27

1M3N16208 (Pages :2) Reg. No:...........................

Name:...........................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

MT3C13 - Topology II

(2015 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

Part A

Answer **all** questions

Each questions has weightage 1

1. Prove that the intersection of any family of boxes is a box.
2. Prove that if a product is non-empty, then each projection function is onto.
3. Prove that in general the box topology is stronger than the product topology.
4. Show that first countability is preserved under continuous open functions.
5. Give an example of a metric space which is not second countable.
6. Show that the evaluation function is continuous if and only if each f_i is continuous.
7. Prove that every continuous, real valued function on a countably compact space is bounded and attains its extrema.
8. Let X becoutably compact. Prove that every infinite subset of X has an accumulation point.
9. Prove that continuous image of a countably compact space is countably compact.
10. Show that a first countable, countably compact space is sequentially compact.
11. Prove that a topological space is compact if and only if there exists a base B for it such that every cover of X by members of B has a finite subcover.
12. Show that sequential compactness is a weakly hereditary property
13. Prove that a closed subset of a complete metric space is complete with respect to the induced metric.
14. Show by an example that total boundedness is not topologically invariant.

$(14 \times 1 = 14 weightage$

Part B

Answer any **seven** questions

Each question has weightage 2

15. Let C_i be a closed subset of a space X_i for $i \in I$. Prove that $\prod_{i \in I} C_i$ is a closed subset of $\prod_{i \in I} X_i$ with respect to the product topology.
16. Show that the projection functions are open.
17. Prove that a topological product is second countable if and only if each coordinate spaces are second countable and all except finitely many coordinate spaces are indiscrete.
18. Show that a topological space is completely regular if and only if all continuous real valued functions on it distinguishes points from closed sets.

19. Prove that a space is embeddable in the Hilbert cube if and only if it is second countable and T_3.
20. Prove that the map $p: R \to S^1$ given by $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering ma
21. Let $\{X_i : i \in I\}$ be an indexed family of non empty compact spaces and let X be their topological product.Show that X is compact.
22. Let X be a Hausdorff space and Y be a dense subset of X. If Y is locally compact ir the relative topology on it, prove that Y is open in X.
23. Show that a metric space is compact if and only if it is complete and totally bounde
24. Prove that every metric space can be isometrically embedded as a dense subspace o complete metric space.

$(7 \times 2 = 14$*weightag*

Part C

Answer any **two** questions

Each question has weightage 4

25. a) Let X be a topological space and (Y,d) be a matric space. Suppose a sequence of continuous functions $\{f_n\}$ converges to f uniformly, prove that f is continuous.
    b)Let A be a closed subset of a normal space X and suppose $f: A \to [-1,1]$ is a continuous function.Then show that there exist a continuous function $F: X \to [-1,1]$such that $F(x) = f(x)$ for all $x \in A$.
26. Prove that a product of spaces is locally connected if and only if each coordinate space is locally connected and all except finitely many of them are connected.
27. Prove that the fundamental group of the circle is infinite cyclic.
28. State and prove Alexander Sub-base Theorem.

$(2 \times 4 = 8$ *weightag*

28

M3N16209 (Pages :2) Reg. No:...............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

MT3C14 - Linear Programming and Its Applications

(2015 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

Part A

(Answer all. 1 weightage each)

1. Define convex set. Give an example.
2. Give an example of a convex set in which all boundary points are vertices.
3. Find $\nabla f(X)$ and $H(X)$ for $f(X) = x_1^2 + 3x_1x_2 - 4x_2^2 + 4x_1 + 5x_2x_3 - x_3^2$.
4. Find the unit vector in the direction of the steepest ascent of $f(X) = x_1^2 + 2x_1x_2 + x_1x_3 + x_2x_4 + x_4^2$ At the point $(1, 0, -1, 1)$.
5. Find the volume of the largest rectangular solid inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
6. Show graphically that the following problem has no feasible extreme points. Maximize $3x_1 + 4x_2$ subject to $4x_1 + 3x_2 \geq 12,\ x_1 + 2x_2 \leq 2, x_1 \geq 0, x_2 \geq 0$.
7. Define loop in a transportation array.
8. Define integer vector and mixed integer vector.
9. Explain a matrix game.
10. Simplify the following payoff matrix using the notion of dominance.
$$\begin{bmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{bmatrix}$$
11. What is the basic solution of a LPP?
12. Explain simplex multipliers in an LPP.
13. Give various applications of duality.
14. State implicit function theorem.

