2M2M21413 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2021

MMT2C06 – Algebra II

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all questions. Each question carries 1 weightage

1. Find all $c \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x]/_{< x^2 + c >}$ is a Field.
2. Find $irr(\alpha, \mathbb{Q})$ and $deg(\alpha, \mathbb{Q})$ where $\alpha = \sqrt{2} + i$.
3. Find a basis for $\mathbb{Q}\left(2^{\frac{1}{2}}, 2^{\frac{1}{3}}\right)$ over $\mathbb{Q}$.
4. Show that a finite extension E of a finite field F is a simple extension of F .
5. Describe all extension of the identity map over $\mathbb{Q}$ to an isomorphism mapping $\mathbb{Q}(\sqrt[3]{2})$ onto a subfield of $\overline{\mathbb{Q}}$.
6. Find the order of $G\left(\mathbb{Q}(\sqrt[3]{2}, i\sqrt{3})/_{\mathbb{Q}}\right)$.
7. Show that $x^3 - 1$ is solvable by radicals over $\mathbb{Q}$.
8. Is regular 20-gon is constructible? Justify your answer.

($8 \times 1 = 8$ *weightage*)

Part B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Let F be a field and $f(x), g(x) \in F[x]$. Show that $f(x)$ devides $g(x)$ if and only if $g(x) \in < f(x) >$.
10. Let E be a simple extension $F(\alpha)$ of a field F and let α be algebraic over F. Let the degree of $irr(\alpha, F)$ be $n \geq 1$. Then show that every element β of $E = F(\alpha)$ can be unequally express in the form $\beta = b_0 + b_1\alpha + \cdots + b_{n-1}\alpha^{n-1}$ where $b_j$are in F.
11. Show that trisecting the angle is impossible. That is there exist an angle that cannot be trisect with a straightedge and a compass.

Unit II

12. If F is any finite field, then show that for every positive integer n there is an irreducible polynomial in $F[x]$ of degree n.
13. If E is a splitting field of finite degree over F then show that $\{E: F\} = \left|G\left(^{E}/_{F}\right)\right|$.
14. Show that every finite field is perfect.

Unit III

15. Find $\emptyset_{12}(x)$ in $\mathbb{Q}[x]$.

16. Prove that let $s_1 \dots s_n$ be the elementary symmetric functions in the indeterminates $y_1 \dots . y_n$. Then every symmetric function of $y_1 \dots . y_n$ over F is a rational function of the elementary functions. Also $(y_1 \dots . y_n)$ is a finite normal extension of degree $n!$ of $F(s_1$. and the Galois group of this extension is isomorphic to S_n.

17. Let K be the splitting field of $(x^2 - 2)(x^2 + 2)$ over $\mathbb{Q}$. Describe the Galois group $G\left(K\right.$

$(6 \times 2 = 12$ *weig*

Part C

Answer any two questions. Each question carries 5 weightage.

18.

a. Let R be a commutative ring with unity and let $N \neq R$ be an ideal in R then show R/N is an integral Domain if and only if N is a prime ideal.

b. Let R be a finite commutative ring with unity, show that every prime ideal in R i maximal ideal.

19.

a. If E is a finite extension field of a field F and K is a finite extension of E, then sh K is a finite extension of F and $[K:F] = [K:E][E:F]$.

b. If E is an extension field of F, $\alpha \in E$ is algebraic over F and $\beta \in F(\alpha)$then show $deg(\beta, F)$ divides $deg(\alpha, F)$.

20.

a. State and prove conjugation isomorphism theorem.

b. Show that complex zeros of polynomials with real coefficients occur in conjugat

21.

a. Let F be a field of Characteristic 0 and let $a \in F$ if K is the splitting field of x^n - F. Then prove that $G\left(K/_F\right)$ is a solvable group.

b. Let F be a field of Characteristic 0 and let $F \leq E \leq K \leq \bar{F}$ where E is a normal extension of F and K is an extension of F by radicals. Then prove that $G\left(E/_F\right)$ is solvable group.

$(2 \times 5 = 10$ we

43

2M2M21414 (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester MSc Degree Examination, March/April 2021

MMT2C07 – Real Analysis II

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part- A

Answer *all* questions. Each question has *one* weightage.

1. Define $\sigma-$algebras. Give one example.
2. If $m^*E = 0$, then prove that E is measurable.
3. Show that any real-valued continuous function with a measurable domain is measurable.
4. Prove that any bounded measurable function f defined on a set E of finite measure is integrable over E.
5. If f is a measurable function on E, then show that f^+ and f^- are integrable on E if and only if $|f|$ is integrable over E.
6. If $f_n \to f$ in measure on E, then show that there is a subsequence f_{n_k} that converges pointwise *a.e.* on E to f .
7. Show that a function of bounded variation can have at most a countable number of discontinuities.
8. Define absolute continuity. What is the relation between absolute continuity and continuity?

