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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March /April 2019

MT4E02 – Algebraic Number Theory

(2017 Admission onwards)

Time: 3 hours Max. weightage : 36

PART A

Answer all questions.

Each questions has one weightage 1

1. Show that the coefficients of the field polynomial are rational numbers.
2. Find the minimal polynomial of $i+\sqrt{2}$ over Q, the field of rational.
3. What are the units in $\mathbb{Q}(\sqrt{-1}\,)$?
4. Find an integral basis for $Q(\sqrt{5})$.
5. If $\{a_1, a_2, a_3, \dots, a_n\}$ is a basis of K consisting of integers, then show that $\Delta[a_1, a_2, a_3, \dots, a_n]$ is a rational integer, not equal to zero.
6. If $K = Q(\xi)$ where $\xi = e^{\frac{2\pi i}{5}}$ find $N_K(\xi^2)$.
7. Define norm and trace of an element α of the number field.
8. Prove that an associate of an irreducible is irreducible.
9. Let R be a ring and a an ideal of R. Then show that a is maximal if R/a is a field.
10. If x and y are associates, prove that $N(x) = \pm N(y)$.
11. Let K be a number field of class number h, and a an ideal of the ring of integers, then show that a^h is principal.
12. Sketch the lattice in R^2 generated by (0,1) and (1,0).
13. Define the volume v(X) where $X \subset R^n$.
14. Define class group.

(14x1 = 14 weightage)

PART B

Answer any seven from the following ten questions.
Each questions has weightage of 2

15. Show that an algebraic number α is an algebraic integer if and only if its minimum polynomial over $\mathbb{Q}$ has coefficient in $\mathbb{Z}$.
16. Express the polynomials $t_1^2 + t_2^2 + t_3^2$ and $t_1^3 + t_2^3$ in terms of elementary symmetric polynomials.
17. Prove that the set **A** of algebraic numbers is a subfield of the complex field **C**.
18. If $\{a_1, a_2, a_3, \dots, a_n\}$ is any $Q - basis$ for K, then $\Delta[a_1, a_2, a_3, \dots, a_n] = \det(T(a_i a_j)$
19. Find the minimal polynomial of $\xi = e^{\frac{2\pi i}{p}}$, p is an odd prime, over Q and find its degree
20. Prove that factorization into irreducibles is not unique in $Q(\sqrt{-26})$.
21. Evaluate $| \mathbb{Z}[\sqrt{-17}/< 18 > |$.
22. If **D** is the ring of integers of a number field **K**, and if **a** and b are non-zero ideals of **D**, then show that **N(ab)=N(a) N(b).**
23. Show $x^4 + y^4 = z^2$ has no integer solutions with $x, y, z \neq 0$.
24. Define a circle group. Show that $\mathbb{R}/\mathbb{Z} \cong S$.

(7x 2 = 14 weightag

PART C

Answer any two from the following questions.
Each questions has weightage of 4

25. a)If K is a number field, Then prove that $K = Q(\theta)$ for some algebraic number θ.

    b) Express $Q(\sqrt{2}, \sqrt{3})$ in the form of $Q(\theta)$.
26. Prove that every subgroup H of a free Abelian group G of rank n is a free of rank $\leq n$. Also prove that there exists a basis $u_1, u_2, \dots. u_s$ for G and positive integers $\alpha_1, \alpha_2, \dots. \alpha_n$ such that $\alpha_1 u_1, \alpha_2 u_2, \dots. \alpha_s u_s$ is a basis for H.
27. a) Define an integral basis. Show that every number field possess an integral basis.

    b)Let $K = \mathbb{Q}(\theta)$ be a number field, where θ has minimum polynomial p of degree n. Show that the $\mathbb{Q} - basis\ \{1, \theta, \theta^2, \dots, \theta^{n-1}\}$ has discriminant.

    $\Delta[1, \theta, \theta^2, \dots, \theta^{n-1}] = (-1)^{\frac{n(n-1)}{2}} N(D_p(\theta))$ where D_p is the formal derivative of p.
28. a)Show that an additive subgroup of $\mathbb{R}^n$ is a lattice if and only if it is discrete.

    b)Define a fundamental domain. Sketch fundamental domain of the lattice generated by $\{(1,0), (0,1)\}$ in $\mathbb{R}^2$.

(2 x 4 = 8 weight

Name: ....

FARook COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March /April 2019

MT4E14 – Differential Geometry

(2017 Admission onwards)

Time: 3 hours

Max. weightage: 36

Part A

Answer All questions.

