

The smallest unit mentioned in the Tanjāvūr inscription is $3/4 \times 1/20$ of *Kīzh Kīzh Kīzh Muntiri* of a véli (of 2000 *kuzhi* units); if this is based on 16 span rod, in metric measure it is = 0.081 square mm, which is practically impossible to measure on the ground, and it should have been obtained by reduction.

b) Calculation based on 12 span rod

Calculation up to kīzh muntiri

1 *kuzhi* = 12 x 12 square $c\bar{a}n$ = 144 square $c\bar{a}n$ s

1 $m\bar{a} = 100 kuzhi = 144$ square $c\bar{a}n \ge 100 = 14,400$ square $c\bar{a}ns$.

 $20 \ m\bar{a}=1 \ v\acute{e}li=288,000 \ square \ c\bar{a}ns$

Therefore, one muntiri of a véli is 288,000/320 = 900 square $c\bar{a}n$, which is 30 x 30 square $c\bar{a}n$.

There are references to fractions further below 900 square $c\bar{a}ns$ in the inscriptions.

Kīzh muntiri equals to $1/320 \ge 1/320 = 1/320$ of 900 square $c\bar{a}n$ which is equal to 2.8125 square $c\bar{a}n$ s.

If we have to go further, we have to divide $c\bar{a}n$ into its subunit angulam.

As mentioned earlier, measuring 2.8125 square + is reasonable and appears relevant.

Calculation beyond kīzh muntiri

If we need to understand the measurements beyond *muntiri*, we have to go to the units smaller than a $c\bar{a}n$, i.e. *angulam*.

1 cān is equal to 12 angulams.

1 square $c\bar{a}n = 12 \ge 12 = 144$ square *angulams*

Therefore, $K\bar{i}zh$ Muntiri (1/320 x 1/320) of a véli is = 144 x 2.8125 = 405 square *angulams*.

Kīzh Kīzh Muntiri $(1/320 \times 1/320 \times 1/320) = 1/320 \times 405 = 1.265$ square *angulams*.

Calculation beyond kīzh kīzh muntiri

To go beyond *kīzh kīzh muntiri*, we need to further and reduce an *angulam*.

If 1 angulam is equal to 8 torai,

Then 1 square angulam equals to 64 square torai units.

 $K\bar{i}zh K\bar{i}zh Muntiri = 1.265$ square $angulams \ge 64 = 80.96$ square $t\bar{o}rai$ units

Kīzh Kīzh Kīzh Muntiri = 1/320 x 80.96 square *tōrai* units.

= 0.253 square *tōrai*.

Which means 1/320 X 1/320 X 1/320 x 1/320 = 1/10485760,000 of a *véli* =0.253 square *tōrai*.

If an *angulam* is 1.76 cm in an average, 1 torai is about 2.2 mm

1 square $t\bar{o}rai$ is = 4.84 sq mm.

0.253 square $t\bar{o}rai$ is = 1.22 square mm

It is highly difficult to measure such small area on the ground.

Comments

The above analysis reveal that by using 16 feet and 12 feet rods, the medieval land surveyors could measure areas smaller than one square mm. If we use the definition of a *véli* as equal to 128 *kuzhi* units, there would be variations in these measurements. What is clear from the above discussion is that in land administration very minute fractions were used, and the medieval surveyors were very particular that even the smallest areas had to be accounted for without any exception. In addition, as we noticed earlier that various measurement rods from as small as en*-piti* to larger 22 or even 54 span rods were used during the medieval times in different micro regions. Therefore, when the ar-

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eas that were measured using smaller rods were converted to the areas that were based on longer measurement rods, such small fractions were produced. Therefore, as pointed out by Y. Subbarayalu (2001a), these minuscule fractions perhaps resulted due to theoretical conversions. The jewelry and the bronze images donated to the Brihadīswara temple at Tanjāvūr reveal the use of torai units in actual terms. Now the question is if the surveyors used these smaller units for the land survey. We could assume that the measurement rods used by the land surveyors of the medieval period had markings of angulams and even torai units. Perhaps when the dimension of the area that they measured was smaller than an *angulam*, they might have meticulously documented even the smallest unit of length. Perhaps such a rigour in measurement was dictated by the standard of surveying practices and also perhaps by the insistence that no part of land should be left out during the measurement. It is still a question if they measured the units less than an angulam for land measurement.

Area and Reduced Areas

Apart from the size of the measurement rods, several other factors also conditioned the assessment of lands for tax. The terms such as *virivu* (area), *matakku* (reduced area) and *taram* were used for land assessment.