(8 × 1 = 8 Weightage)

Part- B

Answer any *two* from each unit. Each question has *two* weightage

Unit - I

9. Show that Lebesgue outer measure is countably subadditive.

10. Show that the Cantor set C is a closed, uncountable set of measure zero.

11. Define measurable functions. Show that a real valued function f defined on a measurable set E is measurable if and only if for each open set O, $f^{-1}(O)$ (the inverse image of O under f) is measurable.

Unit –II

12. If $\{f_n\}$ is a sequence of bounded measurable functions on a set E of finite measure, and if $\{f_n\} \to f$ uniformly on E, then show that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

13. State and prove Monotone Convergence Theorem. Give an example to show that Monotone Convergence Theorem may not hold for decreasing sequence of functions.

14. If E is of finite measure and if the sequence of functions $\{f_n\}$ is uniformly integrable over E and if $\{f_n\} \to f$ point wise a.e. on E, then prove that f is integrable over E and $\lim_{n\to\infty} \int_E f_n = \int_E f$.

Unit – III

15. Define $f(x) = \sin x$, on $[0, 2\pi]$. Find two increasing functions h and g for which $f = h - g$ on $[0, 2\pi]$.

16. If f is absolutely continuous on the closed, bounded interval $[a, b]$ then show that f is differentiable almost everywhere (a, b) and $\int_a^b f' = f(b) - f(a)$.

17. Show that, $\| f \|_p = \left[\int_E | f |^p \right]^{1/p}$ for $1 < p < \infty$, and $f \in L^p(E)$, defines a norm on $L^p(E)$.

(6× 2 = 12 Weightage)

Part- C

Answer any *two* from the following four questions.
Each question has *Five* weightage.

18. (a) Define measurable sets. Show that the collection of all measurable sets is a σ - algebra that contains the σ - algebra of Borel sets .

(b) Show that any subset of R with positive outer measure contains a non-measurable subset.

19. (a) If g is a measurable real valued function defined on a measurable set E and f is a continuous real valued function defined on $(-\infty, \infty)$, then show that $f \circ g$ is measurable.

(b) If f is a real valued measurable function defined on a measurable set E, then show that for each $\varepsilon > 0$ there is a continuous function g on $(-\infty, \infty)$ and a closed set F contained in E for which $f = g$ on F and $m(E-F) < \varepsilon$.

20. Show that any bounded function f defined on the closed, bounded interval $[a,b]$ is Riemann integrable over $[a,b]$ if and only if the set of points in $[a,b]$ at which f fails to be continuous has measure zero.

21. (a) State and prove Jordan's theorem.

(b) If the function f is continuous on the closed, bounded interval $[a,b]$, then show that f is absolutely continuous on $[a,b]$ if and only if the family of divided difference functions $\{\text{Diff}_h f\}_{0<h\leq 1}$ is uniformly integrable over $[a,b]$.

$(2 \times 5 = 10$ Weightage)

44

2M2M21415 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2021

MMT2C08 – Topology

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part-A

Answer all questions. Each question carries 1 weightage.

1. Describe the convergence of sequences in a discrete topological space.
2. Find the derived set of integers in the real line $\mathbb{R}$ with usual topology.
3. Let $X = \{a, b, c\}$ with the topology $\{X, \emptyset, \{a\}, \{a, b\}\}$. Write down the closed sets in X.
4. Define strong topology determined by a family of functions.
5. What do you mean by divisible Property? Give an example.
6. Define embedding of a topological space into another.
7. Give an example of a topological space that is T_1 but not T_2 .
8. State Urysohn's lemma.

(8× 1 = 8 weightage)

Part-B

Answer any two questions from each Unit. Each question carries 2weightages.

Unit - I

9. Prove that if a space is second countable then every open cover of it has a countable subcover.
10. Prove that second countability is a hereditary property.
11. Prove that a subset A of a space X is dense in X iff for every nonempty open subset B of X,$A \cap B \neq \emptyset$.

Unit - II

12. Prove that the product topology is the weak topology determined by the projection function.
13. Prove that every separable space satisfies the countable chain condition.
14. Prove that every path connected space is connected.

Unit – III

15. Prove that every regular, Lindel öf space is normal.
16. Prove that all metric spaces are T_4.
17. Let X be a completely regular space. Suppose F is a compact subset of X,C is a clo subset of X and $F \cap C = \emptyset$.Prove that there exists a continuous function from X i the unit interval $[0,1]$which takes the value 0 at all points of F and the value 1 at a points of C.

