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1M1N22292 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2022

MMT1C01 – Algebra – 1

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Answer all questions. Each carries 1 weightage

1. Verify whether the map $\phi(x,y) = (x+1, y)$ is an isometry of the plane which maps the X-axis to X-axis.
2. Find the order of the element (8, 10) in the direct product $\mathbb{Z}_{12} \times \mathbb{Z}_{18}$.
3. Describe all abelian groups of order 36 upto isomorphism.
4. Find the reduced form and the inverse of the reduced form of the word $a^2a^{-3}b^3a^4c^4c^2a^{-1}$.
5. Find the asceding series of S_3.
6. Let ϕ_2 : Q[x] $\to \mathbb{Q}$ be the evaluation homomorpism with $\phi_2(x) = 2$. Find the Kernal of ϕ_2.
7. Find the units in $\mathbb{Z}[x]$.
8. What is group presentation.

($8 \times 1 = 8$ weightage)

Part B

Answer any two questions from each unit. Each carries 2 weightage

Unit I

9. Prove that if m divides the order of a finite abelian group G, then G has a subgroup of order m.
10. State the converse of the Lagrange's theorem. Prove that the converse of the Lagrange's theorem is false.

11. Find the order of the element $(3, 1) + \langle(1, 1)\rangle$ in the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_4)/\langle(1, 1)\rangle$.

Unit II

12. Let H and N be normal subgoups of a group G. Show that HN is a normal subgroup of G.

13. Prove that every group of prime power order is solvable.

14. Show that every group of order 15 is cyclic.

Unit III

15. Prove that a nonzero polynomial $f(x) \in F[x]$ of degree n can have at most n zeros in a field F.

16. Prove that if F is a field, then every nonconstant polynomial $f(x) \in F[x]$ can be factored in $F[x]$ into a product of irreducible polynomials, the irreducible polynomials being unique except for order and for unit factors in F.

17. Show that an intersection of ideals of a ring R is again an ideal of R.

($6 \times 2 = 12$ weightage)

Part C

Answer any two questions. Each carries 5 weightage

18. (a) Let X be a G-set and let $x \in$X. Prove that $|Gx| = (G : G_x)$.

    (b) State and prove Burnside's Formula.

19. (a) Define Sylow p-subgroup of a group G.

    (b) State and prove Third Sylow Theorem.

    (c) A group of order 24 must have either ——— or ——Sylow 2-subgroups.

20. (a) Let H be subgroups of a group G and let N be a normal sugroups of G. Show that HN/N is isomorphic to $H/(H \cap N)$.

    (b) State and prove Eisenstein criterion for irreducibility of a polynomial.

21. (a) Show that the multiplicative group $< F^*, . >$ of nonzero elements of a finite field F is cyclic.

    (b) List all elements in the group algebra FG where F is the field $\mathbb{Z}_2$ and G is the cyclic group of order 2. Give addition and multiplication table for thr product FG.

($2 \times 5 = 10$ weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2022

MMT1C02 – Linear Algebra

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A
Answer all questions.
Each carries 1weightage

1. let F be the subfield of the complex numbers and T be the function from F^3 in to F^3 defined as $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$, if (a, b, c) is a vector in F^3 what are the conditions on a,b,c so that (a, b, c) is in the range of T?
2. Let W_1 and W_2 be subspaces of a finite dimensional vector space V. Prove that $(W_1+W_2)^0= W_1^0 \cap W_2^0$.
3. Prove that every n- dimensional vector space over the field F is isomorphic to the space F^n.
4. Let A and B be n ×n matrices over the field F. Prove that if (I- AB) is invertible then

   I- BA is invertible.
5. Let T be a linear operator on$\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 - x_2 + x_3)$, is T invertible? If so find T^{-1}
6. Prove that every finite dimensional inner product space has an orthonormal basis.
7. Let V be an n-dimensional vector space and let T be a linear operator on V. Suppose that there exist some positive integer k so that $T^k=0$. Prove that $T^n=0$
8. Prove that an orthogonal set of non zero vectors in an inner product space is linearly independent

(8x1=8weightage)

Part B

Answer any two questions from each unit.
Each carries 2 weightage

Unit -1

9. Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space over the field F then prove that the space L(V,W) is finite dimensional and has dimension mn.

10. Let W_1 and W_2 be subspaces of a vector space V such that $W_1 + W_2 = V$ and $W_1 \cap W_2 = \{0\}$. Prove that for each vector $\alpha \in V$ there are unique vectors $\alpha_1 \in W_1$ and $\alpha_2 \in W_2$ such that $\alpha = \alpha_1 + \alpha_2$.

11. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If T is invertible then prove that the inverse function T^{-1} is a linear transformation.

Unit -2

12. If W is a k –dimensional subspace of an n dimensional vector space V, then prove that W is the intersection of (n-k) hyperspaces in V.

13. Let V be a finite dimensional vector space over the field F and let B={ $\alpha_1, \alpha_2, \ldots \alpha_n$},

$B' = \{\alpha_1', \alpha_2', \ldots. \alpha_n'\}$ be ordered bases for V . Suppose T is a linear operator on V. If

P = [$P_1, P_2, \ldots, P_n$] is the n×n matrix with columns $P_j = \left[\alpha_j'\right]_B$ then prove that

$[T]_{B'} = P^{-1}[T]_B P$. Alternatively if U is the invertible operator on V defined by

$U\alpha_j = \alpha_j', j = 1,2,\ldots,n,$ then prove that $[T]_{B'} = [U]_B^{-1} [T]_B [U]_B$

14. Let the linear operator on $\mathbb{R}^3$ which is representation the standard ordered basis by the matrix.

$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ check whether T is diagonalizable or not

Unit -3

15. (a) Let T be a linear operator on a finite dimensional space V. if T is diagonalizable and $c_1, c_2, \ldots, c_k$ be the distinct characteristic vector of T then prove that there exist linear operators $E_1, E_2, \ldots E_k$ on V such that

    (i) $E_i E_j = 0,\ i \neq j$

    (ii) E_i is a projection

    (iii) The range of E_i is the characteristic space for T associated with c_i.

16. Apply Gram-Schmidt process to the vectors $\beta_1=(1, 0, 1)$, $\beta_2=(1, 0, -1)$, $\beta_3=(0,3,4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.

17. Let W be a finite dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then Prove that E is an idempotent linear transformation of V onto W, $W^\perp$ is the null space of E and $V = W \oplus W^\perp$

(6x2=12 weightage)

Part C
Answer any two questions
Each carries 5 weightage

18. (a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. suppose V is finite dimensional then prove that rank(T)+ nullity(T)=dim V.

    (b) Let V be a finite dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same field F and let $\beta_1, \beta_2, \ldots, \beta_n$ be any vectors in W then prove that there is precisely one linear transformation T from V into W such that $T\alpha_j = \beta_j,\ j = 1, 2, \ldots, n$

19. (a) Let g, $f_1, \ldots f_r$ be linear functional on a vector space V with respective null spaces $N, N_1, N_2, \ldots, N_r$ then prove that g is a linear combination of $f_1, \ldots f_r$ iff N contains the intersection $N \cap N_1 \cap N_2 \cap \ldots \cap N_r$.

    (b) Let V and W be finite dimensional vector spaces over the field F. Let B be an ordered basis for V with dual basis B^* and let B' be an ordered basis for W with dual basis B'^* Let T be a linear transformation from V into W; let A be the matrix of T relative to B, B' and let B be the matrix of T^t relative to B'^*, B^* Then prove that $B_{ij}=A_{ji}$.

20. Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, then prove that f(T)=0.

21. (a) If f is a non zero linear functional on the vector space V then prove that the null space of f is a hyperspace in V. conversely , every hyperspace in V is the null space of a (not unique) non zero linear functional on V

(b) let V and W be vector spaces over the field F and let T be a linear transformation from V into W, then prove that there exist a unique linear transformation T^t from W^* in to V^* such that $(T^t g)(\alpha)=g(T\alpha)$ for every g in W^* and α in V.

(2 x 5 = 10 weightage)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2022

MMT1C03 – Real Analysis – I

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part-A

Answer all questions. Each question carries 1 weightage

1) For $x, y \in \mathbb{R}$, define $d(x,y) = \frac{|x-y|}{1+|x-y|}$. Determine whether d is a metric on $\mathbb{R}$.
2) Prove that every infinite subset of a compact set K has a limit point in K.
3) If f is continuous mapping of metric space X into a metric space Y, prove that $f(\bar{E}) \subset \overline{f(E)}$, for any set $E \subset X$.
4) Prove that, if a function f defined on $[a,b]$ is differentiable at a point $x \in [a,b]$, then f is continuous at x.
5) Suppose f is differentiable on (a,b). Prove that if $f'(x) \leq 0$ for all $x \in (a,b)$, then f is monotonically decreasing on (a,b).
6) If $f, g \in \mathcal{R}(\alpha)$ on $[a,b]$, prove that $fg \in \mathcal{R}(\alpha)$ and $|f| \in \mathcal{R}(\alpha)$.
7) If $\{f_n\}$ converges to f uniformly on $[a,b]$, then $\{f_n'\} \to f'$. Prove or disprove the statement.
8) Let $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}$. Check whether the family of function $\{f_n\}$ is equicontinuous or not.

