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1M4M18223 (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March 2018

ST4C13 – Multivariate Analysis

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

PART A

Answer all questions

Weightage 1 for each question

1. Distinguish between singular and non-singular multivariate normal distribution.
2. If $X \sim N_p(\mu, \Sigma)$, show that $CX \sim N_p(C\mu, C\Sigma C')$, where C is non-singular matrix of order p.
3. What is partial correlation coefficient? How it is connected to simple correlation coefficient?
4. Describe how you construct confidence region for the mean vector of a multivariate normal distribution when its dispersion matrix is known.
5. Explain the additive property of Wishart matrices.
6. What is generalized variance?
7. Define Hotelling's T^2 -statistic. How it can be related interpreted as an extension of univariate Student's t-statistic?
8. Describe briefly Fisher-Behrens problem.
9. Define Mahalanobis D^2 -statistic. Establish its relationship with Hotelling's T^2.
10. What is Fisher's discriminent function? What it is used for?
11. What are principal components? How data reduction can be attained using them?
12. Write a short note on the concept of factor analysis.

(12 × 1=12 Weightage)

PART B

Answer any 8 questions

Weightage 2 for each question

13. Show the a p-dimensional random vector X has multivariate normal n distribution if and only if every linear combination of X has univariate n distribution.

14. Find the distribution of $x_1 + 2x_2 - 3x_3$ where x_1, x_2, x_3 have a trivariate n distribution with mean zero and dispersion matrix Σ, where $\Sigma^{-1} = \begin{bmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$.

15. Define multiple correlation coefficients. Obtain its relationship with s correlation coefficients.

16. If $X_1, X_2, ..., X_N$ is a random sample of size N from $N(\mu, \bar{z})$, show that the san $\bar{X}$ is independent of the sample dispersion matrix S.

17. Derive the sampling distribution of partial correlation coefficient in the null case

18. Let Y be a p-component vector following multivariate normal distribution N Then prove that $Y'\Sigma^{-1}Y$ is distributed according to χ^2 with p degrees of freedo

19. What are the uses of Hotelling's T^2-statistic? Explain any two of it.

20. Derive the test for independence of two sub vectors of a multinormal vector.

21. Describe sphericity test. What are the asymptotic tests available in this case?

22. Describe the problem of classifying a new observation in one of the K di populations.

23. Derive the relationship of principal components with eigen values eigen vectors dispersion matrix of a multinormal population.

24. Explain the procedure of estimating principal components.

(8 × 2=16 Weig

PART C

Answer any 2 questions

Weightage 4 for each question

25. If $X = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix}$ follows $N_p(\mu, \Sigma)$ distribution with $\mu = \begin{bmatrix} \mu^{(1)} \\ \mu^{(2)} \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$, obtain the distribution conditional distribution of $X^{(1)}$ given $X^{(2)} = x^{(2)}$.Show that $X^{(1)}$ and $X^{(2)}$ are independent if only if $\Sigma_{12} = \Sigma_{21} = 0$.

26. (a) Obtain the test for equality of mean vectors in two multivariate normal distribution with same dispersion matrix, (b) Let $X_1, X_2, ..., X_N$ be independently distributed according to $N(\mu, \Sigma)$. Then prove that the distribution of $S = \frac{1}{n}\sum_{\alpha=1}^{N}(x_\alpha - \bar{x})(x_\alpha - \bar{x})'$ is Wishart $W\left(\frac{\Sigma}{n}, n\right)$, where $n = N - 1$.

27. Describe the procedure of testing the equality of the covariance matrices of two multivariate distributions.

28. Formulate the general classification problem. Show that under p-variate normal populations with common dispersion matrix the rule for discriminating between two populations is a linear function of the component random variables.

(2 × 4=8 Weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March 2018

ST4E10 – Statistical Decision Theory & Bayesian Analysis

(2016 Admission onwards)

Max. Time: 3 hours Max. weightage: 36

Part A

(Answer all questions. Each question carries 1 weightage.)

