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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2017

ST2C09– Design and Analysis of Experiment

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

Part A

(Answer **ALL** the questions. Weightage **1** for each question)

1. What is meant by estimability of a parametric function in linear model?
2. Elaborate how 'local control'helps to increase the efficiency of a design?
3. What is meant by model adequacy checking in ANOVA?
4. Distinguish between orthogonal and non-orthogonal data
5. Describe the basic principles of experimentation employed in LSD.
6. What are the objectives and rationale behind the technique of ANCOVA?
7. What is a complete block design?
8. Define a Balanced Incomplete Block Design (BIBD).
9. What is the use of Youden square design?.
10. What is the need for'confounding' in design of experiments?
11. Identify the interaction confounded in the following 2^4-factorial design.

Block-1	a	b	c	abc	ad	bd	cd	abcd
Block-2	(1)	ab	ac	bc	d	abd	acd	bcd

12. Describe the main features of a split-plot design.

(12x 1=12 weightage)

Part B

(Answer any **EIGHT** questions. Weightage **2** for each question)

13. Show that in a linear Model $(\mathbf{Y}, \mathbf{A}\boldsymbol{\theta}, \boldsymbol{\sigma}^2\mathbf{I})$ the number of independent estimable parametric functions is equal to the *rank of A*.
14. For a single factor ANOVA with a fixed linear model establish the fundamental identity (in the usual notation): $TSS = SS_{Treatment} + SSE$.
15. Present a brief account of Kruskal-Wallis test, stating clearly the underlying problem.
16. Describe how efficiency of RBD is estimated compared to CRD.
17. Describe the linear model used in the analysis of data from Latin Square Design (LSD). Based on the model, present an outline of the ANOVA-table specifying the sources of variations, sum of squares, degrees of freedom and F-ratio.

18. Suppose that in a RBD one observation is missing, how do you estimate it? What are the modifications to be made then in the ANOVA?
19. Explain a method for the construction of BIBD.
20. 'Youden square design is a symmetrical BIBD'- discuss.
21. Prove that in a PBIBD with two associate classes (in the usual notation),

$$\text{(i)} \ \sum_{i=1}^{2} n_i \lambda_i = r(k-1) \ \text{and} \ \text{(ii)} \ n_i p_{jk}^{\,i} = n_j p_{ik}^{\,j}.$$

22. Explain confounding in factorial experiments. Illustrate confounding of the interaction effect 'AC' with reference to 2^3– factorial experiments, having A,B,C as factors.
23. Explain the concept of fractional factorial design with a suitable example.
24. What is a 3^2 –factorial design? Explain how the sums of squares are calculated in this case and present the ANOVA table.

(8 x 2 =16 weightage)

Part C

(Answer any **TWO** questions. Weightage **4** for each question)

25. Describe the analysis of variance for two-way cross classified data with more than one observation per cell.
26. a) Define a Graeco-Latin square Design. Develop the ANOVA for this design.
    b) Present an outline of the ANCOVA in the case of a RBD with one concomitant variable.
27. Discuss the analysis of a BIBD with recovery of inter- block information.
28. An experiment involving five factors each at two levels, is to be laid out in blocks of size 16, two blocks per replicate by totally confounding the highest order interaction. Construct such a design and give the outline of the ANOVA table.

(2 x 4=8 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2017

ST2C06 – Estimation Theory

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

Part A

(Answer ALL questions. Weightage 1 for each question.)

1. Define complete sufficient statistic.
1. Show that sample mean is always unbiased for population mean.
2. State Rao Blackwell theorem.
3. Distinguish between pivot and ancillary statistic.
4. Prove that maximum likelihood estimators need not be unique.
5. Let $X_1, X_2, ..., X_n$ be a random sample from $U(0,\theta)$, prove that $X_{(n)}$ is a consistent estimator for θ.
6. Let X_1, X_2 be a random sample from $Poisson(\lambda)$. Show that the statistic $T = X_1 + 2X_2$ is not sufficient for λ.
7. Explain the method of percentiles for estimation of population parameter.
8. Define Bayes' risk.
9. Let X be an observation form B(1,p), then show that based on X, unbiased estimator for p^2 does not exist.
10. What do you mean by efficiency of estimators .
11. Define one parameter exponential family. Examine whether the Poisson distribution is a member of this class.
12. Define coverage probability and confidence coefficient.

(12 x 1=12 weightage)

Part B

(Answer any EIGHT questions. Weightage 2 for each question.)

