17

1M3N16234 (Pages :2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

ST3C11 - Stochastic Process

(2015 Admission onwards)

Max. Time: 3 hours Max. Weightage:36

PART - A

Answer all questions. Weightage 1 for each question

1. What is a stochastic process? Give an example of a stochastic process clearly specifying its index set and the state space.
2. Is it true that a strictly stationary process is always weakly stationary? Justify your answer.
3. Describe the gambler's ruin problem.
4. Define a Galton-Watson branching process. Is it Markovian?
5. When do you say that $\{X_t, t \geq 0\}$ is a Poisson process?
6. Justify the statement "a queueing process is an example of a Birth-Death process".
7. What do you mean by regenerative process?
8. State the key renewal theorem.
9. Explain the meaning of inspection paradox in the context of renewal process.
10. What do you mean by a queue discipline? Explain it using stanard notations.
11. State Chapman-Kolmogorov equations. Cite one of its uses.
12. What do you mean by pricing of stock options?

(12 x 1 = 12 weightage)

PART - B

Answer any 8 questions

Weightage 2 for each question

13. Define a random walk. Show that a one dimensional symmetric random walk is recurrent.
14. Let $\{X_n\}$ be a Markov chain on the state space S = { 0, 1, 2, 3} with TPM

$$P = \begin{pmatrix} 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Assuming $P(X_0 = i)=\frac{1}{4}, i = 0,1,2,3$ compute (i) $P(X_1 = 0)$ and (ii) $P(X_0 = 0, X_2 = 0)$

15. Let $\{X_n,\ n = 0, 1, 2,\ldots\}$ with $X_0 = 1$ be a branching process with offspring distribution $\{p_j, j= 0, 1, 2\}$. Obtain the probability of extinction when $p_0 = p_1 = 1/4$, $p_2 = 1/2$.

16. Show that the sum of two independent Poisson processes is again a Poisson process.

17. Define Yule-Furry process $\{N(t); t \geq 0\}$ and obtain $P(N(t) = n)$ for $n = 0, 1, 2, \ldots\ldots$

18. Define a stochastic process with independent increments. Show that a process $\{X(t); t \geq 0\}$ with independent increments is Markovian, if $\{X(0) = 0\}$.

19. Define a renewal function m(t) and show that $m(t) = \sum_{n=1}^{\infty} F_n(t)$, where $F_n(t)$ is the distribution function of the sum of n inter-arrival times.

20. Let R(t) be the total reward earned by time t in a renewal reward process with inter-arrival time $\{X_n\}$. Assuming the existence of the moments involved, show that $\frac{R(t)}{t} \to \frac{E(R)}{E(X)}\ as\ t \to \infty$

21. Define a semi-Markov process. Does it contain the ordinary Markov chain as a special case? Justify your answer.

22. Stating the required conditions obtain the steady state probabilities for an M/M/1 queue.

23. Define a Brownian motion. Obtain the mean, variance and covariance function of a Brownian motion.

24. What do you mean by reflected Brownian motion? Obtain the probability distribution of a Brownian motion reflected at the origin.

(8 x 2 = 16 weightage)

PART - C

Answer any two questions

Weightage 4 for each question

25. Prove that P the state *j* of a Markov chain $\{X_n\}$ is recurrent if and only if $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$ *where* $p_{jj}^{(n)} = P(X_n = j | X_0 = i)$

26. State the postulates of birth and death process $\{N(t), t \geq 0\}$. Obtain the set of difference- differential equations for $p_n(t) = P(N(t) = n)$ and state the conditions under which the solutions exist.

27. (a) Show that the renewal function satisfies the renewal equation
(b) Obtain the limiting mean of the residual life in a renewal process as $t \to \infty$

28. Define a Brownian motion absorbed at a point *x*? Derive the probability distribution of a Brownian motion absorbed at a point *x*.

(2 x 4 = 8 weightage)

18

1M3N16235 (Pages :2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

ST3C12 - Testing of Statistical Hypotheses

(2015 Admission onwards)

Max. Time: 3 hours Max. Weightage:36

PART A

(Answer All questions. Weightage 1 for each question.)

