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1M4M21531 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2021

MMT4C15 - Advanced Functional Analysis

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer *all* questions. Each carries 1 weightage

1. If $A_n \to A$ and $B_n \to B$ in $BL(X)$ then prove that $A_nB_n \to AB$ in $BL(X)$.
2. Give an example of an operator whose eigen spectrum and approximate eigen spectrum are not equal.
3. Give an example to show that $x_n \xrightarrow{w} x$ in X does not imply that $x_n \longrightarrow x$ in X.
4. Define reflexive normed spaces. Show that if X is reflexive and separable, then X' is separable.
5. Is every continuous linear map compact? Justify your answer.
6. Let X be an inner product space and $E \subset X$. If $F = \overline{\text{span}\, E}$, prove that $F^{\perp} = E^{\perp}$.
7. Show that every orthogonal projection is a positive operator.
8. Show that if $A \in BL(H)$ is a Hilbert-Schmidt operator, so is A^*.

($8 \times 1 = 8$ weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. If $A \in BL(X)$ is of finite rank, prove that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.
10. Show that the dual space of c_0 with $\|\cdot\|_\infty$ is linearly isometric to ℓ^1.
11. If X, Y and Z are normed spaces, and if $F \in BL(X,Y)$ and $G \in BL(Y,Z)$, then show that $(GF)' = F'G'$. Also prove that $\| F \| = \| F' \| = \| F'' \|$ and $F''J_X = J_Y F$.

Unit 2

12. If A is a compact operator, prove that every non zero spectral value of A is a value of A.

13. State projection theorem. Give an example to show that projection theorem n hold for an incomplete inner product space.

14. Show that every Hilbert space is reflexive.

Unit 3

15. If $A \in BL(H)$ is self-adjoint, then prove that $\| A \| = \sup \{ |\langle A(x), x \rangle| : x \in H, \| x$

16. If $A \in BL(H)$, prove that $\sigma_e(A) \subset \sigma_a(A)$ and $\sigma(A) = \sigma_a(A) \cup \{ k : \bar{k} \in \sigma_e(A^*)$

17. If $A \in BL(H)$ is compact, show that $A*$ is also compact.

(6 × 2 = 12 wei

Part C

Answer any *two* questions. Each carries 5 weightage

18. (a) If X is a nonzero Banach space over C and if $A \in BL(X)$, then show that non empty.

    (b) State and prove spectral radius formula.

19. (a) Let X and Y be normed spaces and $F \in BL(X,Y)$. If $F \in CL(X,Y)$, then sh $F' \in CL(Y', X')$. Also, show that the converse holds if Y is a Banach space.

    (b) State and prove Riesz representation theorem.

20. (a) If A is a compact operator on a normed space X, prove that the eigen spect the spectrum of A are countable sets and have 0 as the only possible limit point

    (b) Let H be a Hilbert space, G be a subspace of H and g be a continuou functional on G. Prove that there is a unique continuous linear functional f such that $f|_G = g$ and $\| f \| = \| g \|$.

21. (a) State and prove generalized Schwarz inequality.

    (b)Define numerical range. If $A \in BL(H)$ prove that $\sigma_e(A) \subset \omega(A)$ and contained in the closure of $\omega(A)$.

(2 × 5 = 10 wei

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2021

MMT4E06 –Algebraic Number Theory

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Answer all the questions. Each question carries 1 weightage.

1. Define symmetric polynomials and elementary symmetric polynomials. Give an example for each polynomials
2. Define an algebraic number and algebraic integer. Which of the following numbers are algebric integers? a, $\frac{355}{113}$, b, $\sqrt{17}+\sqrt{19}$
3. Find the norm and trace of $1-\zeta$ in $K=Q(\zeta)$, where $\zeta = e^{\frac{2\pi i}{5}}$
4. Show that the group of units U of the integers in $Q(\sqrt{-1})$ is $U=\{1, -1, i, -i\}$.
5. Let **R** be a ring and **a** an ideal of **R**. Then show that **a** is maximal if and only if **R**/**a** is a field.
6. Define the $n-$ dimensional torus T^n. If L is an $n-$ dimensional lattice in R^n then show that R^n/L is isomorphic to T^n
7. Define class number and class group.
8. Define the norm of an element in L^{st}

(8×1=8 Weightage)

PART B

Answer any two from each unit. Each question carries 2 weightage

Unit I

9. Let G be a free abelian group of rank r, and H a subgroup of G. Then show that G/H is finite if and only if the ranks of G and H are equal. If this is the case, and if G and H have Z-bases $x_1, x_2, , x_r$ and $y_1, y_2, ..., y_r$ with $y_i = \sum a_{ij}x_j$, then prove that $|G/H| = |det(a_{ij})|$.
10. Let $K = Q(\theta)$ be a number field of degree n over Q. Show that there are exactly n distinct monomorphisms $\sigma_i : K \to C$ and the elements of $\sigma_i(\theta) = \theta_i$ are the distinct zeros in C of the minimum polinomial θ over Q.

