1B4M21530 (Pages : 1) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March/April 2021

BMT4C04 – Mathematics – 4

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

PART A

Answer all questions. Each question carries 2 marks.
Maximum mark from this section is 20.

1. Find the order and degree of the differential equation $\frac{d^3y}{dx^3} = ((1 + (\frac{dy}{dx})^2)^{3/2}$.
2. Give an example for a linear and a non-linear differential equation.
3. Define an autonomous differential equation and give an example.
4. Find a solution to the differential equation $x\frac{dy}{dx} = 4y$.
5. Find the integrating factor in simplified form for the differential equation
6. $\frac{dy}{dx} + y \tan x = \sin 2x$
7. Write the general form of Bernoulli's differential equation and explain how it can be solved
8. Define the Dirac Delta function and WRITE its Laplace transformation.
9. Find the inverse Laplace transformation of $F(s) = \frac{s+1}{s^2+1}$.
10. State the superposition principle for homogenous differential equations.
11. Verify whether the functions $f_1(x) = 5$, $f_2(x) = cos^2x$ and $f_3(x) = sin^2x$ are linearly dependent or independent in the interval $(-\infty,\infty)$
12. Find the wroskian of the two functions $f_1(x) = e^x$, $f_2(x) = e^{-x}$

PART B

Answer all questions. Each question carries 5 marks.
Maximum mark from this section is 30.

13. Derive the formula for the Laplace transformation of $f'''(t)$, where $L(f(t)) = F(s)$ is given.
14. Find the Fourier cosine series of the function $f(x) = x$.
15. Using the method of separation of variables, solve the partial differential equation $\frac{\delta^2 z}{\vartheta x^2} - 2\frac{\vartheta z}{\partial x} + \frac{\partial z}{\partial y} = 0$
16. Solve the differential equation $x^2y'' + xy' - y = 0$
17. Show that the differential equation $(e^{2y} - y\cos(xy))dx + (2xe^{2y} - x\cos(xy) + 2y)dy = 0$ is exact and hence solve the same.
18. Solve the initial value problem $\cos x\,(e^{2y} - y)\frac{dy}{dx} = e^y \sin(2x)$, $y(0) = 0$
19. Using the method of undetermined coefficients, solve $y'' + 4y = 8x^2$.

PART C

Answer any ONE question. One question carries 10 marks.

20. Using the Laplace transformation, solve the initial value problem $y'' - y = t$, $y(0) = 1$, $y'(0) = -1$
21. Find the Fourier Series of the function $f(x) = x$, $-\pi < x < \pi$

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March/April 2021

BST4B04 – Testing of Hypothesis

(2019 Admission onwards)

Time: 2.5 hours Max. Marks : 80

PART A

Each question carries 2 marks

1. Define critical region and power.
2. Explain null and alternative hypothesis.
3. Define uniformly most powerful test.
4. Define standard error and sampling distribution.
5. Distinguish between small sample tests and large sample tests.
6. What are the assumptions made for the application of students t test?
7. Point out the difference between one tailed and two tailed test.
8. What is analysis of variance and where it is used?
9. Explain the procedure for testing the significance of correlation coefficient.
10. Define sequential analysis.
11. Find the value of Chi square statistic for a contingency table

	B_1	B_2
A_1	2	28
A_2	13	7

after Yate's correction.

12. The standard deviation of a sample of size 15 from a normal population was found to be 7. Examine whether the hypothesis that the standard deviation is 7.6 is acceptable.
13. Define a run in a sequence of symbols.
14. If the observed and theoretical cumulative distribution functions are

Observed cdf: 0.038, 0.066, 0.093, 0.177, 0.288, 0.316, 0.371

Theoretical cdf: 0.036, 0.042, 0.129, 0.159, 0.243, 0.275, 0.238

Find the value of K – S test statistic.

15. Why do you consider Wilcoxon signed rank test superior to sign test?

Maximum Mark = 25

PART B

Each question carries 5 marks

16. It is decided to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{2}{3}$ where p denotes the probability of getting head when a coin is tossed by tossing the coin 4 times and rejecting the hypothesis if all the four throws result in heads. Obtain the level if significance and power.

