1B5N21183 (Pages : 2) Reg. No:……………………..

Name: ……………………….

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2021

BMT5B05 – Abstract Algebra

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks: 80

Part A
All questions can be attended.
Each questions carries 2 marks.

1. Find the multiplicative inverse of [14] in Z_5.
2. Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 3 & 2 & 1 \end{pmatrix}$ in S_6.
3. Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$ as a product of disjoint cycles.
4. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 7 & 5 & 4 & 8 & 6 & 3 \end{pmatrix}$ in S_8
5. Give an example of non abelian group.
6. Define order of an element in a group.
7. Find the order of subgroup generated by [3] in $\square_{15}$.
8. Find the order of $Z_{15}^{\times}$
9. Check whether $\mathbf{Z_2 \times Z_4}$ is cyclic.
10. Is $Z_5^{\times}$ isomorphic to $Z_8^{\times}$.
11. Draw the subgroup diagram of $\mathbf{Z_{16}}$.
12. Define commutative group.
13. Give an example of an integral domain.
14. Find all the units in the ring of integers **Z.**
15. True or false: Every finite integral domain is a field.

(Ceiling 25marks)

Part B

All questions can be attended.
Each questions carries 5 marks.

16. Show that $\phi(1)+\phi(p)+\phi(p^2)+\ldots+\phi(p^\alpha)= p^\alpha$.
17. State and prove Euler theorem.
18. Let **S** be a set and let ~ be an equivalence relation on S. Prove that each element of **S** belongs to exactly one of the equivalence classes of S determined by the relation ~.
19. Let **G** be a group. Prove that $(ab)^{-1}=b^{-1}a^{-1}$ for every $a,b\in$ **G**.
20. Let **G** be a group. If $x^2= e\ \forall x\in$ **G**. Prove that **G** is abelian.
21. Let **G** be a group. Prove that **H** is a subgroup of **G** if and only if **H** is nonempty and $ab^{-1}\in$ **H** $\forall a,b\in$ **G**.
22. Show that $\mathbf{D} =\left\{\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} : ad \neq 0\right\}$ is a subgroup of **$GL_2(R)$**, where **$GL_2(R)$**. is the set of all invertible 2 x 2 matrices with entries in **R**..
23. Prove that **R** with addition and **R^+** under multiplication are isomorphic.

(Ceiling 35marks)

Answer any two questions.
Each questions carries10 marks.

26. (a) Prove that every permutation in **S_n** can be .written as a product of disjoint cycles.
    (b) Prove that the set **$GL_n(R)$** forms a group under matrix multiplication, where **$GL_n(R)$** is the set of all invertible n x n matrices with entries in **R**.
27. (a) State and prove Lagranges theorem.
    (b) Prove that any cyclic group is abelian.
28. (a) Let **G_1** and **G_2** be groups. If $a_1\in$ **G_1** and $a_2\in$ **G_2** have orders n and m respectively. Prove that order of (a_1,a_2) in **G_1 x G_2** is lcm[n,m].
    (b)Prove that **Z_m x Z_n**is isomorphic to **Z_{mn}** whenever gcd(m,n)=1.
29. (a)Prove that infinite cyclic group **G** is isomorphic to **Z**.
    (b) Prove that every group is isomorphic to a permutation group.

(2x10=20 marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2021

BMT5B06 – Basic Analysis

(2019 Admission onwards)

Time: 2 ½ hours Max. Marks: 80

Section A
ALL questions can be answered.
Each question carries two marks. Ceiling is 25.