13. State and prove Fisher-Neyman Factorization theorem.
14. State and prove invariance property of maximum likelihood estimators.
15. Obtain minimal sufficient statistic for the parameter θ of Cauchy distribution with p.d.f $f(x,\theta) = \dfrac{1}{\pi(1+(x-\theta)^2)}$ based on a sample of size n.

16. Show that $\dfrac{\frac{1}{2}\left(\sum X_i^2\right)^{1/2}\sqrt{\frac{n}{2}}}{\sqrt{\frac{n+1}{2}}}$ is unbiased for θ where $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N(0, \theta^2)$

17. Explain the method of moments for estimation of parameters. Estimate the parameters m and p in the case of Gamma distribution with mean p/m by the method of moments.
18. State and prove Lehman Scheffe theorem.
19. Let $X_1, X_2, \ldots, X_n$ be a sample from $U(0, \theta)$. Find a consistent estimator which is sufficient for θ
20. Explain shortest length confidence interval. Let $X_1, X_2, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$, σ^2 unknown. Find shortest length confidence interval for μ.
21. If n independent observations $X_1, X_2, \ldots, X_n$ are drawn from the p.d.f $f(x, \theta) = e^{(\theta - x)}$, $x > \theta$. Examine whether $X_{(1)}$ is unbiased and consistent for θ.
22. Obtain large sample confidence interval for the parameter of Poisson distribution.
23. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, \theta^2)$. Show that $T = \left(\sum X_i, \sum X_i^2\right)$ is sufficient for θ but not complete.
24. Let $X_1, X_2, \ldots, X_n$ be a random sample from the p.d.f $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$, $x \in R$.

    Obtain maximum likelihood estimator of θ .

(8 x 2=16 weightage)

Part C

(Answer any TWO questions. Weightage 4 for each question.)

25. State and prove Cramer-Rao inequality. Obtain the condition for attainment of this bound.
26. Explain the maximum likelihood method of estimation. Show that with probability approaching one as $n \to \infty$ likelihood equation admits a consistent solution. Obtain the asymptotic distribution of maximum likelihood estimators.
27. Explain the term Risk. What is meant by minimax estimator. Show that Bayes estimator with constant risk is minimax.
28. (a) Derive necessary and sufficient condition for an unbiased estimator to be UMVUE.

    (b) Show that sample mean and sample variance are independently distributed if the sample is drown from a normal population with mean μ and variance σ^2 .

(2 x 4=8 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2017

ST2C08– Probability Theory

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage:

Part A

Answer all the questions

Weight **1** for each question

1. Distinguish between a field and a σ-field.
2. If φ(t) is a characteristic function, show that its real part is also a characteristic function.
3. Let X be a random variable such that

$$F(x)=\begin{cases}1, x>0\\0, x\leq 0\end{cases}$$

 Examine whether F(x) is a distribution function.

4. Define a tail function. Give an example.
5. State Jordan decomposition theorem of distribution functions.
6. Define mutual independence. Prove that pair-wise independence need not imply mutual independence.
7. State Slutsky's theorem and state one of its uses.
8. Show that cumulative distribution function of a random variable is non-decreasing.
9. Give an example of a distribution, which is (i) infinitely divisible and (ii) not infinitely divisible
10. Show that Boral functions of independent random variables are independent.
11. If $\{X_n\}$ and $\{Y_n\}$ are Martingales with respect to $\{F_n\}$, show that X_n-Y_n is a Martingale with respect to $\{F_n\}$.
12. State Kolmogorov's three series theorem.

(12x1=12 weightage)

Answer any **eight** questions
Weight **2** for each question

13. Define conditional probability. Show that conditional probability obeys the axioms of probability measure.
14. Let φ_1 and φ_2 be the characteristic functions of X and Y. Then which of the following are characteristic functions?
(i) $\varphi_1 + \varphi_2$ (ii) $\varphi_1 - \varphi_2$ (iii) $\varphi_1\varphi_2$ (iv) φ_1/φ_2 (v) ½($\varphi_1 + \varphi_2$). Justify your claim in each case.
15. Examine convergence in probability for the sequence of random variables $\{X_n\}$ with $P(X_n=0) = 1 - \frac{1}{n}$, $P(X_n=1) = \frac{1}{n}$, n=1,2,.... Does it satisfy convergence in distribution?
16. State and prove Markov inequality.
17. Let $\{X_n\}$ be a strictly decreasing sequence of random variables which convergence in probability. Then show that $\{X_n\}$ converges almost surely.
18. State and prove Lindberg-Levy form of Central Limit Theorem.
19. State and establish Khintchine's weak law of large numbers.
20. Explain central limit theorem. Determine whether the central limit theorem holds for the sequence of independent random variables $\{X_n; n=1,2,...\}$ where X_n is normal with $E(X_n) = 0$ and $V(X_n) = 2^{-n}$.
21. If $E(X^2) < \infty$, then prove that $Var(X) = Var(E\{X/Y\}) + E(Var\{X/Y\})$.
22. Obtain a necessary and sufficient condition for the holding of weak law of large numbers for independent and identically distributed random variables.
23. State Lindberg- Feller central limit theorem and deduce Liaponov's central limit theorem from this.
24. Show that if $X_n \xrightarrow{P} X$ implies $X_n \xrightarrow{L} X$, as $n\to\infty$. When does the converse hold?