($6 \times 2 = 12$ weightag

Part-C

Answer any two question. Each question carries 5 weightages.

18. (a) Let X be a set, $\mathcal{T}$ a topology on X. Then prove that $\mathcal{S}$ is a sub base for $\mathcal{T}$ if and if $\mathcal{S}$ generates $\mathcal{T}$.

    (b)For a subset A of a space X, prove that

    $\bar{A} = \{ y \in X : \text{every neighbourhood of y meets A non} - \text{vacuously}\}$
19. (a)Prove that a subset of $\mathbb{R}$ is connected iff it is an interval.

    (b)Every continuous real valued function on a compact space is bounded and attai its extrema.
20. (a) State and prove Lebesgue covering lemma

    (b)Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces and $f: X \to Y$ be a function. T prove that f is continuos if and only if $f^{-1}(V)$ is open in X for every open subset in Y .
21. (a) Suppose a topological space X has the property that for every closed subset A X,every continuous real valued function on A has a continuous extension to X. Th show that X is normal.

    (b) Let A be a closed subset of a normal space $X$and suppose $f: A \to [-1,1]$ is a continuous function. Then prove that there exists a continuous function $F: X \to [-1,1]$such that $F(x) = f(x)$ for all $x \epsilon A$.

($2 \times 5 = 10$ weight

45

2M2M21416 (Pages : 3) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester MSc Degree Examination, March/April 2021

MMT2C09 – ODE and Calculus of Variations

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all the questions. Each question has weightage 1.

1. Find a power series solution of the differential equation $y' = 2xy$.
2. Locate and classify the singular points on the X axis of the differential equation $(3x+1)xy'' - (1+x)y' + 2y = 0$.
3. Find the first three terms of the Legendre series of f(x) $= e^x$.
4. Describe the phase portrait of the following system.

$$\begin{cases} \dfrac{dx}{dt} = x \\ \dfrac{dy}{dt} = 0 \end{cases}$$

5. Find the critical points of the system $\begin{cases} \frac{dx}{dt} = y^2 - 5x + 6 \\ \frac{dy}{dt} = x - y \end{cases}$
6. Starting with $y_0(x) = 0$, apply Picard's method to calculate $y_1(x), y_2(x)$ and$y_3(x)$ of the initial value problem $y' = 2x(1+y), \quad y(0) = 0$.
7. Find the point on the plane $ax + by + cz = d$ that is nearest to the origin.
8. State Sturm Seperation theorem.

$(8 \times 1 = 8\, weightage)$

Part B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Prove that the equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenious series solution and find it.
10. Show that the Gauss hypergeometric equation has regular singular points 0,1 & ∞.with corresponding exponents $0 \,\&\, 1-c, 0 \,\&\, c-a-b$ and $a \,\&b$.
11. Find the general solution of the equation $(x^2 - x - 6)y'' + (5+3x)y' + y = 0$ near its singular point $x = 3$.

Unit II

12. Solve the system $\begin{cases} \frac{dx}{dt} = 3x - 4y \\ \frac{dy}{dt} = x - y \end{cases}$

13. Determine the nature and stability properties of the critical point of the system

$$\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$$

14. Determine whether the function $f(x,y) = -x^2 - 4xy - 5y^2$ is positive definite, negative definite or neither.

Unit III

15. Prove that the eigen functions of the boundary value problem

$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + \lambda q(x)y = 0, \qquad y(a) = y(b) = 0$satisfy the relation

$\int_a^b q y_m y_n dx = 0$ if $m \neq n$.

16. Obtain Eulers differential equation $\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y}=0$

17. Let $y(x)$ and $z(x)$ be non-trivial solutions of $y'' + q(x)y = 0$ and $z'' + r(x)z = 0$ where $q(x)$ and $r(x)$ are positive functions such that $q(x) > r(x)$. Show that $y(x)$ vanishes atleast once between any two successive zeros of $z(x)$.

$(6 \times 2 = 12$ *weightag*

Part C

Answer any two questions. Each question carries weightage 5.

18. (a) Show that if $P_n(x)$ is the Legendre polynomial of degree n, then

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 \ if \ m \neq n \\ \frac{2}{2n+1} \ if \ m = n \end{cases}$$

(b). Show that $P_{2n+1}(0) = 0$and $P_{2n}(0) = \frac{(-1)^n 1.3 \ldots (2n-1)}{2^n n!}$.

19. (a). Obtain $J_p(x)$ as a solution of Bessel's equation$x^2y'' + xy' + (x^2 - p^2)y = 0$ where p is a non negative constant.