1. Define denumerable set and give an example.
2. State the Trichotomy property of **R**.
3. If a∈**R** and $\in > 0$, define the $\in$ - neighborhood of a.
4. State the completeness property of **R**.
5. Find the supremum of the set $\{ 2 - 1/n : n \in \mathbf{N} \}$.
6. Define the nested interval property of R.
7. Define the convergence of a sequence of real numbers.
8. State the squeeze theorem.
9. Give examples of increasing and decreasing sequences.
10. State the divergence criteria of sequence of real numbers.
11. Give an example of a bounded sequence which is not contractive.
12. Write the multiplicative inverse of the complex number z = 2 + i3.
13. Find the principal argument Argz if z = 1 + i.
14. Find the polar form of $z = 1 - i\sqrt{3}$.
15. Define continuity of a complex function f(z) at $z = z_0$.

Section B
ALL questions can be answered
Each question carries 5 marks. Ceiling is 35

16. Show that the set **Q** of all rational numbers is countable.
17. If a ∈ **R** is such that $0 \le a < \in$ for every $\in > 0$, then prove that a=0.
18. If a and b are positive real numbers, prove the Arithmetic-Geometric inequality for a and b.
19. Prove the Archimedean property of R: If x∈**R**, then there exists n ∈**N** such that $x \le n$.
20. Prove the density theorem: If x and y are any real numbers with x < y, then there exists a rational number r ∈**Q** such that x< r <y.

21. Prove that every convergence sequence of real numbers is bounded.

22. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$.

23. Using de Moivre's formula, show that $(\frac{\sqrt{3}}{2} + i\,\frac{1}{2})^3 = i$.

Section C
Answer any TWO questions
Each question carries 10 marks

24. (a) Prove that the set **R** of real numbers is not countable.

(b)State and prove the Bernoulli's inequality.

25. Prove the Bolzano -Weierstrass theorem: A bounded sequence of real numbers has a convergent subsequence.

26. Prove the Cauchy convergence criterion: A sequence of real numbers is convergent if and only if it is a Cauchy sequence.

27. (a) Find the cube root of the complex number i.

(b) Define the complex exponential function e^z and find the values of e^z at $z = 2 + i\pi$.

(2 x 10 = 20 Marks)

1B5N21185 (Pages : 2) Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2021

BMT5B07 – Numerical Analysis

(2019 Admission onwards)

Time: 2 hours Max. Marks: 60

Section A

All questions can be attended.

Each question carries 2 marks.

1. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1,2]$. Also find p_2 using bisection method
2. Let g(x) be a continuously differentiable function on $[a, b]$. What are the conditions that, there is exactly one fixed point in [a, b].
3. Let $f(x) = -x^3 - \cos x$ with $p_0 = -1$ and $p_1 = 0$. Use Secant method to find p_2.
4. Determine the linear Lagrange interpolating polynomial that passes through the points (2,4) and (5,1).
5. Write three point end point formula and three point midpoint formula to approximate $f'(x_0)$.
6. Approximate $f''(4.0)$ for the following data

x	0.2	0.4	0.6
f(x)	0.9798652	0.9177710	0.808038

7. Approximate $\int_0^{0.35} \frac{2}{x^2-4}\, dx$ using Simpson's rule.
8. Find the degree of precision of the quadrature formula

$$\int_{-1}^{1} f(x)dx = f\left(-\frac{\sqrt{3}}{2}\right) + f\left(\frac{\sqrt{3}}{2}\right).$$

9. Show that the initial-value problem $y' = y \cos t$, $0 \le t \le 1$ $y(0) = 1$ has a unique solution
10. Use Taylor's method of order two to approximate $y(0.5)$ for the initial-value problem.
$y' = te^{3t} - 2y, 0 \le t \le 1, y(0) = 0$, with $h = 0.5$
11. Use open formula with $n = 0$ approximate $\int_0^{\frac{\pi}{4}} \sin x \, dx$.
12. Write Runge kutta formula for order 4.

(Ceiling .... 20 Marks)

Section B

All questions can be attended.

Each question carries 5 marks.

13. Using fixed point iteration, find a real root of the equation $x^3 + x^2 - 1 = 0$. On the interval $[0,1]$ with an accuracy of 10^{-3}.