11. If $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is any Q-basis of K, then show that $\Delta[\alpha_1, \alpha_2, ..., \alpha_n]=$ det $(T(\alpha_i, \alpha_j))$.

Unit II

12. Show that factorization into irreducibles is not unique in the ring of integers of $Q(\sqrt{10})$.

13. In $Z[\sqrt{-6}]$, obtain the prime factorization of $< 6 >$

14. In $Z[\sqrt{-17}]$, prove that the elements 2, 3 are irreducible but not prime.

Unit III

15. Define lattice . Sketch the lattice in R^2 generated by $\{(1,2),(2,-1)\}$

16. Factorize the following principal ideals $< 2 >$, $< 5 >$ in the ring of integers of $Q(\sqrt{5})$.

17. Find all the solutions of the equation $x^2 + y^2 = z^2$

(6×2=12 Weightage)

PART C

Answer any two questions. Each question carries 5 weightage

18. a, Find the order of the group G/H where G is free abelian with Z-basis $x.y.z$ and H is generated by: $41x + 32y - 999z$, $16y + 3z$, $2y + 111z$
b, Express $Q(\sqrt{2}, \sqrt[3]{5})$ in the form $Q(\theta)$.
c, Compute an integral bases and discriminent for $Q(\sqrt{2}, \sqrt{3})$

19. Show that the ring of integers of $Q(\zeta)$ is $Z[\zeta]$

20. a, Show that every Euclidean domain is a unique factorization domain.
b, Define a noetherian ring. Find a ring which is not noetherian.

21. a, Show that an additive subgroup of R^n is a discrete if it is a lattice.
b, State and prove Minkowski's theorem

(2×5=10 Weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2021

MMT4E09 –Differential Geometry

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PARTA

Answer ALL questions. Each question has1 weightage.

1. Show that the graph of any function $f: R^n \to R$ is the level set for some function $F: R^{n+1} \to R$
2. Find and sketch the gradient field of the function $f(x_1, x_2) = x_1 + x_2$.
3. Show that the set $S$of all unit vectors at all points of R^2forms a 3-surface in R^4
4. Prove that the straight line segment $\alpha(t) = p + tv$ is a geodesic S.
5. Compute$\nabla_v f$ where $f(x_1, x_2) = x_1^2 - x_2^2, \boldsymbol{v} = (1,1, \cos\theta \sin\theta)$.
6. Find the length of the parametrized curve $\alpha: I \to R^4$where $I = [0,2\pi]$ and $\alpha(t) = (\cos t, \sin t, \cos t, \sin t)$
7. Find the Gaussian curvature at a point $\boldsymbol{p}$of a surface Swhose principal cuvatures are $k_1(p) = 1$ and $k_2(\boldsymbol{p}) = \frac{1}{2}$.
8. Let $\phi: U_1 \to U_2$ and $\varphi: U_2 \to R^k$ be smooth,where $U_1 \subset R^n$ and $U_2 \subset R^m$.Verify the chain rule $d(\varphi o \phi) = d\varphi\ o\ d\phi)$.

$(8\times 1 = 8\, weightage)$

PART B

Answer any two questions from each unit. Each question carries 2 weightage.

Unit I

9. Find the integral curve through $p(a, b)$ of the vector field **X**on R^2 given b $\boldsymbol{X}(p) = (p, X(p))$ where $X(x_1, x_2) = (-x_2, -x_1)$.
10. Show by an example that the set of vectors tangent at a point of a level set need not be in general be a vector subspace of R_p^{n+1}.
11. Describe the spherical image of the sphere $x_1^2 + x_2^2 + \cdots \ldots\ldots + x_{n+1}^2 = 1$,chosing the orientation as$N = \frac{\nabla f}{\|\nabla f\|}$.

