2M2M17202 (Pages : 2) Reg. No:...........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, March 2017

MT2C06 - Algebra II

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A (Short Answer Questions)

(Answer **all** questions. *Each question has 1 weightage*)

1. Find all prime ideals and all maximal ideals of Z_2 x Z_4.
2. Define simple extension and give an example.
3. Show that the polynomial x^2+1 is irreducible in $Z_3[x]$.
4. Find a basis of $Q[\sqrt{2},\ \sqrt{6}]$ over $[\sqrt{3}]$.
5. What are conjugates of $\sqrt{2}$ +i over Q.
6. Let α be a real number such that $[Q(\alpha):Q] = 4$. Is α constructible. Justify your answer.
7. Find the number of primitive 18^{th} roots of unity in GF(19).
8. Prove that a finite extension field E of a field F is an algebraic extension of F.
9. Define separable extension of a field F. Give one example.
10. Find the index {Q(α):Q} where $\alpha = 2^{1/3}$.
11. Is $Z_5[x]/ < x^3 + 3x + 2 >$ a field. Justify your answer.
12. Give an example for an extension E of Q($\sqrt[4]{2}$) such that [E:Q] = {E:Q} = $|G(G/Q)|$. Justify your answer.
13. Find $\varphi_3(x)$ over Z_3.
14. Is x^5-2 is solvable by radicals over Q.

(14 x 1=14 weightage)

Part B

*(Answer any **seven** from the following **ten** questions. Each question has weightage 2)*

15. If F is a field then prove that every ideal in F[x] is principal.
16. Prove that $Q(2^{1/2}, 2^{1/3}) = Q(2^{1/6})$.
17. Let E be an extension of a field F, $\alpha \in$ E and let $\varphi_\alpha : F(x) \to E$ be the evaluation homomorphism, Show that α is transcendental over F if and only if φ_α is one to one.
18. Prove that trisecting the angle is impossible.
19. State and prove conjugation isomorphism theorem.
20. If E is a finite extension of field F then prove that {E:F} = [E:F].
21. Prove that every finite extension of a field of characteristic zero is a simple extension.
22. Describe all elements of the Galois group G(K/Q) where K is the splitting field of x^3+2 over Q.
23. Prove that the Galois group of the n^{th} cyclotonic extension of Q has $\varphi(n)$ elements.
24. Is the regular 150-gon constructible.

(7 x 2=14 weightage)

Part C

*Answer any **two** from the following **four** questions. Each question has weightage 4*

25. (a) Prove that an ideal $< P(x) > \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible over F.

    (b) If F is a field Show that every ideal in F[x] is a principal ideal.
26. (a) If E is a finite extension field of a field F and K is a finite extension field of E then prove that K is finite extension of F and [K:F] = [K:E][E:F].

    (b) A finite field $GF(P^n)$ of P^n elements exists for every prime power P^n.
27. Define perfect field and prove that every finite field is perfect.
28. Let F be a field of characteristic 0 and let a$\in$ F, if K is the splitting field of x^n-a over F then prove that G(K/F) is a solvable group.

(2 x 4 =8 weightage)

1M2M17206 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, March 2017

MT2C10- Number Theory

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A (Short Answer Questions) (1 – 14)

Answer **all** questions.
Each question carries 1 weightage.

1. Define residue class a modulo m.
2. Define cryptosystem.
3. Show that $\sum_{d|n} \mu(d) = I(n)$
4. Prove that $\varphi(10^n) = 4 \times 10^{n-1}$
5. Evaluate i) $\varphi(88)$.

   ii) $\mu(35)$
6. Show that $\psi(x) = \sum_{m \le log_2 x} \sum_{p \le x^{\frac{1}{m}}} log\ p$.
7. Let $a \equiv b \pmod m$. Prove that if $d|m$ and if $d|a$, then $d|b$.
8. Find the quadratic residues modulo 13 and non residues modulo 13.
9. Encipher the message" HELP ME" by using the affine transformation with key $a = 13, b = 9$ in the 27 letter alphabet with blank = 26.
10. What is Diffie Hellman assumption?
11. What is knapsack problem?
12. Prove that Fermat number F_5 is composite.
13. Find all solutions of the linear congruence $18x \equiv 30 \pmod{42}$.
14. If $\{x\}$ is the fractional part of x, then what are the possible values of $\{x\} + \{-x\}$?