(8 x 2=16 weighatge)

Part C

Answer any **two** questions
Weight **4** for each question

25. (a) Establish Kolmogorov 0-1 laws.
(b) Can you give an example to show $\sum P(A_k)$ diverges when $P(\lim supA_k) = 0$. Justify.
26. (a) Explain moment problem?
(b) Prove or disprove: a random variable X possess moments of any order if and only if $\limsup_{n\to\infty}[P\{|X| > \alpha^n\}]^{\frac{1}{n}} = 0$, for $\alpha > 1$.
27 (a) State and prove inversion theorem of characteristic function.
(b) Find the probability density function of X, if X has characteristic function:
$$\varphi(t) = \begin{cases} 1-|t|, |t|<1 \\ 0, otherwise \end{cases}.$$
28. State and prove Kolmogorov's strong law of large numbers for independent and

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2017

ST2C07 – Sampling Theory

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage: 36

PART A

(Answer **ALL** questions. Weightage 1 for each question)

1. Explain probability sampling and non-probability sampling with suitable examples
2. Distinguish between SRSWR and SRSWOR
3. Explain Linear systematic sampling
4. What is sampling design? Give an example.
5. What do you mean by auxiliary variable techniques in estimation?
6. What do you mean by regression estimator?
7. Show that the coefficient of variation of the auxiliary variable is the upper bound for the relative bias of ratio estimator under Simple Random Sampling.
8. Distinguish between two stage and two phase sampling
9. What do you mean by Des Raj ordered estimator?
10. Explain cluster sampling with an example.
11. What do you meant by PPS sampling?
12. Explain non - sampling error

(12 x 1=12 weightage)

PART B

(Answer any **EIGHT** questions. Weightages 2 for each question)

13. Distinguish between census and sampling. What are the advantages of sampling over census?
14. From a finite population a Simple Random Sample has to be drawn. How will you determine the sample size? What are the assumptions used?
15. Show that under SRSWOR, the sample mean is an unbiased estimator of population mean. Also find its variance.
16. Derive Hartley-Rose unbiased ratio type estimator.
17. Discuss Murthy's unordered estimator in the case of PPS sampling.
18. What is proportional allocation? Write down the expression for sample size in each stratum under this allocation.

19. Show that mean of cluster means in the sample is not an unbiased estimator of population mean in cluster sampling if clusters are of unequal size. Give an unbiased estimator in this case.
20. Give an unbiased estimator of population mean by Two-stage sampling with SRSWOR at both the stages. Derive the sampling variance
21. Describe Sen-Midzuno scheme of sampling and derive the expression for the first order (π_i) and 2nd order (π_{ij}) inclusion probabilities.
22. Explain Lahiri's method of drawing a PPS sample without replacement.
23. Obtain the efficiency of cluster Sampling (with equal size) with SRSWOR in terms of intra class Correlation Coefficient. Discuss the situation when you would prefer cluster sampling to SRSWOR.
24. With usual notation, show that systematic sampling is efficient than simple random sampling if and only if $S_{wsy}^2 \geq S^2$.

(8 x 2=16 weightage)

PART C

(Answer any **TWO** questions. Weightages 4 for each question)

25. (a) How would you estimate sample size n required for estimating population mean in simple random sampling if the margin of error permitted is d with confidence coefficient $1-\alpha$?
    (b) Carry out a comparison between SRSWR and SRSWOR in estimating populations mean.
26. Explain various allocation methods used in Stratified Random Sampling. Also compare the variances of the unbiased estimators of population mean under these allocation methods.
27. (a) Define Hansen-Hurwitz estimator of population mean. Derive an unbiased estimator of its variance.
    (b) Derive Yates-Grundy form of estimated variance of Horvitz-Thomspon estimator of population mean under PPSWOR
28. (a) "Cluster sampling is a special case of two-stage sampling". Justify. Obtain 3 estimators of population mean in two-stage sampling where the units are of unequal sizes.
    (b) Describe how do you control non- sampling errors in a sample survey.

(2 x4=8 weightage)