1M1N21079 (Pages : 2) Reg. No:...........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2021

MMT1C01 – Algebra – I

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART -A

Answer all questions each question carries one weightage

1. Do the translations, together with the identity map, form a subgroup of the group of plane isometries? Why or why not?
2. Find the order of the element (3, 10, 9) in the direct product $Z_4 \times Z_{12} \times Z_{15}$.
3. Show that $\mathbf{Z/nZ} \cong \mathbf{Z_n}$
4. If $w_1 = a_2^3 a_1^{-5} a_3^2$ and $w_2 = a_3^{-2} a_1^2 a_3 a_2^{-2}$. Find the readuced form and the inverse of the readuced form $w_1.w_2$.
5. Find an isomorphic refinement of the two normal series, $\{0\} < (3) < Z_{24}$ and $\{0\} < (8) < Z_{24}$
6. Show that $x^4 - 22x^2 + 1$ is irreducible over **Q**
7. Show that $\{x, y : y^2x = y, yx^2y = x\}$ is a presentation of the trivial group.
8. Give an example for a factor ring of an integral domain may not be a field.

(8x1 =8 weightage)

PART -B

Answer any two from each unit (Each question carries 2 weightage)

UNIT I

9. Show that a subgroup M of a group G is a maximal normal subgroup of G if and only if G/M is simple.
10. a, Find the order of the element (2,0)+$< (4,4) >$ in the factor group $\mathbf{Z_6} \times \mathbf{Z_8} / < (4,4) >$.

    b, Let X be a G-set. For each $g \in G$, the function $\sigma_g : X \to X$ defined by $\sigma_g(x) = gx$ for $x \in X$. Show that σ is a permutation of X.

11. Find the number of distinguishable ways the edges of an equilateral triangle can be painted if four different colors of paint are available, assuming only one color is used on each edge, and the same color may be used on different edges.

UNIT II

12. Show that the center of a finite nontrivial p-group G is nontrivial.

13. Find a composition series of $S_3 \times S_3$. Is $S_3 \times S_3$ solvable?

14. State and prove Cauchys Theorem

UNIT III

15. State and prove Eisenstein Criterion for irreducibilty of Polynomials.

16. Let H be a subring of the ring R. Show that multiplication of additive cosets of H is well defined by the equation $(a+H)(b+H) = ab+H$ if and only if $ah \in H$ and $hb \in H$ for all $a, b \in R$ and $h \in H$.

17. Show that a non-zero polynomial $f(x) \in F[x]$ of degree n can have at most n zeros in a field F.

(6x2 =12 weightage)

PART -C

Answer any two questions(Each question carries 5 weightage)

18. a, Show that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.
b, How many abelian groups (up to isomorphism) are there of order 24? of order 25? of order (24)(25)?
c, If m is a square free integer, then show that every abelian group of order m is cyclic.

19. a, Let H be a subgroup of a group G. Show that left coset multiplication is well defined by the equation $(aH)(bH) = (ab)H$ if and only if left and right cosets coincide.
b, If p and q are distinct primes with $p < q$, then show that every group G of order pq has a single subgroup of order q and this subgroup is normal in G. If q is not congruent to 1 modulo p, then show that G is abelian and cyclic.

20. a, Define Sylow p-subgroup of a group G. Find all Sylow 3-subgroups of Z_{12}.
b, State and prove first Sylow theorem.
c, Show that no group of order 15 is simple

21. a, If F is a field, then show that every nonconstant polynomial $f(x) \in F[x]$ can be factored in $F[x]$ into a product of irreducible polynomials, the irreducible polynomials being unique except for order and for unit (that is, nonzero constant) factors in F.
b, Is $2x^3 + x^2 + 2x + 2$ an irreducible polynomial in $Z_5[x]$? Why? Express it as a product of irreducible polynomials in $Z_5[x]$.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2021

MMT1C02 – Linear Algebra

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A: Answer *all* questions. Each carries 1 weightage

