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N17087 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2017

MT3C11 - Complex Analysis

(2016 Admission onwards)

Time: 3 hours Max. Weightage: 36

Part A

Answer all questionsEach question carries 1 weightage

1. Find the fixed points of the linear transformation $w = \frac{2z}{3z-1}$.
2. Find the linear transformation which maps the point$(0, 1, \infty)$to $(\infty, 1, 0)$.
3. If $z = x + iy$, prove that $|e^z| = e^x$.
4. Prove that, the integral over an arc depends only on its end points if and only if the integral over any closed curve is zero.
5. Compute $\int_r \; ydz$ where r is the directed line segment from0 to i.
6. Let n be a positive integer. Prove that $\int_\gamma \; (z-a)^n dz = 0$ for any closed curve γ.
7. Evaluate $\int_{|z|=1} \frac{dz}{z^2+1}$
8. State and prove Morera's theorem.
9. Determine the nature of singularity of the function $\frac{sinz}{z}$ at z=0. Justify your answer.
10. Find the residue of the function $f(z) = \frac{e^z}{(z-a)^2}$ at $z = a$.
11. Define: Simply connected region. Give an example of a simply connected region.
12. Prove: the argument θ is harmonic wherever it can be defined
13. Find the Taylor series expansion of the function $\frac{1}{z-2}$ at $z = 1$.
14. Prove that an elliptic function without poles is constant.

(14 x 1 = 14 weightage)

Part B

Answer any seven questionsEach question carries 2 weightage

15. Let Ω be a region $\mathbb{C} - \{z: z \leq 0\}$, the complement of the negative real axis. Define a continuous function $f: \Omega \to \mathbb{C}$ satisfying $f(z) = z^2$ for all $z \in \Omega$ and $f(1) = 1$. Show th f is analytic in Ω.
16. Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant. The same is true if either the real part, the imaginary part, the modulus or the argument is constant.
17. Prove that the cross ratio is invariant under bilinear transformation.
18. Let f be a continues complex valued function defined on the closed interval $[a, b]$. Pro that $\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|\, dt$.
19. If $pdx + qdy$ is locally exact in Ω then prove that $\int_\gamma p\, dx + q\, dy = 0$ for every cycle $\gamma \sim 0$ in Ω.
20. Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.
21. How many roots does the equation $z^7-2z^5+7z^3-z+1=0$ have in the disc $|z| < 1$.
22. State and prove generalized form of Argument Principle.
23. Find the Laurent series expansion of the function $f(z) = \frac{1}{(z-1)(z-2}$, in the regions $0 < |z-1| < 1$ and $1 < |z-2| < \infty$.
24. Derive Legendre's relation : $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$ **(7 x 2 = 14 Weightage)**

Part C

Answer any two questionsEach question carries 4 weightage

25. (i)The line integral $\int_\gamma pdx + qdy$ defined in Ω depends only on the end point points γ if and only if there exist a function $U(x, y)$ in Ω with partial derivatives $\frac{\partial U}{\partial x} = p$ an $\frac{\partial U}{\partial y} = q$.

    (ii) Compute $\int_\gamma x\, dz$ where γ is the directed line segment from 0to $1 + i$.
26. Let the function f be analytic in a region Ω and let $a \in \Omega$. Suppose that $f(a)$ and all derivatives $f^{(n)}(a)$ vanish. Prove that f is identically zero in Ω.
27. Discuss the evaluation of integrals of the type $\int_{-\infty}^{\infty} R(x)\, e^{ix} dx$ using the residues and compute $\int_0^{\infty} \frac{x \sin x}{a^2+x^2}\, dx$, a real.
28. Derive the formula for the Weierstrass elliptic function in the form,

$$P(z) = \frac{1}{z^2} + \sum_{w\neq 0} [\frac{1}{(z-w)^2} - \frac{1}{w^2}].$$

(2 x 4 = 8 Weigh

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2017

MT3C12 - Functional Analysis - I

(2016 Admission onwards)

Time: 3 hours Max. Weightage: 36

Part A(Short Answer Type Questions)

Answer all the questions

Each question has weightage 1.

