23

1M4M20202 (Pages : 3) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester MSc Degree Examination, March/April 2020

MT4E05 – Measure and Integration

(2018 Admission onwards)

Time: 3 hours Max. Weightage : 36

Section A

Answer ALL questions. Each question carries 1 weight.

1. Is $\mathfrak{M}$, the set of all finite subsets of **N**, the set of natural numbers, a σ-algebra on **N** ?

   Give reason.

2. Prove that the intersection of two σ-algebras is a σ-algebra.

3. Give example of a G_δ-set that is not open.

4. Define a simple measurable function. Give one example.

5. If μ is a positive measure on a meaurable space X and E is a measurable set, then *prove or disprove* that $\int_E f\,d\mu = 0 \Rightarrow f \equiv 0$ on E.

6. Prove that a set function μ defiend as follows is a positive measure on **R**.

$$\mu(A) = \begin{cases} 1 & \text{if } 0 \in A \\ 0 & \text{otherwise.} \end{cases}$$

7. Does there exist an infinite σ-algebra which has only countably many members ?

   Give reason.

8. Prove that $f \longrightarrow \int_X f\,d\mu$ is a linear functional on $L^1(\mu)$.

9. Explain with example the difference between compact and σ-compact spaces.

10. State Lusin's theorem.

11. If a measure λ is concentrated on a set A, prove that $|\lambda|$ also is concentrated on A.

Let λ_1, λ_2 be measures and μ be a positive measure. If $\lambda_1 \perp \mu$ and $\lambda_2 \perp \mu$ then prove that $\lambda_1 + \lambda_2 \perp \mu$.

Define **monotone class** in $\mathbf{R}^1$. Give example of such a class that is not a σ-algebra on $\mathbf{R}^1$.

. State the Fubini's theorem. **$14 \times 1 = 14$ Weights.**

Section B

Answer any SEVEN questions. Each question carries 2 weights.

. Let $\{E_n\}$ be a decreasing sequence of measurable sets.

Can you conclude that $\mu(\cap E_n) = \lim \mu(E_n)$? Give reason.

;. Obtain the sum of two measurable simple functions $f(x)$ and $g(x)$ as a simple function.

$$f(x) = \begin{cases} 1 & 0 \le x \le \frac{1}{2} \\ 2 & \frac{1}{2} < x \le \frac{3}{2} \\ 3 & \frac{3}{2} < x \le 2 \end{cases} \quad g(x) = \begin{cases} 4 & 0 \le x \le 1 \\ 5 & 1 < x \le 2 \end{cases}$$

7. Prove that the limit superior of a sequence of measurable functions is a measurable function.

8. If $a_{i,j} \ge 0 \, \forall i, j = 1,2,3,\cdots$, then prove that $\sum_i \sum_j a_{i,j} = \sum_j \sum_i a_{j,i}$.

9. State Fatou's Lemma. Give one example for strict inequality in Fatou's Lemma.

0. Define a **Lower Semi Continuous Function.**

Prove that the characteristic functions of open sets are lower semi continuous.

.1. If P_n is the set of all $\bar{x} \in \mathbf{R}^k$ whose co-ordinates are integral multiples of 2^{-n} and Ω_n is the collection of all 2^{-n} boxes with cornerss at points of P_n, then prove that every non-empty open set in $\mathbf{R}^k$ is a countable union of disjoint open sets belonging to $\Omega_1 \cup \Omega_2 \cup \Omega_3 \cdots$.

22. If μ & λ are measures on a σ-algebra $\mathfrak{M}$, prove that "$\lambda \ll \mu$ & $\lambda \perp \mu$" $\Rightarrow \lambda = 0$.

23. State and prove the minimum property of Jordan decomposition.

fine Lebesgue Point of an $f \in L^1(\mathbf{R}^k)$. Prove that for an $f \in L^1(\mathbf{R}^k)$, almost every $\bar{x} \in \mathbf{R}^k$ is
.ebesgue Point of an f. **$7 \times 2 = 14$ Weights.**

Section C

Answer any TWO questions. Each question carries 4 weights.