14. Using Newton's method to establish the formula $p_{i+1} = \frac{1}{2}\left(p_i + \frac{n}{p_i}\right)$ to find $\sqrt{n}$

15. Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

x	0.0	0.2	0.4	0.6	0.8
f(x)	1.00000	1.22140	1.49182	1.82212	2.22554

16. For a function $f(x)$, the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table.

17. Given $f(0.5) = 0.4794, f(0.6) = 0.5646$, and $f(0.7) = 0.6442$ Approximate $f'(0.5)$ and $f'(0.6)$. If $f(x) = \sin x$ find error bounds using the error formula.

18. Approximate $\int_1^{1.5} x^2 \ln x \, dx$ using Midpoint rule. Also find a bound for the error.

19. Use Euler's method to approximate the solution for the initial-value problem
$y' = t^2 e^t + \frac{2y}{t}, 1 \le t \le 2, y(1) = 0$ with $h = 0.25$.

(*Ceiling* ….30 *Marks*)

Section C

Answer any One question.

20. a) Find an approximation to $\sqrt{3}$ correct to within 10^{-2} using the Bisection method.
    b) Let $P_3(x)$ be the Lagrange's interpolating polynomial for the data (0,0), (0.5, y), (1, 3), and (2,2). The coefficient of x^3 in $P_3(x)$ is 6. Find y.

21. Approximate the solution for the initial-value problem $y' = 1 + \frac{y}{t}, 1 \le t \le 2, y(1) = 0$ with $h = 0.5$

    Using a) Midpoint method b) Modified Euler's method.

($1 \times 10 = 10$ *Marks*)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2021

BMT5B08 – Linear Programming

(2019 Admission onwards)

Time: 2 hours Max. Marks: 60

Section A

All questions can be attended, Each question carries 2 marks

1. Explain the terms objective function, constraint set and feasible solution with an example
2. An appliance company manufactures heaters and air conditioners. The production of one heater requires 2 hours in the parts division of the company and 1 hour in the assembly division of the company; the production of one air conditioner requires 1 hour in the parts division of the company and 2 hours in the assembly division of the company. The parts division is operated for at most 8 hours per day and the assembly division is operated for at most 10 hours per day. If the profit realized upon sale is 30$ per heater and 50$ per air conditioner .Formulate as a mathematical problem
3. Pivot 4 in the following table

x	y	−1	
1	2	3	$= -t_1$
4	5	6	$= -t_2$
7	8	9	$= f$

4. Define hyper plane and closed half space of R^n with example.
5. Explain the canonical slack minimization and canonical slack maximization linear programming problem and also identify the slack variables
6. Explain the concept cycling in a linear programming problem.
7. Write the Tucker table for the problem

$$Max\, f = 20x_1 + 15x_2$$

Subj to $x_1 + x_2 \leq 2$, $2x_1 - 3x_2 \leq 6$, $7x_1 + 4x_2 \leq 9$, $x_1, x_2 \geq 0$. Also identify the dependent and independent variables.

8. Sketch the constraint and find the optimal solution $Min\ g = x + 2y$

$$Subj\ to\ \ x + y \geq 1, \qquad 4x - 4y \geq -1, \ \ x, y \geq 0$$

9. Explain unbalanced transportation problem with an example and convert it to a balanced problem.
10. Find the transportation cost of the following table and also check whether it optimal or not

(2)^{20}	(1)^{10}	(2)^{20}	0	50
9	(4)^{40}	7	(0)^{30}	70
(1)^{20}	2	9	0	20
40	50	20	30	

11. Compare the transportation problem and assignment problem
12. Discuss permutation set of zeros of an assignment problem with example.