1. If X is a separable metric space and $Y \subseteq X$, then prove that Y is separable in the induced metric.
2. Show that the metric space l^{∞} is complete.
3. Define a normed space.
4. Prove or disprove.
   The normed space $(K^n, \| \ \|_1)$ is strictly convex.
5. Let E be a measurable subset of R. Define the metric space $L^{\infty}(E)$.
6. Let X and Y be normed spaces where X is infinite dimensional and $Y \neq \{0\}$. Prove that there is a discontinuous linear map from X to Y.
7. Let E be a convex subset of an inner product space X. Prove that there exists at most one best approximation E to any $x \epsilon X$.
8. Show that among all the norms, $\| \ \|_p, 1 \leq p \leq \infty$ on $L^p(E)$, only the norm $\| \ \|_2$ is induced by an inner product.
9. Show that $C[a, b]$ is not closed in $L^p([a, b]), 1 \leq p \leq \infty$.
10. State Riesz representation theorem.
11. Show that the dual space X' of every normed space X is a Banach space.
12. Define a Schauder basis for a normed space and give an example.
13. Show that the linear space C_{00} cannot be a Banach space in any norm.
14. Show that if the domain space X is not a Banach space, then the uniform boundedness principle may not hold.

(14 x 1 = 14 weightage)

Part B(Paragraph Type Questions)

Answer any seven questions

Each question has weightage 2.

15. Let X be a normed space such that the closure of $U(0,1)$is compact. Then prove t X is finite dimensional.
16. Prove that $\|\ \|_1, \|\ \|_2$ and $\|\ \|_\infty$ are equivalent on K^n.
17. Prove that the set of all polynomials in one variable is dense in$C([a, b])$ with the supmetric.
18. Prove that the metric space $L^\infty([a, b])$ is separable for $1 \le p < \infty$.
19. Prove or disprove.
    A linear map on a linear space X may be continuous with respect some norm on
    but discontinuous with respect to another norm on X.
20. Let X and Y be normed spaces and $F: X \to Y$ be a linear map. Then show that F is bounded on $\overline{U(0,r)}$ for some $r > 0$ if and only if $||F(x)|| \le \alpha||x||$ for all $x \in X$ for some $\alpha > 0$.
21. Let E be a non-empty closed convex subset of a Hilbert space H. Then prove tha there exists a unique element in E of minimal norm.
22. State and prove Hahn- Banach separation theorem.
23. Let $X = K^2$ with the norm$\|\ \|_1$. Consider $Y = \{(x(1), x(2)) \in X : x(2) = 0\}$ and define $g \in Y'$ by $g(x(1)x(2)) = x(1)$. Determine all the Hahn Banach extensior g to X.
24. Let X be a normed space and E be a subset of X. Show that E is bounded in X and only if $f(E)$ is bounded in K for every $f \in X'$.

$(7 \times 2 = 14$ weight

Part C(Essay Type Questions)

Answer any two questions

Each question has weightage 4.

25. Prove that every closed and bounded subset of a normedspace X is compact if an only if X is finite dimensional.

26. Let X and Y be normed spaces and $BL(X, Y)$ be the set of all bounded linear map from $X$to$Y$. For $F \in BL(X, Y)$, define $\|F\| = \sup\{\|F(x)\|: x \in X, \|x\| \le 1\}$. Prov that $\|\ \|$ is a norm on $BL(X, Y)$. Also prove that $\|F(x) \le \|F\|\|x\|$ for all$x \in X$.
27. (a).Let $\{U_\alpha\}$ be an orthonormal set in a Hilbert space H. Then prove that $\{U_\alpha\}$ is orthonormal basis for H if and only if $x \in H$ and $< x, u_\alpha \ >= 0$ for all α, then $x = 0$.

    (b). Let $H = L^2\ [-\pi, \pi]$ and for $n = 0, \pm1, \pm2, \cdots\ U_n\ \ (t) = \frac{e^{int}}{\sqrt{2\pi}}$, $t\epsilon[-\pi, \pi]$. Sh that $\{U_n\ :\ n = 0,\ \pm1,\ \pm2, \cdots\}$ is an orthonormal basis for H.