Each question carries 1 weightage

1. Sketch the vector field on R^2: Where $X(x_1, x_2) = (x_2, x_1)$.
2. Show that the gradiant of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.
3. Show that the graph of a smooth function $f: U \subset R^n \rightarrow R$ is an n – surface in R^{n+1}.
4. Find and sketch the gradiant field of the function $f(x_1, x_2) = x_1 - x_2^2$.
5. Show that if $\alpha: I \rightarrow R^{n+1}$ is a parametrized curvewith constant speed then $\ddot{\alpha}(t) \perp \dot{\alpha}(t)$ for all $t \in I$.
6. Let$f: U \rightarrow R$ be a smooth function and $\alpha: I \rightarrow U$be an integral curve of ∇f. Show that $\frac{d}{dt}(fo\alpha)(t) = \|\nabla f(\alpha(t))\|^2$ for all $t \in I$.
7. Find the velocity, the acceleration and the speed of the curve $\alpha(t) = (t,\ t^2)$.
8. Let S be an n-surface in R^{n+1}, let $\alpha: I \rightarrow S$ be a parametrized curve and let X and Y are vector fields tangent to S along α. Verify that $(fX)' = f'X + fX'$.
9. Let X and Y be smooth vector fields along parametrized curve $\alpha: I \rightarrow R^{n+1}$ and let $f: I \rightarrow R$ be smooth function along α. Verify that $(X\,.\,Y)' = \dot{X}.Y + X.\dot{Y}$.
10. Prove that the geodesic have constant speed.
11. Compute $\nabla_v f$ where $f: R^{n+1} \rightarrow R$ and $v \in R_p^{n+1}, p \in R^{n+1}$ are given by
$$f(x_1, x_2, x_3) = x_1 x_2 x_3^2, v = (1,1,1,a,b,c).$$
12. Find the length of the parametrized curve $\alpha: I \rightarrow R^{n+1}$ given by $\alpha(t) = (t^2, t^3), I = [0,2], n = 1$.
13. Find the Gaussian curvature $K: S \rightarrow R$ where S is given by $x_1^2 + x_2^2 - x_3^2 = 0, x_3 > 0$.
14. Define Geodesic in an n-surface.

(14 x 1 = 14 weightage)

Answer any seven questions.

Each question carries 2 weightage.

15. Find the integral curve through p=(1,1) of the vector field X(p) = p.

16. Let S be an n – surface in R^{n+1},S =$f^{-1}(c)$ where $f: U \to R$ is such that $\nabla f(q) \neq 0$ for all $q \in S$. Suppose $g: U \to R$ is a smooth function and $p \in S$ is an extreme point of g on S, Then Show that there exist a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$.

17. Show that the unit n- sphere $x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = 1$ is connected if n > 1.

18. Let $S \subset R^{n+1}$ be a connected n-surface in R^{n+1}. Then show that there exist on S exactly two smooth unit normal vector fields N_1 and N_2 and $N_2(p) = -N_1(p)$ for all $p \in S$.

19. Let S be an n-surface in R^{n+1},let $p, q \in S$, and let α be a piecewise smooth parametrized curve from p to q .Then show that parallel transport $P_\alpha: S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.

20. Let S denote the cylinder $x_1^2 + x_2^2 = r^2$ of radius r > 0 in R^3. Show that α is a geodesic of S if and only if α is of the form
$\alpha(t) = (r\cos(at+b), r\sin(at+b), ct+d)$ for some $a, b, c, d \in R$.

21. Show that the Weingarten map at each point of a parameterized n-surface is self adjoint.

22. Show that on each compact oriented n-surface S in R^{n+1}, there exist a point p such that the second fundamental form at p is definite.

23. Prove that, in an n-phase, parallel transport is path independent.

24. Show that the spherical image of an n-surface with orientation N is the reflection through the origin of the spherical image of the same n-surface with orientation –N.

(7 x 2 = 14 weightag

Part C

Answer any two questions.

Each question carries 4 weightage

25. Let S be an n-surface in R^{n+1}, let $p \in S$and $v \in Sp$. Then show that there exist an open interval containing 0 and a geodesic $\alpha: I \to S$ such that
    (i) $\alpha(0) = p$ and $\dot{\alpha}(o) = v$
    (ii) If $\beta: \tilde{I} \to S$ is any other geodesic in S with $\beta(0) = p$ and $\dot{\beta}(0) = v$, then $\tilde{I} \subset I$ and $\beta(t) = \alpha(t)$ or all $t \in \tilde{I}$.

26. Let $\varphi: U \to R^{n+1}$ be a parametrized n-surface in R^{n+1} and let $p \in U$. Then show that there exist an open set $U_1 \subset U$ about p such that $\varphi(U_1)$ is an n-surface in R^{n+1}.

27. Let S be a compact connected oriented n-surface in R^{n+1} whose Gauss- Kronecker curvature is nowhere zero. Then show that
    (i)The Gauss map $N: S \to S^n$ is one to one and onto. (ii)S is strictly convex.

28. Let C be an oriented plane curve. Then show that there exists a global parametrization of C if and only if C is connected.

(2 x 4 = 8 weightage

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March /April 2019

MT4E08 – Probability Theory

(2017 Admission onwards)

Time: 3 hours Max. weightage: 36

Part A

Answer All questions

Each question has weightage 1.