Unit II

12. Let **X**and **Y** be smooth vector fields along the parametrized curve $\alpha: I \to R^{n+1}$.Verify that $(\boldsymbol{X}.\boldsymbol{Y})' = \boldsymbol{X}'.\boldsymbol{Y} + \boldsymbol{X}.\boldsymbol{Y}'$.
13. Show that the Weingarten map at each point p of an oriented n-surface in R^{n+1} is self adjoint.
14. Prove that the local parametrization of a plane curve is unique upto reparametriztion.

UNIT III

15. Show that the normal curvature of the sphere$x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = r^2(N =$

any point in any direction is the constant $\frac{1}{r}$..

16. Describe a parametrized torus in R^4.

17. Let S be an n-surface in R^{n+1} and let $p \in S$ .Prove that there exists an ope

about p inR^{n+1} and a parametrized n-surface $\varphi: U \to R^{n+1}$ such that φ is a one

open map from U onto $V \cap S$.

(6× 2 = 12 weigh

PART C

Answer any Two questions. Each question carries 5weightage.

18. a) Let S be an n-surface inR^{n+1}, $S = f^{-1}(c)$ where $f: U \to R$ is such that $\nabla f($

for all $q \in S$. Suppose $g: U \to R$ is a smooth function and $p \in S$ is an extreme

g on S,then prove that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$.

b)Show that the maximum and minimum values of the function

$g(x_1, x_2) = ax_1^2 + 2bx_1x_2 + cx_2^2$ where $a, b, c \in R$ on the unit circle $x_1^2 +$

are the eigen values of the matrix$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$.

19. a) Let $S \subset R^{n+1}$ be a connected n-surface inR^{n+1}. Prove that there exist on S

two smooth unit normal vector fields N_1 and N_2, and that $N_2(p) = -N_2(p)$for

$p \in S$.

b) Let S be a compact, connected oriented n-surface in R^{n+1}. Prove that th

map maps S onto the unit n-sphereS^n .

20. Let S be an n-surface inR^{n+1}, let $\alpha: I \to S$ be a parametrized curve in S,let t_0

let$v \in S_{\alpha(t_0)}$.Then prove that there exist a unique vector field $\boldsymbol{V}$ tangent to S al

which is parallel and has $V(t_0)= v$

b)Prove that in the n-plane$a_1x_1 + a_2x_2 + \cdots + a_{n+1}x_{n+1} = b$ in R^{n+1},the par

transport is path independent.

21. a) Let S be an n-surface in R^{n+1} and let $f: S \to R^k$.Then prove that f is smoo

only if $fo\varphi: U \to R^k$ is smooth for each local parametrization $\varphi: U \to R^k$.

b) State and prove the inverse function theorem for n-surfaces.

(2× 5 = 10 weig

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2021

MMT4E14 –Computer Oriented Numerical Analysis

(2019 Admission onwards)

Time: 1 ½ hours

Max. Weightage : 15

Section A

Answer ALL questions. Each question carries 1 weight.

1. Wrie the output of the statement "print 5/2" in Python.

   Is the output same as the actually calculated value ?

   If our answer is "NO", write a suitable Python Program so as to get the output to be the actual value 2.5.

2. Write a Python program, which while executed will ask for your height in metres and will print it as output.

3. Write a Python program to get the print out of multilication table of 7 upto $10 \times 7 = 70$ using a "WHILE" loop.

4. What is a conditional execution in Python ? Explain using a suitable example.

4×1 = 4 Weights.

Section B

Answer any THREE questions. Each question carries 2 weights.

5. Write a Python program to do the following. Asking the input of a natural number "n" and the first "n" Fibonacci numbers.

6. Explain the problem and power method to find the largest eigen value of a given square matrix.

7. Distinguish between 'Local' and 'Global' definitions (of varables/functions) in Python Explain with a suitable example (or sample program).

8. Write a note on the use and precautions in the use of 'functions' in Python. Write a Python program to find the average of five numbers which you input by defining a suitable function.

9. What is the use of modules in Python ? What is the Pickle Module ? Explain with a suitable example.

$3 \times 2 = 6$ **Weights**

Section C

Answer any ONE question. Each question carries 5 weights.

10. a) Write a Python Program to find the Greatest Common Divisor of two positive integers you input.

    b) Explain the problem, the method, the algorithm and a Python program for the Lagrange's Interpolation.

11. Explain the problem, the method, the algorithm and a Python program for the Runge Kutta Mehod (order 4).

$1 \times 5 = 5$ **Weights**