$(14 \times 1 = 14 weightage)$

Part B

(Answer any 7. 2 weightage each.)

15. Define closed set. Give an example. Prove that any intersection of closed sets is closed.
16. Prove that a point X_ν of a polytope is a vertex if and only if X_ν is the only member of the intersection set of all the generating hyper planes containing it.
17. Let $f(X)$ be defined in a convex domain $K \subset E_n$ and be differentiable. Then $f(X)$is a convex function if and only if $f(x_2) - f(x_1) \geq (x_2 - x_1)\ \nabla f(x_1)$ for all $x_1, x_2 \in K$.
18. Find the extrema of $x_1x_2^2x_3^3$ under the constraints $x_1 + x_2 + x_3 = 6,\ x_1 > 0,\ x_2 > 0,\ x_3 > 0$.
19. Solve the following problem by dual simplex method. Minimize $2x_1 + 3x_2$ subject to $2x_1 + 3x_2 \leq 30,\ x_1 + 2x_2 \geq 10, x_1 \geq 0, x_2 \geq 0$.
20. Prove that the transportation problem has a triangular basis.

21. Solve the transportation problem for minimum cost with the cost coefficients, deman supplies as given in the following table. Obtain three optimal solutions.

	D1	D2	D3	D4	
O1	1	2	-2	3	70
O2	2	4	0	1	38
O3	1	2	-2	5	32
	40	28	30	42	

22. A batch of four jobs can be assigned to five different machines. The set up time for ea on each machine is given in the following table. Find an optimal assignment of jobs to machines which will minimize the total set up time.

Machines

	1	2	3	4	5
1	10	11	4	2	8
2	7	11	10	14	12
3	5	6	9	12	14
4	13	15	11	10	7

23. Explain cutting plane method to solve an ILP.
24. State and prove minimax theorem.

$(7 \times 2 = 14$ *weight*

Part C

(Answer any two. 4 weightage for each.)

25. a. Prove that dual of the dual is primal.

b. For the problem Minimize $x_1 + x_2$ subject to $2x_1 + x_2 \geq 8,\ 3x_1 + 7x_2 \geq 2,\ x_1 \geq 0,$ $x_2 \geq 0$, find the dual, solve the primal and dual graphically and verify that the optimal v of primal and dual are equal.

26. Show that the simplex method can be used to solve a system of linear equations or inequalities, and use this approach to solve the following sets.

1. $2x_1 + x_2 - x_3 + 2x_4 = 4,\ x_1 - x_2 + x_3 + x_4 = 2,\ x_1 - x_3 + 3x_4 = 3,\quad x_1, x_2, x_3, x$ $0.$

2. $2x_1 - 4x_2 = 1, 2x_1 - 3x_2 - 2x_3 \geq 3, x_1 - x_2 - 6x_3 \leq 5,\ \ x_1, x_2, x_3 \geq 0.$

27. What is a loop? Prove that the necessary and sufficient condition for a set of column vec P_{ij} in the matrix $\overline{T}$ which is obtained by deleting a row from the transportation matrix T be linearly dependent is that the corresponding variable x_{ij} in the transportation array occupy cells a subset of which constitutes a loop.

28. Write both the primal and the dual LP problems corresponding to the rectangular games with the following payoff matrices. Solve the game by solving the LP problem by simple method.