1. Give an example for an infinite dimensional vector space. Justify your example.
2. Find a basis for the plane $x+2y-z=0$.
3. If T is a linear operator on an n – dimensional vector space V whose range and null space are identical, then show that n is even.
4. Let $u=(1,1,0), v=(0,1,1,)$ and $w=(1,0,-1)$. If T is a mapping from $\Re^3$ to $\Re^4$ such that $T(u)=T(v)=0$ and $T(w)=(1,0,-1,0)$. Is T linear? Justify your answer.
5. If T is a linear operator on V and if every subspace of V is invariant under T then show that T is a scalar multiple of the identity operator.
6. Prove or disprove: If a diagonalizable operator has only the characteristic values 0 and 1, it is a projection.
7. If A and B are $n\times n$ complex matrices, show that $AB-BA=I$ is impossible, where I is the identity matrix of order n.
8. For any $\alpha\in\Re^2$, with standard inner product, show that $\alpha=(\alpha\mid e_1)e_1+(\alpha\mid e_2)e_2$, where $\{e_1,e_2\}$ is the standard basis for $\Re^2$.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. Define vector space isomorphism. Show that any vector space of dimension n over F is isomorphic to F^n.
10. Define basis for a vector space. Illustrate with an example. Show that any linearly independent subset of a vector space can be extended to a basis for the space.
11. Show that a linear operator T on a finite dimensional vector space is one – one if and only if it is onto. Give examples to show that this is not true for infinite dimensional vector spaces.

Unit 2

12. If V is a finite dimensional vector space over the field F, and if $B=\{\alpha_1,\alpha_2,\ldots,\alpha_n\}$ is a basis for V, then show that there is a unique dual basis $B^*=\{f_1,f_2,\ldots,f_n\}$ for the dual space V^* such that $f_i(\alpha_j)=\delta_{ij}$. Also, show that for each linear functional f on V we have $f=\sum_{i=1}^{n} f(\alpha_i)f_i$ and for each vector α in V, we have $\alpha=\sum_{i=1}^{n} f_i(\alpha_i)\alpha_i$.

13. Define the transpose of a linear transformation T from V into W. If V and W are finite dimensional, then show that both T and its transpose have the same rank.

14. If T is a linear operator on a finite dimensional vector space, prove that the minimal polynomial for T divides the characteristic polynomial for T.

Unit 3

15. What do you mean by independent subspaces of a vector space? Let T be a linear operator on a finite – dimensional vector space V. Let R be the range of T and let N be the null space of T. Prove that R and N are independent if and only if $V = R \oplus N$.

16. State and prove Bessel's inequality.

17. Explain Gram-Schmidt orthogonalization process. Apply it to the vectors $u_1 = (3,0,4)$, $u_2 = (-1,0,7)$ and $u_3 = (2,9,11)$, to obtain an orthonormal basis for $\Re^3$.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage

18. (a) State and prove the rank – nullity theorem.

(b) Let T be the linear operator on R^3, whose matrix in the standard ordered basis is

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$. Find the basis for the range of T and a basis for the null space of T.

19. (a) Let V be an n-dimensional vector space over F and let W be an m-dimensional vector space over F. Show that there is a one to one correspondence between the set of all linear transformations from V into W and the set of all $m \times n$ matrices over the field F.

(b)If a finite number of vectors spans a vector space V, then show that V is finite dimensional.

20. (a) If f is a non-zero linear functional on the vector space V, then prove that the null space of f is a hyperspace in V. Also prove that every hyperspace in V is the null space of a (not unique) non-zero linear functional on V.

(b) Let T be a linear operator on an n-dimensional vector space V. Show that the characteristic and minimal polynomials for T have the same roots except for multiplicities.

21. Define inner product space. Illustrate with an example. If V is a real or complex vector space with an inner product $(|)$, then for any α, β in V, prove that

(a) $(\alpha \mid \beta) = 0,\ \forall \beta \in V \Rightarrow \alpha = 0$.

(b) $|(\alpha \mid \beta)| \le \|\alpha\| \, \|\beta\|$, and

(c) $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$

(2 × 5 = 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2021

MMT1C03 – Real Analysis – I

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer ALL questions. Each question carries 1 weight.

1. Prove that the set of all irrational numbers between 0 and 1 is uncountable.

2. Give with reason example of a non-compact set.

3. If $E^\circ = E$, for a set E of real numbers then prove that E is an open set.

4. Define $\lim_{x\to a} f(x)$. Prove that the limit, if it exists, is unique.

5. A real valued function f on $[a,b]$ is such that $|f|$ is continuous on $[a,b]$.

   Is it true that f is continuous on $[a,b]$? Give reason.