1. What is p-value?
2. A sample of size 1 is taken from exponential distribution with unknown mean θ. To test, $H_0 : \theta = 1$ against $H_1 : \theta > 1$, the test is $\phi(x) = 1$, if $x > 2$ and $\phi(x) = 0$, if $x \leq 2$. What is the size of the test?
3. Define Uniformly Most Powerful (UMP) test.
4. Describe likelihood ratio test.
5. Define α -similar and UMP α -similar tests.
6. Describe how Baye's test differ from classical tests.
7. Distinguish between randomized tests and non-randomized tests.
8. How do you distinguish parametric and non-parametric tests.
9. What is Kendaull's τ ? Given a random sample how do you estimate it?
10. Describe chi-square test for homogeneity.
11. What is Sequential Probability Ratio Test (SPRT)? Explain.
12. Define Average Sample Number (ASN) function associated to SPRT.

(12 x 1 = 12)

PART B

(Answer any EIGHT questions. Weightage 2 for each question.)

13. State and prove Neyman - Pearson fundamental lemma.
14. Let α be a real number, $0 < \alpha < 1$, and ϕ^* be a most powerful size α test of H_0 against H_1. Let $\beta = E_{H_1}(\phi^*(X)) < 1$. Show that $1 - \phi^*$ is a most powerful test for testing H_1 against H_0 at level $1 - \beta$
15. Define unbiased test. Give an example to a UMP unbiased test. Justify your claim.
16. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. For testing $H_0 : \mu = \mu_0, \sigma^2 > 0$ against $H_1 : \mu \neq \mu_0, \sigma^2 > 0$ describe the UMP unbiased test.
17. Find the likelihood ratio test of a simple H_0 versus a simple H_1. Is this test equivalent to the one obtained from the Neyman-Pearson lemma? Justify your answer.
18. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from an exponential distribution with pdf:

    $f(x:\theta) = \exp(-(x-\theta)), x \geq \theta, -\infty < \theta < \infty$

    Derive the likelihood ratio test for $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$ where θ_0 is a specified value θ

19. Illustrate Baye's test through a simple example.
20. Describe two-sample Kolmogorov-Smirnov test. Is it distribution free? Justify.
21. Illustrate Wilcoxon's signed-ranks test.
22. Describe chi-square test for independence.
23. Let X be a Bernoulli random variable with parameter θ, to test $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta = \frac{3}{4}$, construct an SPRT of size of your choice.
24. For the SPRT with stoping bounds A and B, A > B, and strength (α, β), prove that $A \leq \frac{1-\beta}{\alpha}, and\ B \geq \frac{\beta}{1-\alpha}$

(8 x 2 =16

PART C

(Answer any TWO questions. Weightage 4 for each question.)

25. (a) Define the following terms:
    i. Type I and Type II errors
    ii. Power of a test
    iii. Size of a test
    iv. Most Powerful test

    (b) Give an example to a two sided UMP test. Justify your answer.
26. (a) Define α -simlar test and UMP α -similar test. Let the power function of every test ϕ of $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$ be continuous in θ. Then prove that a UMP α -similar test is UMP unbiased, provided that its size is α for testing H_0 against H_1.

    (b) State generalized Neyman-Pearson lemma.
27. a)Define Kendal's tau and derive the test based on it.

    (b) Describe the chi-square test for goodness of fit.
28. (a) State and prove Wald's inequality.

    (b) Prove that SPRT terminates with probability one.

(2 x 4 = 8

19

1M3N16236 (Pages :2) Reg. No:......................

Name:..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

ST3E02 - Econometric Models

(2015 Admission onwards)

Max. Time: 3 hours Max. Weightage:36

PART-A

*Answer **all** Questions*

Each question carries a weightage of 1.

1. Explain the demand, revenue and cost functions.
2. Define equilibrium of a market.
3. What is a homogenous production function?
4. Define Cobweb growth model.
5. State any four properties of ML Estimators of regression coefficients.
6. What do you mean by forecasting?
7. What is an instrumental variable?
8. What is simultaneous equation model?
9. Give the expression of ARMA(p, q) time series model.
10. Define MA(q) process.
11. Explain unit root test.
12. Define ARIMA (p, d, q) time series model.