ı) Prove that every non-negative extended real valued meaurable function on a measurable space is the limit of an increasing sequence of non-negative simple functions.

›) State and prove Lebesgue's Monotone Convergence Theorem.

a) Prove that $L^1(\mu)$ is a complex vector space and $\int_X (\alpha f + \beta g) d\mu = \alpha \int_X f d\mu + \beta \int_X g d\mu$.

b) Prove that $|\int_X f d\mu| \leq \int_X |f| d\mu$ for $f \in L^1(\mu)$.

ate and prove Lebesgue Radon Nikodym theorem.

:t $I = [a, b]$, $f : I \longrightarrow \mathbf{R}^1$ be continuous and non-decreasing.

ove that the following are equivalent.

a) f is absolutely continuos on I.

b) f maps sets of measure zero to sets of measure zero.

c) f is integrable almost everywhere on I, $f' \in L^1$ and

$\int_a^x f'(t)\,dt = f(x) - f(a), a \leq x \leq b.$ **$2 \times 4 = 8$ Weights.**

24

4M20201 (Pages : 1) Reg. No:………………………..

Name: …………………………

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2020

MT4E12 – Computer Oriented Numerical Analysis

(2018 Admission onwards)

ne: 1 ½ hours Max. Weightage : 18

PART A (Short Answer Questions)

(Answer* all *Questions. Each question has weightage 1)

1. Write a Python program to display the following statement on the screen "National Mathematical year 2012"
2. Write the output of 10>>2 and 5^3.
3. Write a python program to finding the area of a triangle, whose sides are a, b and c.
4. What is the uses of *break* statement in Python.
5. Explain Simpson's rule of integration.
6. Write a python program that uses *if...else* statement.

(6 x 1 = 6 weightage)

PART B

(Answer any* fourf*rom the following six questions. Each question has weightage 2)

7. What are the features of Python.
8. Write a Python program that uses *while* statement.
9. Write an algorithm for generating Fibonacci numbers less than or equal to 100.
10. Write a Python program to find GCD of two numbers.
11. Write a Python program to evaluate $\int_a^b f(x)dx$ using Trapezoidal rule.
12. Explain *init* method with an example.

(4 x 2 = 8 weightage)

PART C

(Answer any* onef*rom the following two questions. Each question has weightage 4)

13. Write an algorithm and corresponding Python program for finding a real root of an equation $f(x) = 0$ using Bisection method.
14. Write a computer oriented algorithm and the corresponding Python program to solve the differential equation $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ using Range-Kuttamethod.

(1 x 4 = 4 weightage)

M4M20199 (Pages :2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester MSc Degree Examination, March/April 2020

MT4E02 – Algebraic Number Theory

(2018 Admission onwards)

Time: 3 hours Max. Weightage : 36

Part A: Answer all 14 questions. Each question has weightage 1

1. Define an R- module. Give an example for the same.
2. Evaluate the order of the group G/H where basis of G is $\{x, y, z\}$ and the basis of H is $\{x + 3y - 5z, 2x - 4y, 7x + 2y - 9z\}$
3. Find norm and trace of $p + q\sqrt{7}$ in $K = Q(\sqrt{7})$
4. Define unit in a ring R. Show that the units of a ring $U(R)$ form a group under multiplication.
5. Show that the ring of integers in a number field K is noetherian.
6. Show that every principal ideal domain is a unique factorization domain.
7. Let R be a ring and α an prime ideal of R, then show that R/α is a domain.
8. Let R be a ring and α an ideal of R, then show that $\alpha.\alpha^{-1} = \alpha^{-1}\alpha$
9. In $Z(\sqrt{-7})$, prove that the element 2 is irreducible but not prime.
10. Define class number and class group.
11. Explain the volume of $X \subset R^n$ and locally volume preserving.
12. Find monomorphisms $\sigma_i : K \to C$ for $K = Q(\zeta)$, where $\zeta = e^{\frac{2\pi i}{7}}$
13. Define the $n-$ dimensional torus T^n. If L is an $n-$ dimensional lattice in R^n then show that R^n/L is isomorphic to T^n
14. For each $\alpha \in Z[\zeta]$, there exist $a \in Z$ such that $\alpha^p \equiv a(\text{mod } l^p)$, where $l = <\lambda>$. Prove the statement