(b). Show that$\frac{2p}{x}J_p(x) = J_{p-1}(x) + J_{p+1}(x)$.

20. (a). State and prove Liapunov's stability theorem for an isolated critical point of the autonomous system $\begin{cases} \frac{dx}{dt} = F(x,y) \\ \frac{dy}{dt} = G(x,y) \end{cases}$

(b). Show that (0,0) is a stablecritical point of the system $\begin{cases} \frac{dx}{dt} = -2xy \\ \frac{dy}{dt} = x^2 - y^3 \end{cases}$

21. (a). Let $f(x,y)$ be a continuous function that satisfies the Lipchitz condition $|f(x,y_1) - f(x,y_2)| \leq k|y_1 - y_2|$ on a strip defined by

$a \leq x \leq b$and $-\infty < y < \infty$.

Prove that if(x_0, y_0) is any point of the strip, then the IVP

$$y' = f(x,y), \qquad y(x_0) = y_0$$

has one and only one solution $y = y(x)$ on $a \leq x \leq b$.

(b). Solve the initial value problem by Picard's method

$$\begin{cases} \frac{dy}{dx} = z & y(0) = 1 \\ \frac{dz}{dx} = -y & z(0) = 0 \end{cases}$$

($2 \times 5 = 10$ *weightage*)

2M2M21417 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2021

MMT2C10 – Operations Research

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer question (1-8)

Answer all questions. Each question has 1 weightage.

1. Prove that the sum of two convex function is again a convex function.
2. Why do we introduce new variables in linear programming problem ?
3. Write the dual of the linear programming problem: Maximise $x_1 + 6x_2 + 4x_3 + 6x_4$ subject to

   $2x_1 + 3x_2 + 17x_3 + 80x_4 \leq 48, \quad 8x_1 + 4x_2 + 4x_3 + 4x_4 = 2, \; x_1, x_2 \geq 0$ *and*

   x_3 *and* x_4 *are unrestricted in sign.*
4. What are simplex multipliers in a linear programming problem.
5. Define transportation matrix . Write an example for transportation matrix.
6. What is Caterer Problem in Operation Research.
7. Explain the terms mixed strategy, pure strategy, and optimal strategy with reference to any matrix game.
8. Describe minimum path problem in network analysis. **(8 x 1 = 8 weightage)**

Part B

Answer any two questions from each units (9-17)

Each question has weightage 2.

UNIT I

9. State and prove necessary and sufficient conditions for a differentiable function $f(x)$ defined in a convex domain to be a convex function.
10. Prove that a vertex of the convex set of feasible solution is a basic feasible solution.
11. Solve graphically the linear programming problem: Minimize $z = x_1 + 3x_2$ subject to

    $x_1 + x_2 \geq 3, \quad -x_1 + x_2 \leq 2, \quad x_1 - 2x_2 \leq 2, \quad x_1, x_2 \geq 0$

UNIT II

12. If the primal problem is feasible, then prove that it has an unbounded optimum iff the dual has no feasible solution and vice versa.
13. State and prove complimentary slackness conditions.
14. Prove that the transpotation array has a triangular basis.

UNIT III

15. Let $f(X,Y)$ be such that both $\max_X \min_Y f(X,Y)$ and $\min_Y \max_X f(X,Y)$ exist . Then pro necessary and sufficient condition for the existence of a saddle point (X_0, Y_0) of $f(X,Y)$ $f(X_0,Y_0) = \max_X \min_Y f(X,Y) = \min_Y \max_X f(X,Y)$.
16. Describe the generalised problem of maximum flow.
17. Describe cutting plane method to solve an integer linear programming problem.

(6 x 2 = 12 weightag

PART C

Answer any two questions from the following four questions (18-21)
Each question has weightage 5.

18. a) What is mean by canonical form of equations.

    b) Maximize $f(x) = 5x_1 + 3x_2 + x_3$ subject to constrains $2x_1 + x_2 + x_3 = 3; -x_1 + 2x_3$ $x_1, x_2, x_3 \geq 0$.

19. a) Solve the transportation problem for minimum cost with cost coefficients demands and su as in the following table.

	D_1	D_2	D_3	D_4	
O_1	3	2	5	4	25
O_2	4	1	7	6	35
O_3	7	8	3	5	30
	10	18	20	42	

    b) What is mean by an unbalanced transportation problem. How can we solve it ?

20. (a) Describe the branch and bounded method in integer linear programming problem. Illustr an example..

    (b) What is mean by Mixed integer linear programming problem.

21. (a)Solve graphically the game whose pay-off matrix is $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

    (b) Describe rectangular game as a linear programming problem .

(2 x 5= 10 weightage)