(12 × 1 = 12 Weightage)

PART B

*Answer **eight** Questions*

Each question carries a weightage of 2.

13. What is meant by production function? Give an example.
14. Discuss Harold Domer Growth model.
15. Describe Leontiff's input output model
16. Describe the Almon's approach to distributed lag models.
17. Describe Cobweb model.
18. Obtain an unbiased estimator of the variance of the error term in the simple linear regression model.
19. Explain the Goldfield Quandt test.

20. Explain the idea of weighted least square.
21. Distinguish between reduced form and structural models.
22. Comment on the identifiability of the equations,

Demand, $D = a_0 + a_1P + a_2Y + V$,

Supply, $S = b_0 + b_1P + W$,

where, P is the price of the commodity, Y is income of the consumer and V and W are random error.

23. Define MA(1) process. Derive its autocorrelation and state the condition of invertibility of the process.
24. Explain the basic steps in Box-Jenkins iterative three stage modelling and model identification in time series analysis.

(8 × 2 = 16 Weightage)

PART-C

Answer two Questions

Each question carries a weightage of 4.

25. Explain homogeneous production function. Examine the linear homogeneity of Cobb-Douglus production function. Show that for a linearly homogeneous Cobb-Douglus production function, α and β are the output elasticities of capital and labour.
26. Describe the method of least squares for estimating parameters of a simple linear regression model. Show that the estimators are BLUE's, under certain conditions to be stated.
27. Explain the problem of autocorrelation? How do you detect its presence? What are the consequences of autocorrelation?
28. Explain the concept of identification problem. How do you examine identifiability of an equation?

(2 × 4 = 8 Weightage)

20

1M3N16237

(Pages :2)

Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester M.Sc Degree Examination, November 2016

ST3E09 - Life Time Data Analysis

(2015 Admission onwards)

Max. Time: 3 hours

Max. Weightage:36

Part A

Answer all questions. Weightage 1 for each question

1. Define hazard rate. State its properties.
2. Show that for Pareto distribution, the mean residual life function is linear and increasing.
3. Distinguish between type I and type II censoring schemes.
4. Explain bathtub hazard rate models
5. Define actuarial estimator of survival function. State its properties.
6. Obtain 95% confidence interval for the mean of exponential distribution under type II censoring.
7. What do you mean by parametric regression models. Give an example.
8. Explain Peto's method for estimation of survival function in the presence of tied lifetimes.
9. Write down likelihood under random censoring.
10. Define proportional hazards model. Why it is referred so?
11. Describe the method of hazard plots for model checking.
12. What do you mean by accelerated failure time model? Show that Weibull distribution belongs to this class.

(12x1=12)

Part B

Answer any 8 questions. Weightage 2 for each question

13. Discuss the behaviour of hazard rate for log normal distribution
14. Derive non-parametric estimator for distribution function for right truncated data.
15. Obtain maximum likelihood estimator of reliability function of exponential distribution under type I censoring
16. Explain a method for comparing means of two exponential distributions under type I censoring.

17. Find the estimator for the data given below.

1, 2, 2, 3, 6, 6, 10, 11, 12, 13, 13, 14, 15, 15+, 16, 17, 18+, 20+, 21, 23, 24, 25+, 26, 29, 29+, 31, 33, 34+. 35+ (+ denotes censored values)

18. Explain influence analysis of parametric regression models.
19. Describe generalised Mantel- Hansel test for two sample problem.
20. What do you mean by partial likelihood? How it is used to estimate regression parameter?
21. Derive Breslow estimator for cumulative hazard function. State its properties.
22. Explain the method using martingale residuals for model checking.
23. How do you compare two survival functions for grouped data.
24. Explain discrete time hazard based models.

(8 x 2 = 16)

Part C

Answer any 2 questions. Weightage 4 for each question

25. Define Nelson –Aalen estimator of cumulative hazard function. Show that it is uniformly strong consistent.
26. Derive the estimator of survival function using life table procedure. Derive Greenwoods formula.
27. Define Weibull regression model. Derive inference procedure for estimating parameters of the model
28. Define Cox proportional hazards model. Derive Cox likelihood as a marginal likelihood.

(2 x 4 = 8)