Part-B

Answer any two questions from each unit. Each question carries 2 weightage

Unit - I

9) Let (X,d) be a metric space and $E \subset Y \subset X$. Prove that E is open relative to Y if and only if $E = G \cap Y$ for some open subset G of X.
10) Prove that compact subsets of a metric space are closed.
11) If f is a continuous mapping of a compact metric space X into a metric space Y, prove that f is uniformly continuous on X.

Unit - II

12) Let $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$. Check the existence and continuity of f'.

13) Prove that $f \in \mathcal{R}(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$, there exists a partition P of $[a,b]$ such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$.

14) State and prove fundamental theorem of calculus.

Unit - III

15) If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$.

16) State and prove Cauchy criterion for uniform convergence.

17) If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1,2, \ldots$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

Part-C

Answer **any two** questions. Each question carries **5 weightage**

18)

a) Prove that every $k - cell$ is compact.

b) Let f be monotonically increasing on (a,b). Prove that $f(x+)$ and $f(x-)$ exist at every point x of (a,b). Also prove that if $a < x < y < b$, then $f(x+) \leq f(y-)$.

19)

a) Suppose f is continuous on $[a,b]$, $f'(x)$ exists at some point $x \in [a,b]$, g is defned on an interral I which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))$; $(a \leq t \leq b)$, prove that h is differentiable at x.

b) Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Prove that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$ and

$$\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x) dx.$$

20)

a) Prove that a nonempty perfect set in $\mathbb{R}^k$ is uncountable.

b) If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1,2,3, \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

21) State and prove the Stone-Weierstrass theorem

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Name: ………………………

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2022

MMT1C04 – Discrete Mathematics

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

(Short Answer Questions)

(Answer all questions. Each question has weightage1)

1. Define a lattice. Give an example.
2. Let (X, + , . ,′) be a Boolean algebra. Prove that x +x.y = x , for all $x,y \in X$.
3. Define girth of graph and find the girth of Peterson graph.
4. State Euler formula for planar graph and verify for the cycle with four vertices.
5. Prove that $\kappa(K_n) = n-1$.
6. Define identity graph and give an example.
7. Find a dfa that accepts all strings on {0, 1} except those containing the substring 001.
8. Define a language L and L*, the star closure of L. Give an example.

(8x1=8 weightage)

Part B

(Answer any two question from each unit.Each question carries weightage 2)

Unit I

9. Let X be a finite set and ≤ a partial order on X. Define a binary relation R on X by xRy if and only if y covers x (w.r.t.≤). Prove that ≤ is generated by R.
10. Let (X, + , .,′) be a finite Boolean algebra , Prove that every element of X can be uniquely expressed as sum of atoms.
11. Write the following Boolean function in the disjunctive normal form

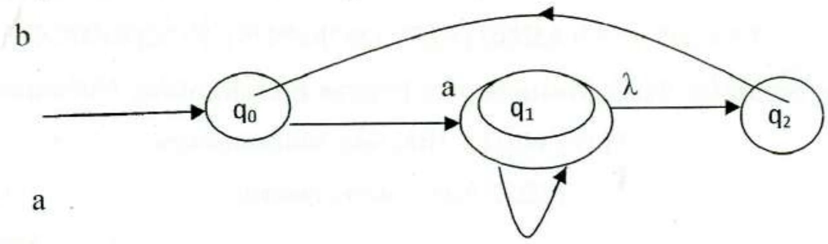
$$F(x_1,x_2,x_3) = (x_1+x_2+x_3)(x_1'+x_2+x_3')(x_1+x_2'+x_3')(x_1'+x_2'+x_3')(x_1+x_2+x_3')$$

Unit II

12. State and prove a necessary and sufficient condition for a graph to be bipartite.
13. Prove that a graph G with at least three vertices is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.
14. Prove that $\delta(G) \le 5$, for any simple planar graph G.