(14 x 1 = 14 weightage)

Part B

Answer any **seven** from the following ten question (15 – 24).
Each question carries 2 weightage.

15. Determine those odd prime p for which $(-3|p) = 1$ and which $(-3|p) = -1$.

16. State and prove Lagrange's theorem for polynomial congruence.

17. For any prime $p \geq 5$, prove that $\sum_{k=1}^{p-1} \frac{(p-1)!}{k} \equiv 0 \pmod{p^2}$.

18. Show that, for $x \geq 2$, $\sum_{p \leq x} \frac{1}{p} = \log\log x + A + O(\frac{1}{\log x})$, where A is a constant.

19. Solve the system of congruence

$$x + 3y \equiv 1 (\text{mod } 26)$$

$$7x + 9y \equiv 1 (\text{mod } 26)$$

20. Prove that for $x \geq 1$ $\left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$with equality holding only if $x < 2$

21. State and prove Euler's summation formula.

22. Explain ElGamal cryptosystem.

23. State and prove Selberge's identity

24. A cryptosystem uses 26-letter alphabet and enciphering transformation, the matrix transformation $AX = B$, where $A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} \in M_2(\mathbb{Z}/26\mathbb{Z})$. Decipher the message "QV".

(7 x 2 = 14 weightage)

Part C

Answer any **two** from the following four questions (25 – 28).
Each question carries 4 weightage.

25. For $n \geq 1$ prove that the nth prime p_n satisfies the inequalities

$\frac{1}{6}\, n \log \text{n} < p_n < 12(n \log \text{n})$ and hence prove that the series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

26. Suppose an advisory is using an enciphering matrix A in the 26 letter alphabet. A cipher text message is "WKNCCHSSJH" and its first word is "GIVE". Find the deciphering matrix A^{-1} and what is the message?

27. Prove that prime number theorem implies $\lim_{x\to\infty} \frac{M(x)}{x} = 0$.

28. State and prove quadratic reciprocity law.

(2 x 4 = 8 weightage)

1M2M17205 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, March 2017

MT2C09- PDE and Integral Equations

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

Answer **all** questions

Each question carries one weightage

1. Give the classification of first order partial differential equations with examples.
2. Show that there exists an Integrating factor for a Pfaffian differential equation in two variables.
3. Give the conditions for the compatibility of the partial differential equations $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$
4. What are the conditions required for $u = F(x,y,z,a,b,c)$ to be a complete integral for $f(x,y,z,u_x,u_y,u_z) = 0$
5. Define Monge cone and give example
6. Define characteristic strip and characteristic curve.
7. Classify all second order partial differential equations in canonical form.
8. State d'Alembert's solution for the vibrations of an infinite string.
9. Define a) domain of dependence and b) range of influence.
10. Define Cauchy Problem and give example.
11. State any two boundary value problems.
12. Write the four properties of green's function which satisfies the Integral equation $y(x) = \int_a^b G(x,\xi)\Phi(\xi)d\xi$
13. Define separable Kernel and give example.
14. Write the formulas used in iterative method for solving $y(x) = F(x) + \lambda\int_a^b K(x,\xi)y(\xi)d\xi$

(14 x 1 = 14 weightage)

Part B

Answer any **seven** questions

Each question carries 2 weightage

15. Show that the singular integral is a solution of the partial differential equation $f(x,y,z,p,q)=0$
16. Find the general solution of $x(y^2-z^2)p-y(z^2+x^2)q=(x^2+y^2)z$
17. By Charpit's Method, find a complete integral of $f=z^2-pqxy=0$
18. Solve $z^2+zu_z-u_x^2-u_y^2=0$ by Jacobi's Method.
19. Find the integral surface of the differential equation $x(z+2)p+(xz+2yz+2y)q=z(z+1)$ passing through the curve $x_0=s^2, y_0=0, z_0=2s$
20. Reduce the equation $(n-1)^2u_{xx}-y^{2n}u_{yy}=ny^{2n-1}u_y$, where $n\in\mathbb{N}$, to a canonical form.
21. Prove that for the equation $u_{xy}+\frac{1}{4}u=0$, the Riemann function is $v(x,y;\alpha,\beta)=J_0(\sqrt{(x-\alpha)(x-\beta)})$
22. Prove that the solution for $u_{tt}-c^2u_{xx}=F(x,t), 0<x<l, t>0$