(14×1=14 weightage)

Part B: Answer any *SEVEN* questions. Each question has weightage 2

15. Express the polinomial $t_1^3 + t_2^3 + t_3^3$ in terms of elementary symmetric polinomial (n=3)
16. Compute an integral bases and discriminent of $Q(\sqrt{2}, \sqrt{3})$
17. If K is a number field then show that $K = Q(\theta)$ for some algebraic number θ.

18. If $\{\alpha_i, \alpha_2, \alpha_3 \ldots \alpha_n\}$ is any $Q-basis$ of K, then show that $\Delta[\alpha_i, \alpha_2, \alpha_3 \ldots \alpha_n] = \det\ (\mathrm{T}(\alpha_i, \alpha_j))$

19. Show that factorization into irreducibles is not unique in the ring of integers of $Q(\sqrt{d})$ for atleast the following values of $d = 10, 15, 26, 30$.

20. In $Z[\sqrt{-5}]$, obtain the prime factorization of $< 6 >$

21. Define lattice . Sketch the lattice in R^2 generated by $\{(1,2), (2,-1)\}$

22. State and prove Minkoswki's theorem.

23. Solve the diophantine equation $x^2 + y^2 = z^2$

24. If $p(t) \in R(t)$ is a monic polynomial, all of whose zeros in C have absolute value 1, then show that every zero is a root of unity.

(7×2 = 14 weightage)

Part C: Answer any *TWO* questions. Each question has weightage 4

25. Show that the ring of integers of $Q(\zeta)$ is $Z[\zeta]$

26. Show that the ring of integers of $Q(\sqrt{d})$ is Euclidean for $d = -1, -2, -3, -7, -11$

27. Let $\sigma : K \to L^{st}$ be the usual map. Show that if $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a basis for K over Q, then $\sigma(\alpha_1), \sigma(\alpha_2), \ldots, \sigma(\alpha_n)$ are linearly independent over R.

28. If X is a bounded subset of R^n and $v(X)$ exists and if $v(V(X) \neq v(X)$, then show that v/X is not injective.

(2×4 = weightage 8)

1M4M20200 (Pages : 3) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester M.Sc Degree Examination, March/April 2020

MT4E14 – Differential Geometry

(2018 Admission onwards)

Time: 3 hours Max. Weightage : 36

Part A

Answer All questions.

Each question carries 1 weightage

1. Show that the graph of any function $f: R^n \to R$ is a level set for some function $F: R^{n+1} \to R$.
2. Show that the gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p
3. Find the velocity, the acceleration and the speed of the curve $\alpha(t) = (cos3t, sin3t)$
4. Find and sketch the gradient field of the function $f(x_1, x_2) = x_1 - x_2$
5. Let $f: U \to R$ be a smooth function and $\alpha: I \to U$be an integral curve of∇f. Show that $\frac{d}{dt}(fod)(t) = \|\nabla f(\alpha(t))\|^2$ for all $t \in I$
6. Show that the unit n-sphere $x_1^2 + x_2^2 + \cdots x_{n+1}^2 = 1$ is connected for $n > 1$
7. Show that if $\alpha: I \to R^{n+1}$ is a parametrized curve with constant speed then $\ddot{\alpha}(t) \perp \dot{\alpha}(t)$ for all $t \in I$
8. Compute $\nabla_v f$ where $f: R^{n+1} \to R$ and $v \in R_p^{n+1}, p \in R^{n+1}$ are given by
$$f(x_1, x_2) = 2x_1^2 + 3x_2^2, v = (1,0,2,1)$$
9. Prove that the geodesic have constant speed.
10. Let S be an n-surface in R^{n+1} .let $\alpha: I \to S$ be a parametrized curve and let X and Y are vector fields tangent to S along α. Verify that $(fX)' = f'X + fX'$
11. Find the Gaussian curvature $K: S \to R$ where S is given by $x_1^2 - x_2^2 - x_3^2 = 0, x_3 > 0$
12. Let S be an n-surface in R^{n+k} and let $p \in S$. Define the tangent space S_p at p.
13. Show that the two orientations on the unit n-sphere $x_1^2 + x_2^2 + \cdots x_{n+1}^2 = 1$ are given by $N_1(p)$= (p, p) and $N_2(p)$=(-p,p)
14. Find the length of the parametrized curve $\alpha: I \to R^{n+1}$ given by
$$\alpha(t) = (cos3t, sin3t, 4t); I = [-1,1], n = 2$$