(Ceiling 20marks)

Section B

All questions can be attended, each question carries 5marks

13. Solve Max $f(x, y) = x + y$

$$Subj\ to\ 2x + y = 5, \qquad x - y = -2, \ \ x + 3y = 6 \quad x, y \geq 0$$

14. An oil company owns two refineries, say refinery A and refinery B. Refinery A is capable of producing 20 barrels of gasoline and 25 barrels of fuel oil per day; refinery B is capable of producing 40 barrels of gasoline and 20 barrels of fuel oil per day. The company requires at least 1000 barrels of gasoline and at least 800 barrels of fuel oil. If it costs \$300 per day to operate refinery A and \$500 per day to operate refinery B, how many days should each refinery be operated by the company so as to minimize costs?
15. Define complementary slackness. Prove that a pair of feasible solutions of dual canonical linear programming problems exhibit complementary slackness if and only if they are optimal solutions.
16. Maximize $f(x, y) = x + y\ subject\ to\ x + 4y \leq 12, x - 4y \leq -4$

17. Solve the assignment problem

7	2	4	10
10	5	9	20
7	3	5	30
20	10	30	

18. Solve the noncanonical linear programming problem

$$Max\ f(x,y,z) = x - y + z$$

$$Subj\ to\ \ x + y \geq 2,\ z - y \geq 3,\ \ 2x + z \leq 8$$

19. Explain Hungarian algorithm

20. Explain Northwest- corner method with an example

(Ceiling 30marks)

Section C

Answer any one question

21. Solve the dual canonical linear programming problems below

(a)

	x_1	x_2	-1	
y_1	-1	-1	-3	$= -t_1$
y_2	1	1	2	$= -t_2$
-1	2	-4	0	$= f$
	$= s_1$	$= s_2$	$= g$	

(b)

	x_1	x_2	-1	
y_1	-1	-1	-3	$= -t_1$
y_2	1	1	2	$= -t_2$
-1	2	-4	0	$= f$
	$= s_1$	$= s_2$	$= g$	

22. A company wishes to assign six of its workers to six different jobs (one worker to each job and vice versa). The rating of each worker with respect to each job on a scale of 0 to 10 (10 being a high rating) is given by the following table:

(jobs)

(workers)		J_1	J_2	J_3	J_4	J_5
	W_1	5	4	2	8	5
	W_2	7	6	4	6	9
	W_3	5	5	3	3	2
	W_4	4	3	5	5	4
	W_5	3	6	4	10	2

If the company wishes to maximize the total rating of the assignment, find the optimal assignment plan and the corresponding maximum total rating.

(1 x 10= 10 marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2021

BMT5B09 – Calculus of Multivariable – I

(2019 Admission onwards)

Time: 2 hours Max. Marks: 60

Section A
All questions can be attended
Each question carries 2 marks

1. Describe the curve represented by $x = 4\cos\theta$ and $y = 3\sin\theta$ where $0 \leq \theta \leq 2\pi$.
2. State the area of a surface of revolution.
3. Plot the point $(4, \frac{\pi}{4})$ with the polar coordinates and $(0,5)$ with the rectangular coordinates.
4. Find parametric equation for the line passing through the point $P_0(-2,1,3)$ and parallel to the vector $v = (1,2,-2)$.
5. Sketch the graph of the cylinder $x^2 + y^2 = 4$
6. Find an equation in rectangular coordinates for the surface $r^2\cos 2\theta - z^2 = 4$.
7. Find the domain of the vector function $\boldsymbol{r}(t) = t\,\boldsymbol{i} + \frac{1}{t}\boldsymbol{j}$.
8. Define the arc length of the space curve.
9. Find $\frac{dr}{dt}$ if $\boldsymbol{r}(s) = 2\cos 2s\,\boldsymbol{i} + 3\sin 2s\,\boldsymbol{j} + 4s\,\boldsymbol{k}$, where $s = f(t) = t^2$.
10. Find the level surfaces of the function f defined by $f(x,y,z) = x^2 + y^2 + z^2$.
11. State the properties of continuous functions of two variables.
12. Find f_x and f_y if $f(x,y) = x\cos xy^2$.