1. $\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$

$(2 \times 4 = 8$ *weighta*

N16206 (Pages :3) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

MT3C11 - Complex Analysis

(2015 Admission onwards)

x. Time: 3 hours Max. Weightage: 36

PartA

Short answer questions 1 − 14. *Answer all questions.*

Each question has 1 *weightage.*

1. Let $T_1 z = \frac{z+2}{z+3}; T_2 z = \frac{z}{z+1}$. Compute $(T_1 \circ T_2)(z)$.
2. Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$ respectively.
3. Is the function $f(z) = z^2 + 2iz + 4$ is differentiable at the point $z = i$ Justify your claim.
4. Compute $\int_c z dz$ where c is the directed line segment from 0 to $(1 + i)$.
5. Show that if f is analytic in a region Ω and if $f \neq 0$, then the zeros of f are isolated.
6. Evaluate $\int_{|z|=1} \frac{e^z}{z^n} dz$.
7. Discuss the singularity of the function e^z at $z = \infty$.
8. Define cycle and give an example.
9. Define residue of $f(z)$ at an isolated singularity $z = a$. Find $Res_{z=a} f(z)$ where $f(z) = \frac{e^z}{(z-a)(z-b)}$.
10. If u is harmonic in Ω , show that $f(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ is analytic in Ω.
11. State the maximum principle for harmonic functions.
12. How many roots does the equation $z^7 - 2z^4 + 6z^2 - z + 1 = 0$ have in the disk $|z| < 1$.
13. Show that an elliptic function without poles is a constant.
14. show that a non-constant elliptic function has equally many poles as it has zeros.

(14×1 = 14 *weightage*)

Turn over

PartB

Answer any **seven** *questions.*

Each question has **2** *weightage.*

15. Define analytic function. State and prove a sufficient condition for a function to be analytic in a region Ω.

16. Define linear transformation. Give an example. Prove that a linear transformation carries circles to circles.

17. Show that the line integral $\int_\gamma pdx + qdy$ defined in Ω depends only on the end points of γ if and only if there exists a function $u(x, y)$ in Ω with the partial derivatives $\frac{\partial u}{\partial x} = p, \frac{\partial u}{\partial y} = q$.

18. State and prove Cauchy's theorem for an open disk.

19. State and prove Taylor's theorem (finite development).

20. Define essential singularity. Prove that an analytic function comes arbitrarily close to any complex value in the neighbourhood of an essential singularity.

21. Suppose that $u(z)$ is harmonic in $|z| < R$, continuous for $|z| \leq R$. Prove that $u(a) = \frac{1}{2}\int_{|z|=R} \frac{R^2-|a|^2}{|z-a|^2} u(z)d\theta$ for all $|a| < R$.

22. Suppose that $f_n(z)$ is analytic in the region Ω_n, and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω uniformly on every compact subset of Ω. Then prove that $f(z)$ is analytic in Ω and $f'_n(z)$ converges uniformly to $f'(z)$ on every compact subset of Ω.

23. Prove that any two bases of the same module are connected by a unimodular transformation.

24. Derive the Legendre's relation $\eta_1\omega_1 - \eta_2\omega_2 = 2\pi i$

($7 \times 2 = 14$ *weightage*)

PartC

Answer any **two** *questions.*

Each question has **4** *weightage.*

25. If the function $f(z)$ is analytic on a rectangle R, prove that $\int_{\partial R} f(z)dz = 0$ where ∂R is the boundary curve of R.

26. a) State and prove Cauchy's integral formula.

    b) Evaluate $\int_c \frac{1}{z(z-3)}dz$ where c is the circle $|z| = 2$.

27. a) State and prove Cauchy's residue theorem.

    b) Prove that $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}dx = \frac{5\pi}{12}$.

28. a) Prove that the Laurent development of an analytic function is unique.

    b) Prove that the modular function $\lambda(\tau)$ is invariant under the congruence subgroup mod 2 .