1. Find the probability of getting 1 head and 3 tails when a fair coin is tossed 4 times.
2. Find the distribution function of the random variable X with pmf

$$f(x) = \begin{cases} \frac{x^2}{30}, & x = 1, 2, 3, 4 \\ 0, & otherwise \end{cases}$$

3. Can $f(x) = \begin{cases} \sin x, & 0 < x < \frac{\pi}{2} \\ 0, & otherwise \end{cases}$ be the density function of a random variable?
4. Let X be a random variable with distribution function

$$F(x) = \begin{cases} k(1-e^{-x})^2, & x > 0 \\ 0, & otherwise \end{cases}$$ Find k.

5. Let X be a random variable. Prove that $|X|$ is also a random variable.
6. What is the significance of second central moment of a random variable?
7. State Lyapunov Inequality?
8. Show that V (X) = 0 only if X is degenerate.
9. A fair coin is tossed three times. Let X denote the number of heads and Y denote the number of tails obtained. Give the joint pmf of X and Y .
10. State Cauchy Schwarz Inequality.
11. If $f(x, y) = \begin{cases} 2, & 0 \le y \le x \le 1 \\ 0, & otherwise \end{cases}$ Find E(XY).
12. If Cov(X,Y)=0, are the random variables X and Y independent? Justify.
13. Does convergence in distribution imply convergence in moments? Justify.
14. Can you find the mgf for a Cauchy pdf ?

(14 x 1 = 14 weightage)

PART-B

Answer any seven from the following ten questions.

Each questions has weightage of 2

15. Let $f(x, y) = \begin{cases} kxy, & 0 < x < y < 1 \\ 0, & otherwise \end{cases}$

Find k and $E(Y|X = x)$.

16. Let X and Y be two identically distributed random variables. Prove or disprove $P\{X = Y\} = 1$.

17. Let X, Y be independent and identically distributed random variables with comm

pdf $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & otherwise \end{cases}$

Show that X + Y and $\dfrac{X}{X+Y}$ are independent.

18. Let X be distributed with pdf $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & otherwise \end{cases}$

Find the upper bound for $P\left\{\left|X - \dfrac{1}{2}\right| \geq k / \sqrt{12}\right\}$.

19. Prove that the set of discontinuity points of a distribution function is atmost cou

20. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a population with pdf

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & otherwise \end{cases}$$

Find the distribution of $\min(X_1, X_2, \ldots, X_n)$.

21. X and Y are jointly distributed discrete random variables with probability functi

$$f(x, y) \begin{cases} \dfrac{1}{4}, & at (x, y) = (-3, -5), (-1, -1), (1, 1)(3, 5) \\ 0, & otherwise \end{cases}$$

Compute E(X), E(Y),E(XY). Are X and Y independent?

22. Prove that X and Y are independant random variables if and only if

$M(t_1, t_2) = M(t_1, 0)\, M(t_2, 0)$, for all $t_1, t_2 \in R$.

23. Let X_n be distributed as χ^2 (n), n = 1, 2,….. Find the limiting distribution of X_n

24. If $X_n \xrightarrow{a.s} X$ then prove that $X_n \xrightarrow{P} X$ **(7 x 2 = 14 weigh**

PART- C

Answer any two from the following questions.
Each questions has weightage of 4

25. A fair coin is tossed 4 times. Let X denote the number of times a head is followe immediately by a tail. Find the probability distribution. mean and variance of X. find the probability distribution if the coin is biased with probability of head 'p'=

26. Let (X_1, X_2) have uniform distribution on the triangle $0 \leq x_1 \leq x_2 \leq 1$. Find the de function of $Y = X_1 + X_2$.

27. State and prove Lindberg-Levy Central Limit Theorem.

28. Let $\{X_n\}$ be a sequence of iid Random variables with a common finite mean μ.

prove that $\dfrac{S_n}{n} \xrightarrow{P} \mu$ as $n \longrightarrow \infty$ where $S_n = \sum_{k=1}^{n} X_k$. **(2 x 4 = 8 weigh**

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M4M191201 (Pages : 1) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March /April 2019

MT4E12 – Computer Oriented Numerical Analysis

(2017 Admission onwards)

Time: 1.5 hours Max. weightage : 18

Part A (Short Answer Questions)

(Answer all questions. Each question has weightage 1)

1. Write the output of 5/3 and 8%3.
2. Explain break statement with examples.
3. Write an algorithm to find the GCD of two numbers.
4. Write a python program that uses while loop.
5. Write a python program to display the first n terms of Fibonacci sequence.
6. Write an algorithm to find the biggest from among n numbers.

(6 x 1=6 weightage)

Part B

(Answer any four from the following six questions. Each question has weightage 2)

7. Write a python program that uses functions.
8. Write an algorithm to find the $\int_a^b f(x)dx$ using trapezoidal rule.
9. Explain Lagrange's interpolation algorithm.
10. Write an algorithm to solve the initial value problem by using Runge Kutta method of order 4.
11. Write a short note on the data structures List and Tuple.
12. Write an algorithm to find the derivative of continuous function.

(4x2=8 weightage)

Part C

Answer any one from the following two questions. Each question has weightage 4

13. Write an algorithm and python programme to find the root of the given continuous function f(x) on [a,b] by using bisection method.
14. Write an algorithm and python programme to find the integral using tabulated values by the method of Simpson rule.

(1x4=4 weightage)