17. Explain briefly the procedure followed in tests of statistical hypothesis.

18. In a sample of 600 men from a certain city 400 are found to be smokers. In 900 from another city 450 are smokers. Does the data indicate that the cities are significantly different as far smoking habits of people are concerned? ($\alpha = 0.05$)

19. Explain the test procedure for testing equality of means of two normal populations with equal variances.

20. The following figures give the prices in rupees of a certain commodity in a sample of shops selected at random from a city A. Assuming the distribution of prices to be normal, examine whether the standard deviation of prices is 0.30.

7.41, 7.77, 7.44, 7.40, 7.38, 7.93, 7.58, 8.28, 7.23, 7.52, 7.82, 7.71, 7.84, 7.63, 7.68.

21. Explain how the Chi square distribution may be used to test goodness of fit.

22. In a production process, the following sequence of defective (D) and good (G) items are obtained.

GGGDDGGDGGGGGDDDGGGGDDGDDD

Use run test at 5 % level of significance to determine the sequence is random.

23. Explain Kruskal Wallis method of analysis for one way classification of data.

Maximum Mark = 35

PART C

Each question carries 10 marks (Answer any **TWO** Questions)

24. State Neyman Pearson Lemma. Use Neyman Pearson theorem to find a most powerful test with significance level α for testing the hypothesis $H_0: \mu = \mu_0$ *against* $H_1: \mu = \mu_1 (\mu_1 > \mu_0)$ using a random sample $x_1, x_2, \ldots, x_n$ drawn from the population with pdf $f(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{1}{18}(x-\mu)^2}$, $-\infty < x < \infty$.

25. Below are given the yield (in kg) per acre for 5 trial plots of 4 varieties of treatment.

	Treatment			
Plot No.	1	2	3	4
1	42	48	68	80
2	50	66	52	94
3	62	68	76	78
4	34	78	64	82
5	52	70	70	66

Carry out an analysis of variance and state your conclusions.

26. 100 students were classified according to their brilliance and community. B_1 and B_2 denote two levels of brilliance and A_1, A_2, A_3 denote 3 communities. Examine whether there is any relationship between community and brilliance.

	B_1	B_2	Total
A_1	215	135	350
A_2	325	175	500
A_3	60	90	150
Total	600	400	1000

27. Explain Mann Whitney test. Given the scores of two groups of persons, the one under placebo and the other under drug are as follows:

Score under placebo (X)	Scores under drug (Y)
10	20
13	14
12	7
15	9
16	17
8	18
6	19
	25
	24

Test that distributions of scores under placebo and under drug are identical using Mann – Whitney U test. [Table value of U for $n_1 = 7, n_2 = 9$ *and* $\alpha = 0.05$ *is* 12].

(2 × 10 = 20 Marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Mathematics Degree Examination, March/April 2021

BST4C04 – Statistical Inference and Quality Control

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

SECTION-A

Each question carries 2 Marks.

Maximum Marks that can be scored in this section is 20.

1. Give any three situations in which c- chart can be used.
2. What is principle of least squares?
3. Write the observational equation in the case of one-way classification
4. Distinguish between null and alternative hypothesis
5. What is meant by interval estimation?
6. How sufficiency is related to conditional distribution?
7. Distinguish between defects and defectives
8. What is median test?
9. Distinguish between assignable causes and chance causes of variation
10. Let $X_1, X_2, ..., X_n$ be a random sample from $N(0, \theta^2)$. Give a point estimator of θ^2.
11. State Crammer- Rao inequality
12. Give the test statistic for testing the discrepancy between observed frequency and expected frequency.

SECTION-B

Each question carries 5 Marks.

Maximum Marks that can be scored in this section is 30.