28. Show that a normed space can be embedded as a dense subspace of a Banach spa

$(2 \times 4 = 8$ weigh

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2017

MT3C13 - Topology II

(2016 Admission onwards)

Time: 3 hours Max. Weightage: 36

Part A

Answer all questions (1 – 14)

Each question has weightage 1

Prove that a box is an intersection of family of walls.

Let H_i be an open subset of topological spaceX_i, for each $i \in I$. Prove that $\prod_{i\in I} H_i$ is open subset of the product $\prod_{i\in I} X_i$ iff $H_i = X_i$ for all except finitely many indeces in I.

Let (X, d) be a metric space. Give a metric e on X which induces the same topology as d induces and $e(x, y) \leq 10 \ \ \forall x, y \in X$. Justify.

Prove that if a product is non-empty, then each projection function is onto.

Prove that every cube is Tychonoff.

Prove that any hereditary property possessed by a product is also possessed by each coordinate space.

If X is a Tychonoff space, prove that the family of all continuous real valued functions on X distinguishes points.

Prove that the evaluation map of a family of functions is one-to-one, iff the family distinguishes points.

Prove that the Euclidian space $\mathbb{R}^n$ is simply connected.

Define strong deformation retraction of a topological space.

Prove that continuous image of a countably compact space is countably compact.

Describe one-point compactification of a topological space.

Prove or disprove that every bounded metric is totally bounded.

Prove that $\mathbb{Q}$ is of first category in $\mathbb{R}$.

$(\mathbf{14 \times 1 = 14}\ weightage)$

Part B

Answer any seven questions $(15 - 24)$

Each question has weightage 2

15. Let A be a closed subset of a normal space X and suppose $f: A \to (-1,1)$ is continuous. Prove that there exist a continuous function $F: A \to (-1,1)$ such that $F(x) = f(x)$ for $x \in A$.
16. If the product is non-empty, prove that each coordinate space is embeddable in it.
17. Prove that regularity is a productive property.
18. Prove that a topological space is Tychonoff iff it is embeddable into a cube.
19. Prove that path homotopy $(\simeq_p)$ is an equivalence relation.
20. Let S^1 be the unit circle and $x_0 \in S^1$. Prove that the inclusion map $j : (S^1, x_0) \to (\mathbb{R}^2 - 0, x_0)$ induces an isomorphism between $\pi_1(S^1, x_0)$ and $\pi_1(\mathbb{R}^2 - 0, x_0)$.
21. Prove that X is countably compact iff every countable family of closed subsets of X having finite intersection property has non-empty intersection.
22. Let A be a subset of a metric space (X, d), such that A is complete in the induced metric on it. Prove that A is closed in X.
23. Countable compactness implies sequential compactness. Prove or disprove.
24. If a metric space X is compact, prove that X is complete.

$(7 \times 2 = 14$ ***weighta***

Part C

Answer any two questions $(25 - 28)$

Each question has weightage **4**

25. Prove that the product of topological spaces is connected iff each co-ordinate space is connected.
26. State and prove Urysohn metrisation theorem.
27. Prove that the fundamental group of the circle is infinite cyclic.
28. Prove that a subset of first category in a complete metric space cannot have any interior point.

$(2 \times 4 = 8$ ***weighta***

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N17090 (Pages : 3) Reg. No:..............................

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2017

MT3C14 - Linear Programming and Its Applications

(2016 Admission onwards)

Time: 3 hours Max. Weightage: 36

Part A (Short Answer Questions)

(Answer **all** questions. Each question has weightage 1)