Unit III

15. Find a dfa equivalent to the following nfa



16. Find a grammer that generate the language $L = \{a^n b^n : n \geq 0\}$.

17. Show that the language L={awa:w∈{a,b}*} is regular.

(6x2=12 weightage)

Part C

Answer any two from the following four questions. Each question has weightage 4

18. (a)Let (X,+,. ,′) be a Boolean algebra. Prove that the relation $\leq$ on X defined by $x \leq y$ if $x.y'=0$ is a lattice and 0 and 1 are the minimum and maximum elements of this lattice.

(b) Prove that the set of all symmetric Boolean functions of *n* Boolean variables $x_1, x_2, \ldots, x_n$ is a sub algebra of the Boolean algebra of all Boolean functions of these variables . Also prove it is isomorphic to the power set Boolean algebra of the set {0, 1,….,n}.

19. (a) If G is a simple graph , then prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

(b)Prove that K_5 is non-planar.

20. (a)Prove that every connected graph contains a spanning tree.

(b)Let G be a connected graph. Prove that G is Eulerian if and only if the degree of each vertex of G is an even positive integer.

21. (a) Are the grammars $G_1 = (\{S\}, \{a,b\}, S, \{ S \to SS, S \to aSb, S \to \lambda, S \to bSa\})$ and $G_2 = (\{S\}, \{a,b\}, S, \{ S \to SS, S \to SSS, S \to aSb, S \to bSa, S \to \lambda \})$ are equivalent.

(b) Let L be the language accepted by a non deterministic finite accepter $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$. Then prove that there exist a dfa $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D\}$ such that $L=L(M_D)$.

(2x5=10 weightage)

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Name: ……………………….

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Mathematics Degree Examination, November 2022

MMT1C05 – Number Theory

(2022 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Answer *all* questions. Each carries 1 weightage

1. Define Mobius function $\mu(n)$ and show that $\sum_{d/n} \mu(d) = 0 \; if \;\; n > 1$.
2. Find all integers n such that $\varphi(n) = \frac{n}{2}$
3. Give an example of a multiplicative function which is not completely multiplicative.
4. With usual notations, prove that $\wedge(n) * u = u'$ and hence derive Selberg identity.
5. Calculate the highest power of 10 that divides 1000!
6. Determine the quadratic residues and non residues modulo 11
7. Prove that product of two shift enciphering transformations is also shift enciphering transformations .
8. What is meant by affine cryptosystems?

(8 × 1 = 8 weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. Let f be a multiplicative function. Show that $f \; is \; completely \; multiplicative$ if and only if $f^{-1}(n) = \mu(n)f(n)$ for all n≥ 1
10. If n≥ 1, prove that $\Delta(n) = \sum_{d/n} \mu(d) \; log\left(\frac{n}{d}\right) \; = \; -\sum_{d/n} \mu(d) \; log(d)$
11. State and prove the Euler's Summation formulae.

Unit 2

12. Show that for $x \geq 2$; $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p \; = \; x \log x - x + O(\log x)$.
13. State and prove the Abel's identity
14. Show that $\frac{1}{6}\frac{n}{\log n} \; < \pi(n) < \frac{6n}{\log n} \; , for \; every \; integer \; n \geq 2.$

Unit 3

15. State and prove Gauss lemma

16. Solve the following system of simultaneous congruences

$$9x + 20y \equiv 10(mod\ 29)$$

$$16x + 13y \equiv 21(mod\ 29).$$

17. Find the inverse of $A = \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \in M_2\left(\frac{Z}{26Z}\right)$

(6 × 2 = 12 weightage)

Part C

Answer any *two* questions. Each carries 5 weightage

18. (a) State and prove the Mobius inversion formula.

(b) Show that for $n \geq 1, the\ n^{th}\ prime\ \ P_n$ satisfies the inequalities

$$\frac{1}{6}\ nlogn < P_n < 12\left(n\log n + n\log\frac{12}{e}\right).$$

19. (a) State and Prove Quadratic reciprocity law.

(b) Prove that 5 is a quadratic residue of an odd prime p if $p \equiv \pm 1(mod\ 10)$.

20. (a) Explain the advantages and disadvantages of public key cryptosystems as compared to classical cryptosystems.

(b) Describe algorithm for finding the discrete logs in the finite fields

21 .Let P_n denotes the n^{th} prime. Prove the following are equivalent:

(a). $\lim_{x\to\infty}\frac{\pi(x)\log x}{x} = 1$

(b). $\lim_{x\to\infty}\frac{\pi(x)\log \pi(x)}{x} = 1$

(c). $\lim_{n\to\infty}\frac{P_n}{n\log n} = 1$

(2 × 5 = 10 weightage)