1. Define the terms: (i) Action (ii) Action Space.
2. What is a decision rule?
3. Distinguish between randomized and non-randomized decision rule.
4. What is prior information? How do you quantify it?
5. What is non-informative prior? Give an example.
6. What are conjugate priors? Give example.
7. What is posterior distribution? Illustrate it through an example.
8. Distinguish between admissible and inadmissible decision rule.
9. Describe Bayes principle.
10. What are statistical games?
11. Describe two person zero-sum game.
12. What are Bayesian credible sets?

(12 x 1 = 12 weightage)

Part B

(Answer any eight questions. Each question carries 2 weightages.)

13. Let X have a $P(\theta)$ distribution, $\Theta = (0, \infty)$ *and* $A = [0, \infty)$. The loss function is $L(\theta, a) = (\theta - a)^2$. Consider decision rules of the form $\delta_c(x) = cx$. Assume $\pi(\theta) = e^{-\theta}$ as the prior density.

    a) Find $R(\theta, \delta_c)$

    b) Show that δ_c is inadmissible if c > 1

    c) Find $r(\pi, \delta_c)$

14. What is utility function? Briefly describe various steps in the construction of utility function.

15. Define loss function. Describe absolute error loss function and 0-1 loss function.

16. Describe the histogram approach to prior selection.

17. What is maximum entropy prior?

18. Let $X \sim N(\theta,1)$ is to be observed, it is known that $\theta > 0$. It is further believed that θ has a prior distribution with mean μ. Derive the prior density of θ which maximize entropy subject to these constraints.

19. For a statistical decision problem the loss function is

$$L(\theta,a)=\begin{cases} k_0(\theta-a), & if\ \theta-a\geq 0 \\ k_1(a-\theta) & if\ \theta-a<0 \end{cases}$$

Prove that any $\left(\frac{k_0}{k_0+k_1}\right)^{th}$ fractile of the posterior density $\pi(\theta|X)$ is a Bayes estimate of θ.

20. What is Jeffrey's prior? Illustrate it through an example.

21. Suppose $X \sim B(n,\theta)$, where θ has a $\beta(\alpha,\beta)$ prior distribution. Show that the posterior distribution of θ given X is again $\beta(\alpha+x,\beta+n-x)$.

22. A large shipment of parts is received, out of which 5 are tested for defects. The number of defective parts, X, is assumed to have a B(5, θ)distribution. From past shipments, it is known that θ has a β (1, 9) prior distribution. Find 95% HPD credible region for θ, if X = 0 is observed.

23. State and explain minimax theorem.

24. If $X_1, X_2, \ldots X_n$ are sample from $N(\theta,1)$ show that $\overline{X}$ is a minimax estimator.

(8 x 2 = 16 weightage)

Part C

(Answer any two questions. Each question carries 4 weightages.)

Define minimax decision rule. A company has to decide whether to accept or reject a lot of incoming parts. These actions are labeled a_1 and a_2 are respectively. The lots are of three types: θ_1, θ_2 *and* θ_3. The loss $L(\theta_i, a_j)$ incurred in making the decision is given in the following table:

	a_1	a_2
θ_1	0	3
θ_2	1	2
θ_3	3	0

The prior belief is that $\pi(\theta_1) = \pi(\theta_2) = \pi(\theta_3) = \frac{1}{3}$. What is the minimax non randomized action.

Define the risk function of a decision rule. When will you say that one decision rule is better than the other? To estimate θ in samples from N(θ, 1) under squared error loss, consider the class of estimators of the form $\delta_c(x) = cx, c \geq 1$. Show that δ_c is inadmissible if c > 1.

Define minimal complete and admissible class of decision rules. If a minimal complete class exists, show that it consist of exactly the admissible decision rules.

Derive (i) Point Estimation Problem (ii) Testing of Hypothesis (iii) Confidence Interval Estimation Problem as particular cases of statistical decision problem.

(2 x 4 = 8 weightage)