13. Outline the various steps for ANOVA testing in two way classification
14. A random sample of size 25 is taken from a Normal population $N(\mu, 9)$. Determine a 95% confidence interval for μ.
15. A random sample of 10 boys had the following IQ's :70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do the data supports the assumption of a population mean IQ of 100 (use $\alpha = 0.05$)
16. Explain the method of moment estimation

17. Test the significance of the difference between the means of the samples from the following data (use $\alpha = 0.01$)

	Size	Mean	SD
Sample A	100	61	4
Sample B	200	63	6

18. What is the need for using Non parametric statistics

19. Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = 1/3$ against $H_1 : p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 2 heads obtained. Find the size and power of the test.

SECTION-C

(Answer *any one* Question and carries 10 marks)

20. (a) Explain F- test of equality of variances of two normal populations

(b) Following are the set of observations from two normal populations. Test the equality of these population variances at 5% level of significance

From first population	39	41	43	41	45	39	42	44
From second population	40	42	40	44	39	38	40	

21. Ten samples each of size 5 are drawn at regular intervals from a manufacturing process. The sample means ($\bar{X}$) and their ranges (R) are given below

Sample No	1	2	3	4	5	6	7	8	9	10
means ($\bar{X}$)	49	45	48	53	39	47	46	39	51	45
range (R)	7	5	7	9	5	8	8	6	7	6

Draw the control chart for means and ranges and comment the result

(for $n = 5, A_2 = 0.577, D_4 = 2.115, D_3 = 0$)

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Name: ………………………….

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March/April 2021

BAS4C04 – Probability Models and Risk Theory

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

PART A (Short answers)

Each question carries two marks. Maximum 20 marks

1. Explain saddle point in a zero- sum two -person game.
2. Define Minimax criterion.
3. Define proportional reinsurance
4. Define retention limit
5. State assumptions under individual risk model.
6. Define insurable interest
7. Define convolution
8. Give general features of an insurance product.
9. Define compound binomial distribution.
10. Explain surplus process
11. Define premium loading factor.
12. Define Lundeberg's inequality.

PART B

Each question carries 5 marks. Maximum 30 marks

13. (i) Explain Zero sum two person game

    (ii) Define the concept of Non proportional reinsurance arrangement.

14. The table gives the payoff matrix for a 2× 2 zero sum two person game.

PLAYER A

PLAYER B

6	-1
0	4

(i) Find the randomized strategy that should be adopted by each player and the value of the game.

(ii) State briefly why the players have adopted randomized strategies in this case.

15. S is a compound Poisson random variable with Poisson parameter λ and individual claim size distribution X with MGF $M_X(t)$. T is a compound Poisson random variable with Poisson parameter μ and individual claim size distribution Y with MGF $M_Y(t)$. Sand T are independent. Let U=S+T . Prove that U has compound Poisson distribution stating its Poisson parameter and individual claim MGF.

16. The distribution of number of claims from a motor portfolio is negative binomial with parameters k=4 and p=0.9. The claim size distribution is pareto with parameters α=5 and λ=1200. Calculate the mean and variance of the aggregate claim distribution.

17. Explain probability of ruin in continuous case.

18. S has compound Poisson distribution with Poisson parameter 4 and the individual claim amounts are either 1 with probability 0.3 or 3 with probability 0.7. Calculate the probability that S = 4.

19. An insurer considers that claims of certain type occur in accordance with compound Poisson process. The claim frequency for the whole portfolio is 100 per annum and individual claims have an exponential distribution with the mean of 8000. Calculate the adjustment coefficient if total premium rate for portfolio is 1000,000 per annum.

PART C

Each question carries ten marks. Maximum 10 marks

20. Claims on a motor insurance policy have a gamma distribution with mean 2000 and standard deviation 100. The insurer effects proportional reinsurance with retains proportion 85%. Determine

    (i) Mean amount paid by insurer

    (ii) Variance of the amount paid by reinsurer

    (iii) MGF of the amount paid by insurer.

21. An insurer knows from past experience that the number of claims received per month has Poisson distribution with mean 15 and the claim amounts have an exponential distribution with mean 500.The insurer uses a security loading of 30%. Calculate the insurers adjustment coefficient and give an upper bound for the insurers probability of ruin if insurer sets aside an initial surplus of 1000.

(1 x 10 = 10 Marks)