(14 × 1 = 14 weightage)

Part B

Answer any seven questions.
Each question carries 2 weightage.

15. Sketch the vector fields on $R^2: X(p) = (p, X(p))$ where $X(x_1, x_2) = (x_2, x_1)$

16. Let S be an n – surface in R^{n+1}, $S = f^{-1}(c)$ where $f: U \to R$ is such that $\nabla f(q) \neq 0$ for all $q \in S$. Suppose $g: U \to R$ is a smooth function and $p \in S$ is an extreme point of g on S, Then Show that there exist a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$

17. Let S be a 2 – surface in R^3 and $\alpha: I \to S$ be a geodesic in S with $\dot{\alpha} \neq 0$. Then show that a vector field X tangent to S along α is parallel along α if and only if both $\|X\|$ and the angle between X and $\dot{\alpha}$ are constant along α.

18. Show that the set S of all unit vectors at all points of R^2 forms a 3-surface in R^4.

19. Let $S \subset R^{n+1}$ be a connected n-surface in R^{n+1}. Then show that there exist on S exactly two smooth unit normal vector fields N_1 and N_2 and $N_2(p) = -N_1(p)$ for all $p \in S$

20. State and prove the Inverse function theorem for n-surfaces.

21. Find the Gaussian curvature $K: S \to \mathbb{R}$, where S is the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, $(a, b and c all \neq 0)$.

22. Let S be the unit n-sphere $\sum_{i=1}^{n+1} x_i^2 = 1$ oriented by the outward unit normal vector field. Prove that the Weingarten map of S is multiplication by -1.

23. Prove that the 1-form η on $\mathbb{R}^2 - \{0\}$ defined by

$$\eta = -\frac{x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2$$

is not exact..

24. Prove that, in an n-phase, parallel transport is path independent.

(7 × 2 = 14 weightage)

Part C

Answer any two questions.

Each question carries 4 weightage

25. Let S be a compact connected oriented n-surface in R^{n+1} exhibited as a level set $f^{-1}(c)$ of a smooth function $f: R^{n+1} \to R$ with $\nabla f(p) \neq 0$ for all $p \in S$. Then show that the Gauss map maps S onto the unit sphere S^n.

26. Let S be an n-surface in R^{n+1}, let $p \in S$ and $\in Sp$. Then show that there exists an open interval containing 0 and a geodesic $\alpha: I \to S$ such that

    (i) $\alpha(0) = p$ and $\dot{\alpha}(o) = v$

    (ii) If $\beta: \tilde{I} \to S$ is any other geodesic in S with $\beta(0) = p$ and $\dot{\beta}(0) = v$, then $\tilde{I} \subset I$ and $\beta(t) = \alpha(t)$ or all $t \in \tilde{I}$

27. Let $\varphi: U \to R^{n+1}$ be a parametrized n-surface in R^{n+1} and let $p \in U$. Then show that there exist an open set $U_1 \subset U$ about p such that $\varphi(U_1)$ is an n-surface in R^{n+1} .

28. (i) Let S be an n-surface in R^{n+1}, oriented by the unit vector field N. Let $p \in S$ and $v \in Sp$. Then show that for every parametrized curve $\alpha: I \to S$, with $\dot{\alpha}(t_0) = v$ for some $t_0 \in I$,

$$\ddot{\alpha}(t_0).N(p) = L_p(v).v$$

(ii)Show that the Weingarten map L_p is self adjoint.

$(4 \times 2 = 8$ weightage$)$