1. Prove that a vertex is a boundary point but not all boundary points are vertices. Give an example of a convex set in which all boundary points are vertices.
2. Prove that the set of all convex linear combinations of points of a set is a convex set.
3. Define supporting hyper plane to a convex set. Give one example.
4. Find $\nabla f(X)$ and H(X) for $f(X) = x_1^3+2x_2^3+3x_1x_2x_3+x_3^2$ at (1, 2, 3).
5. Prove that the linear function f(X) = CX, X$\in E_n$.is both convex and concave.
6. Find the unit vector in the direction of the steepest ascent of $f(X) = x_1^2+2x_1x_2+x_1x_3+x_2x_4+x_4^2$ at the point (1,0,-1,1).
7. Describe the concept of degeneracy in transportation problem.
8. When do we say that the transportation problem reduces to an assignment problem?
9. Is the function $f(x) = x^3 .\ x \in R$, the set of real numbers a convex function? Justify your answer.
10. What type of problems can be solved by the dual simplex method? Explain.
11. What is the role of artificial variable in simplex method.
12. Use notion of dominance to solve the game $\begin{bmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{bmatrix}$.
13. State the minimax theorem on theory of games.
14. Describe rectangular game as a linear programming problem.

(14x1=14 weightage)

Part B

(*Answer any* **seven** *from the following* **ten** *questions. Each question has weightage* 2)

15. Define a convex set. Let $S = \{X \in E_2 : |X| = 1\}$. Is the set convex? Justify your answ

16. Use the method of Lagrange multipliers to find the maxima and minima of $x_2^2 - (x_1+1)^2$ subject to $x_1^2 + x_2^2 \leq 1$.

17. Examine $f(X) = x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_2 + 4x_1x_3 + 16x_2x_3$ for relative extrema.

18. Write the dual of the following problem and verify the dual of the dual is primal

$$\text{Maximize } x_1 + 6x_2 + 4x_3 + 6x_4$$

$$\text{Subject to } 2x_1 + 3x_2 + 17x_3 + 80x_4 \leq 48$$

$$8x_1 + 4x_2 + 4x_3 + 4x_4 = 21$$

$$x_2 \geq 0, x_3 \geq 0, \quad x_1,\ x_4 \text{ are unrestricted in sign}$$

19. Show that the optimum value of the objective function of the primal LP problem, if it exists is equal to the optimum value of the objective function of the dual LP problem.

20. Describe the caterer problem. Formulate the problem as a standard transportation problem.

21. Solve the following transportation problem for minimum cost with the cost coefficien demands and supplies as given in the following table. Obtain three optimal solutions.

	D_1	D_2	D_3	D_4	
O1	1	2	-2	3	70
O_2	2	4	0	1	38
O_3	1	2	-2	5	32

Prove that the transportation problem has a triangular basis.

22. Solve the graphically the game whose payoff matrix is $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$.

23. Define (a) saddle point (b) matrix game (c) optimal strategies (d) mixed integer vecto

(7x2=14 weigh

Part C

nswer any two from the following four questions. Each question has weightage 4

Solve the following using simplex method

$$\begin{aligned} \text{Maximize} \quad & 5x_1 - 3x_2 + 4x_3 \\ \text{subject to} \quad & x_1 - x_2 \leq 1 \\ & -3x_1 + 2x_2 + 2x_3 \leq 1 \\ & 4x_1 - x_3 = 1 \end{aligned}$$

$x_1 \geq 0, x_2 \geq 0,\ x_3$ unrestricted in sign.

(a) Describe the concept of loop in transportation problem.

(b) Four operators A,B,C,D are to be assigned to four machines M_1, M_2, M_3, M_4 with the restriction that A and C cannot work on M_3 and M_2 respectively. The assignment cost given below. Find the minimum assignment cost.

	M_1	M_2	M_3	M_4
A	5	2	-	5
B	7	3	2	4
C	9	-	5	3
D	7	7	6	2

Solve the integer linear programming problem by cutting plane method

$$\begin{aligned} & \text{Maximize } \varphi(X) = 3x_1 + 4x_2 \\ & \text{Subject to } 2x_1 + 4x_2 \leq 13 \\ & \qquad -2x_1 + x_2 \leq 2 \\ & \qquad 2x_1 + 2x_2 \geq 1 \\ & \qquad 6x_1 - 4x_2 \leq 15 \end{aligned}$$

$x_1 \geq 0, x_2 \geq 0, \quad x_1, x_2$ are integers.

(a)State and prove fundamental theorem of games.

(b)Solve the geame whose payoff matrix is $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.

(2 x 4=8 weightage)