(Ceiling 20 Marks)

Section B
All questions can be attended
Each question carries 5 marks

13. Show that the surface area of a sphere of radius r is $4\pi r^2$.

14. Find the angle between the two planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

15. Find the rectangular coordinates of the point $\left(2,0,\frac{\pi}{4}\right)$.

16. Find the antiderivative of $\boldsymbol{r}'(t) = \cos t\, \boldsymbol{i} + e^{-t}\boldsymbol{j} + \sqrt{t}\, \boldsymbol{k}$ satisfying the initial condition $\boldsymbol{r}(0) = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$.

17. Find the velocity vector , acceleration vector and speed of a particie with position vector $\boldsymbol{r}(t) = \sqrt{t}\boldsymbol{i} + t^2\boldsymbol{j} + e^{2t}\boldsymbol{k},\ t \geq 0$

18. Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

19. Suppose z is a differentiable function of x and y that is defined implicitly by $x^2 + y^2 - z + 2yz^2 = 5$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(Ceiling 30 Marks)

Section C
Answer any one question

20. Find the area of the region that lies outside the circle $r = 3$ and inside the cardioid $r = 2 + 2\cos\theta$.

21. (a) A particle moves along a curve described by the vector function $\boldsymbol{r}(t) = t\,\boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$. Find the tangential scalar and normal scalar components of acceleration of the particle at any time t.

    (b) Show that the function f defined by $f(x,y) = 2x^2 - xy$ is differentiable in the plane.

(1 x 10 = 10 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2021

(Open Course)

BMT5D03 – Linear Mathematical Models

(2019 Admission onwards)

Time: 2 hours Max. Marks: 60

Section A

All questions can be attended.

Each question carries 2 marks.

1. Find slope of the line $5x - 9y = 11$.
2. Find $g(\frac{1}{4})$, where $g(x) = 4x - 5$.
3. Write the augmented matrix for the system

$$x + y + 3z = 5$$

$$y - 3z = -2$$

$$2x + 6z = 8$$

4. Find the product AB if $A = \begin{bmatrix} 5 & 3 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 7 & 3 \end{bmatrix}$.
5. Graph the inequality $x + 3y \leq 6$.
6. State corner point theorem.
7. Define corner point of the feasible region.
8. Define point slope form of an equation of a line.
9. State duality theorem.
10. Identify basic variables and pivot element from the table.

x_1	x_2	x_3	s_1	s_2	z	
1	5	0	1	2	0	6
0	2	1	2	3	0	15
0	4	0	1	-2	1	64

11. Write standard form of minimization problem.
12. Find the transpose of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 10 & 0 \end{bmatrix}$.

(Ceiling 20 Marks)

Section B

All questions can be attended.

Each question carries 5 marks.

13. Find the equation of the line passing through the point (3,7) and perpendicular to the line having the equation $5x - y = 4$.

14. Solve the following system of equations using echelon method.

$$x + y = 5$$

$$2x - 2y = 2$$

15. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 4 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

16. Explain the method of solving a linear programming problem graphically.

17. Graph the feasible region for the system of inequalities

$$4x - y < 6$$

$$3x + y < 9$$

Is this region bounded or not?

18. Calculate correlation coefficient

x	1	2	3	4
y	1	2	3	4

19. State dual problem for linear programming problem

Maximize $\quad z = 2x_1 + 7x_2 + 4x_3$

Subject to: $\quad 4x_1 + 2x_2 + x_3 \leq 26$

$$x_1 + 7x_2 + 8x_3 \leq 33$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(Ceiling 30 Marks)

Section C

Answer any one question.

20. Solve the following system of equations using inverse method.

$$2x + y = 1$$

$$3y + z = 8$$

$$4x - y - 3z = 8$$

21. Solve the following linear programming problem using simplex method.

Maximize $\quad z = 40x_1 + 30x_2$

Subject to: $\quad x_1 + x_2 \leq 12$

$$2x_1 + x_2 \leq 16$$

$$x_1 \geq 0, x_2 \geq 0$$

(1× 10 = 10 Marks)