Virivu: Expansion

The size of the cultivable fields was determined according to the length of the scale, in certain instances. An inscription from Chidambaram mentions about the (SII VIII 52). Two lands that were purchased and they each measured 512 *kuzhi* units, with a total of 1024 *kuzhi* units. The inscription mentions the units of 1024 *kuzhi* according to the convention of the village and they form 8 $m\bar{a}$, which means each $m\bar{a}$ in this village meant 128 *kuzhi* unit. In this system a *véli* had 2560 *kuzhi* units. The inscription mentions that *virivu nilam* would be 1/2 *véli* and one *kāni*, which means 10 $m\bar{a}$ and 25 *kuzhi*s; but, actually the units are one *kuzhi* less than 1025 *kuzhi*s. Hence, the term *virivu* is used here in the case of reduced size of $m\bar{a}$ from 128 to 100 *kuzhi* units, and as a result the overall area increases. In some cases, the size of land decreases; therefore, the term *virivu* means conversion to the existing scheme of measurement.

Matakku: Reduction

Matakku means 'to fold' in Tamil and it refers to reduction in the size of the area .In some cases the actual are of the land was reduced

for the purpose of tax calculation and in order to standardize. According to Karashima (2006) this term occurs from the time of Rajendra I. It referred to conversion from a type of measurement to standard unit, according to Subbarayalu (2001b), and Karashima (2006) presents evidence for this from a rare inscription of Sundarapandiyan from Chidambaram. He clearly traces the links between *matakku* and $k\bar{z}th$. In one case, with the use of *matakku*, the land size was reduced to 6.15 times, according to Subbarayalu (2001b: 55). The reference "*taramittu matakki*" occurs in an inscription. The measurement of actual land is different from the measurement that we get after *matakku*. As pointed out by Karashima, even large areas in one scale becomes fraction in another (2006) and therefore, the first case from the Chidambaram inscription was measured using a smaller scale and when the large scale or rod was used these units became very small.

Taram (Quality or standard)

Taram refers to quality of the land and it refers to productivity. According to Subbarayalu 12 standards of lands are mentioned in the inscriptions (Subbarayalu, 2001a; 2001; 2001c). The *taram* referred to the quantity of production from an area. The production rate was also taken into consideration while assessing the land for tax.

III. Discussions and Conclusions

The land measurement system of the medieval period incorporated numerous measurement rods in different micro regions. The size of the rods varied from 4 spans to 32 spans and some during the Nayaka period measured even up to 54 spans (Stephen, 1992). Because of the diversity of measurement rods, the actual area of land also varied on the ground, although similar terms such as *véli* or *mā* or *kāņi* were used in different contexts.

The descriptions found in the inscriptions reveal that the surveyors and accountants used very accurate measurement scales to measure the lands. One is surprised by the fact that land as small as 1/5242,88,00000 of a *véli* was measured. The idea behind such small fractions was perhaps that the land assessment had to be very accurate. The large area of *véli* was used as the main unit and its fractions conveyed area as small as 1 square mm or even less.

In some cases the areas were conveyed in terms of *kuzhi* units. But, in many instances, rather than conveying area in the form of *kuzhi* units or even the smaller square units such as *angulam* and *tōrai*, they conveyed the areas in terms of *véli* and its fractions. The fact that very minute fractions were used in the measurement suggests that length

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even up to the size of a nel (paddy) or torai (a kind of paddy, approximately 2.2 mm) were measured. Can we assume that they adopted the same approach, which they followed for measuring the bronze images, for land measurement too? Perhaps, such small units were marked on the measuring rods. It indicates their care for accuracy of land measurements. The measurement of length up to torai was actually used as indicated by the measurements of bronze images donated to the Brihadīswara temple at Tanjāvūr. In modern times, people may not sometime use such accurate measurements for land, i.e. up to 1 or 2 mm; however, in the medieval times, they had the scales to measure such smaller units on the ground for accurate taxation. They might have measured on the ground, the fractions of an *angulam* such as 3/4, 1/2, 1/4 and 1/8. It is not certain, however, if they used such small units actually, though there is a probability up to 1 sq mm area. As pointed by Y. Subbarayalu, when an area measured by a smaller rod was converted into area units based on longer measurement rods, it resulted in such minute fractions. However, an important point to be noted here is that they used such miniscule fractions in the land measurement, and it reveals their cognitive advancement in arithmetic and land measurement skills and methods, and their concern for accuracy. It also reveals the systematic efforts towards standardization and uniformity. Another point that emerges here is the pragmatics of the medieval administration which did not attempt to completely change or ignore the local measurement rods, and they preferred the mathematic conversion to standardize the measurement units, which was an important effort. The meticulous survey and calculation of the area of entire villages are indeed monumental tasks undertaken by the Chola administration.

In this paper, we have dealt with only a few aspects of the medieval measurement system under the Cholas and more detailed research is necessary to understand the conversion ratios and method of reduction of land areas for tax assessment. The use of traditional fractions, and the mathematical solutions offered by the inscriptions as well as those found in the ancient texts (e.g. *Kaṇakkatikāram*) could be taught in schools in order to develop the cognitive capabilities of the students in the subject of mathematics.

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