6. Prove that a function f that is differentiable at a point c is continuous at c.

7. Define the integral of a vector valued function on an interval $[a,b]$.

8. State the theorem of change of limit and derivative. Explain the related terms. **$8\times1 = 8$ Weights.**

Part B

Answer any TWO questions from each unit. Each question carries 2 weights.

UNIT I

9. Prove that a neighbourhood is a convex set.

10. Prove that a closed subset of a compact set is a compact set.

11. Give example of a discontinuuous monotone function on $\mathbb{R}$. Explain the reason for discontinuity.

UNIT II

12. Let f be defined for all real x, and supose that $|f(x) - f(y)| \le (x-y)^2$ for all real x and y.

    Prove that f is constant.

13. State L'Hospital's Rule. Will the rule work for vector/complex valued functions ? Explain.

14. If $f \in \mathscr{R}(\alpha)$ over $[a,b]$ then prove that $|f| \in \mathscr{R}(\alpha)$ over $[a,b]$. Is the converse true ? Give reason.

UNIT III

15. Let $f(x) = \sum_{n=0}^{\infty} f_n(x)$ where $f_n(x) = \dfrac{x^2}{(1+x^2)^n}, x \in \mathbb{R}$.

Is $f_n(x)$ continuous on $\mathbb{R}$? Is $f(x)$ continuous on $\mathbb{R}$? Is the convergence of the series uniform ?

16. Give example of a sequence of functions converging uniformly in the corresponding domain. Explain the steps.

17. Let $\{f_n\}$ be a pointwise bounded sequence of complex functions on a countable set E. Prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ suc that $\{f_{n_k}(x)\}$ converges for every $x \in E$. **6×2 = 12 Weights.**

Section C

Answer any TWO questions. Each question carries 5 weights.

18. a) Prove that a subset E of $\mathbb{R}^1$ is connected if and only if it satisfies the property that "$x \in E, y \in E$ and $x < z < y$ implies that $z \in E$".

b) Define a compact set. Give one example.

Prove that the continuous image of a compact set is compact.

19. a) State and prove the chain rule of differentiation.

b) State and prove the mean value theorem.

20. a) State and prove the fundamental theorem of calculus.

b) If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|dt$.

21. a) Prove that there exists a real continuous function on $\mathbb{R}$ which is nowhere differentiabble.

b) Define equicontinuity of a family of functions. Give example of one such family.

2×5 = 10 Weights.

1M1N21082 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2021

MMT1C04 – Discrete Mathematics

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions. Each question carries 1 weightage

1. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.
2. If $\{x, y\}$ is a 2-edge cut of a graph G, show that every cycle of G that contains x must also contain y.
3. Prove that a tree with at least two vertices contains at least two pendant vertices.
4. Define a strict partial order. If P is a partial order on the set X, show that $P - \{(x, x) : x \in X\}$ is a strict partial order.
5. Define a lattice and give an example.
6. Prove that every nonzero element of a finite Boolean algebra contains at least one atom..
7. Give a grammar for the set of all regular expressions in Pascal.
8. Design a *dfa* which accepts all binary sequences that ends with the digits 011.

(8 × 1 = 8 weightage)

Part B

Answer any questions from each unit. Each question carries 2 weightage

Unit I

9. Prove that at a party with at least two people, there are two with the same number of friends.
10. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices
11. Prove by the Jordan Curve Theorem that $K_{3,3}$ is non planar.

Unit II

12. Let X be a finite set and $\leq$ be a partial order on X. R is a binary relation on X defined by xRy if, and only if y covers x. Prove that $\leq$ is the smallest order relation containing$R$.
13. Write the function : $f(a, b, c) = a + b + c'$ in its disjunctive normal form.
14. Show that every Boolean function on n variables $x_1, x_2, \ldots, x_n$ can be uniquely expressed as the sum of terms of the form $x_1^{\epsilon_1} x_2^{\epsilon_2} \ldots x_n^{\epsilon_n}$ where $x_i^{\epsilon_i}$ is either x_i or x_i'.

Unit III

15. Find a grammar for that generate the language$\{a^{n+2}b : n \geq 1\}$. Give convincing arguments that the grammar you give generate the language

16. Let L be the language$L = \{ awa : w \in \{a, b\}^*\}$. Show that $L^2$is regular.

17. Let $M = (Q, \Sigma, \delta, q_0, F)$be a *dfa* and let G_M be its associated transition graph. Then show that for any$q_i, q_j \in Q$, and $w \in \Sigma^+$, $\delta^*(q_i, w) = q_j$ if and only if there is in G_Ma walk with label w from q_i to q_j.