($2 \times 4 = 8$ *weightage*)

30

M3N16207 (Pages :2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

MT3C12 - Functional Analysis

(2015 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

PART A

Answer all questions

Each question carries weightage 1

1. Prove that sub set of a separable metric space is separable with the induced metric.
2. Let E_1 be an open subset of a normed space X and $E_2 \subset X$. Prove that $E_1 + E_2$ is open in X.
3. Prove that K^n with $\| \|_1$ is strictly convex.
4. Let X be an inner product space and $x \in X$. Prove that
$$< x, y > = 0, \forall y \in X \text{ if and only if } x = 0.$$
5. Is K^n with $\| \|_1$ is an inner product space. Justify your answer.
6. What you mean by an orthonormal basis for a Hilbert space H? Give an example for an orthonormal basis for l^2.
7. Let $\{x_1, \dots, x_n\}$ be an orthogonal set in X and $k_1, \dots, k_n$ be scalars having absolute value 1 prove that $\|k_1 x_1 + \dots + k_n x_n\| = \|x_1 + \dots + x_n\|$.
8. Let X be an inner product space and E be a convex subset of X. Prove that there exist at most one best approximation from E to any $x \in X$.
9. Let X be an inner product space and E be an orthonormal subset of X.Prove that $\|x - y\| = \sqrt{2}\ all\ x \neq y\ in\ E$.
10. Let $X = K^2$ with $\| \|_1$, $Y = \{(x(1), x(2)) \in X : x(2) = 0\}$ *and* $g \in Y'$ is defined by $g(x(1), x(2)) = x(1)$.Find a Hahn Banach extension of g to X.
11. Prove that every nonzero linear functional on a normed space X over K is an open map.
12. Is C_{00} is a Banach space? Justify your answer.
13. Let X and Y be normed spaces and $F: X \to Y$ be linear. Prove that F is continuous if and only if goF is continuous $\forall g \in Y'$.
14. Let X be a Banach space. Prove that every absolutely summable series of elements in X is summable In X.

$(14 \times 1 = 14$*weightage*$)$

PART-B

Answer any seven questions.

Each question carries weightage 2.

15. Prove that the metric space l^2 is separable.
16. Prove that the set of all polynomials in one variable is dense in $C([a, b])$ with the sup metric.
17. let X be a normed space .Prove that every closed and bounded subset of X is compact if and only if X is finite dimensional.

18. Let X and Y be normed spaces and $F: X \to Y$ be linear and range of F is finite dimensional. Prove that F is continuous if and only if the zero space of F is closed in X

19. State and prove Gram –Schmidt orthonormalization.

20. Let E be a nonempty closed convex subset of a Hilbert space H and $x \in X$. Prove that there is a unique best approximation from E to X.

21. Let X be a normed space over K ,Y be a subspace of X and $a \in X$ *but* $a \notin \bar{Y}$, Prove th there exist $f \in X'$ such that $f|_Y = 0, f(a) = dist\ (a, \bar{Y})$ and $\|f\| = 1$.

22. Let X and Y are normed spaces and $X \neq 0$ Prove that $BL(X, Y)$ is a Banach space in th operator norm if and only if Y is a Banach space.

23. Prove that a Banach space cannot have a denumerable(Hamel) basis.

24. Let $\|\ \|$ and $\|\ \|'$ be norms on a linear space X. Prove that $\|\ \|'$ is equalent to $\|\ \|'$ if and only if there is $\alpha > 0$ *and* $\beta > 0$ such that $\beta\|x\| \leq \|x\|' \leq \alpha\|x\|$ *for all* $x \in X$.

$(7 \times 2 = 14$*weightage*

PART-C

Answer any two questions.

Each question carries weightage 4.

25. Prove that for $1 \leq p \leq \infty$, the metric space $L^P(E)$ *is complete*.

26. Let H be a Hilbert space over K . Prove that H is separable if and only if H has a countable orthonormal basis.

27. Let X be a normed space . Prove that for every subspace Y of X and every $g \in Y'$ ther a unique Hahn –Banach extension of g to X if and only if X' *is strictly convex*.

28. State and prove Uniform boundedness principle.

$(2 \times 4 = 8$ *weightag*