(6× 2 = 12 weightage)

Part C

Answer anytwo questions. Each question carries 5 weightage

18. a) Prove that a graph without odd cycles is bipartite.

    b)Prove that the connectivity and edge connectivity of a simple cubic graph G are equal.

19. For a nontrivial connected graph G, prove that the following statements are equivalent:

    a) G is Eulerian

    b) The degree of each vertex of G is an even positive integer.

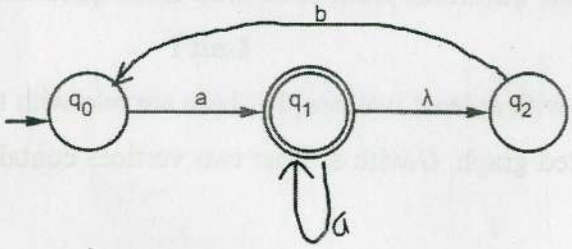
    c) G is an edge disjoint union of cycles.

20. a)Show that if $(X, \leq)$ is a bounded, complemented and distributive lattice then there exists a Boolean algebra on X such that the partial order relation defined by this structure coincides with the given relation $\leq$.

    b)Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

21. a)Find a *dfa* that accepts the set of all strings on $\Sigma = \{a, b\}$ starting with the prefix ab.

    b)Convert the *nfa* given by the transition graph into an equivalent *dfa*.



(2× 5 = 10 weightage)

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Name: ……………………….

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2021

MMT1C05 – Number Theory

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer *all* questions. Each carries 1 weightage

1. Prove that the equation $f(n) = \sum_{d/n} g(d)$ implies $g(n) = \sum_{d/n} f(d).\mu\left(\frac{n}{d}\right)$.
2. Give an example of a multiplicative function which is not completely multiplicative
3. If $f \text{ and } g$ are arithmetical functions , show that $(f * g)' = f' * g + f * g'$.
4. With usual notations, prove that $\wedge(n) * u = u'$ and hence derive Selberg identity
5. Prove that $[2x] + [2y] \geq [x] + [y] + [x + y]$.
6. Determine whether -104 is a quadratic residue or nonresidue of the prime 997.
7. How do we send a signature in RSA?
8. What is an affine cryptosystem ?

(8 × 1 = 8 weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. Show that for $n \geq 1$, $\emptyset(n) = n\prod_{p/n}\left(1 - \frac{1}{p}\right)$.
10. Show that if f is an arithmetical function with $f(1) = 0$, then there is a unique arithmetical function f^{-1} such that $(f * f^{-1}) = (f^{-1} * f) = 1$
11. State and prove the Euler's Summation formulae.

Unit 2

12. Show that for $x \geq 1$; $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$, where C is Euler's constant.
13. State and prove the Abel's identity
14. Show that for $n \geq 1$, the n^{th} prime P_n satisfies the inequalities

$$\frac{1}{6}\, n\log n < P_n < 12\left(n\log n + n\log\frac{12}{e}\right).$$

Unit 3

15. State and prove the Euler's criterion on the Legendre's symbol (n/p).

16. Find the inverse of A = $\begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix} \in M_2\left(\frac{z}{26\,z}\right)$.

17. Explain the advantages and disadvantages of public key cryptosystems as compared to classical cryptosystems.

(6 × 2 = 12 weightage)

Part C

Answer any *two* questions. Each carries 5 weightage

18. State and prove the Shapiro's Tauberian theorem.

19. (a) State and Prove Quadratic reciprocity law.

    (b) Prove that 5 is a quadratic residue of an odd prime p if $p \equiv \pm 1 (mod\ 10)$.

20. (a) If both g and $f * g$ are multiplicative then show that f is also multiplicative.

    (b) Solve the following system of simultaneous congruence's

    $17x + 11y \equiv 7 (mod\ 29)$

    $13x + 10y \equiv 8 (mod\ 29)$.

21. Suppose that our adversary is using a 2 x 2 enciphering matrix with a 29 – letter alphabet, where A – Z have the usual numerical equivalents, blank = 26, ? = 27, ! = 28. We receive the message " GFPYJP X?UYXSTLADPLW" and we know that the last five letters of plaintext are our adversary's signature "KARLA". Decode and read the message.

(2 × 5 = 10 weightage)