    At $t=0, u$ & u_t takes $f(x)$ and $g(x)$ respectively, $0\leq x\leq l$ $u(0,t)=u(l,t)=0$, $t\geq 0$

    is unique.
23. Transform $y''+xy=1, y(0)=0, y(l)=1$ to an integral equation.
24. Show that the characteristic numbers of a Fredholm equation with real symmetric Kernel are all real.

(7 x 2 = 14 weightage)

Part C

Answer any **two** questions

Each question caries 4 weightage

25. Derive the necessary and sufficient condition for a Pfaffian differential equation of three variables to be integrable.
26. For the non-linear partial differential equation $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$, find the solution which passes through the x-axis.
27. Solve $u_t=u_{xx}$, $0<x<l,\ t>0$

    $u(0,t)=u(l,t)=0$

    $u(x,0)=x(l-x)$, $0\leq x\leq l$
28. a) Write a short note on Neumann series.

    b) Show that the resolvent Kernel $\Gamma(x,\xi;\lambda)$ is the solution of $y(x)=F(x)+\lambda\int_a^b K(x,\xi)y(\xi)d\xi$, when F is replaced by K.

(2 x 4 = 8 weightage)

FAROOK COLLEGE (AUTONOMOUS), JUNE 2017
SECOND SEMESTER M.Sc. DEGREE EXAMINATION
MT2C07 Real Analysis II

2M2M17203 Scheme.

Part A (Short Answer Type Questions)

Answer all questions. Each question has 1 weight.

1. Grade C for each part.
   Theorem 95, Page 207 Rudin.
2. Theorem 9.24, Page 221 Rudin. Grade as per the steps.
3. $\nabla f(x,y) = (6x^2 - 6x)\hat{i} + (6y^2 - 6y)\hat{j}$.
   Gradient at (2,3) is $12\hat{i} + 72\hat{j}$.
4. Corollary to Theorem 9.2, Pages 205-206 Rudin. Grade as per the steps.
5. Definition. Grade as per the steps.
6. Grade D for statement. Grade B for proof.
7. Grade C for Proof. Grade C for "Converse is not true" and justification by an example.
8. Grade D for defiition of measurable function. Grade B for proof.
9. Definition. Grade as per the steps.
10. Proof. Grade as per the steps.
11. Grade D for Example Page 95, Royden. Grade as per the steps.
12. Grade D for proof. Grade as per the steps.
13. Statement of Question needs correction ?
    Grade C for an attempt of the question.
14. Statement of Theorem 17, Page 92, Royden. Grade as per the steps.

$14 \times 1 = 14$ Weights.

Part B

**Answer any Seven from the Nine questions.
Each question has 2 weights.**

15. Theorem 9.7, Page 208, Rudin. Grade as per the steps.
16. Theorem 9.36, Page 233, Rudin. Grade as per the steps.
17. Proposition 15, Page 63, Royden. Grade as per the steps.
18. Lemma 11, Page 60, Royden. Grade as per the steps.
19. Proof.
    Grade D for the definition of a measurable set. Grade B for the rest.

20. Lemma 16, Theorem 17, Page 64, 65 66 Royden. Grade as per the steps.

21. Grade D for statement and Grade B for proof. Grade as per the steps.

22. Lemma 10, Page 107, Royden. Grade as per the steps.

23. Lemma 11, Page 108, Royden. Grade as per the steps.

24. Theorem 3, Page 100, Theorem 5, Page 103 and Corpllary 6, Page 104, Royden.

$7 \times 2 = 14$ Weights.

Part C
Answer any Two from the Four questions.
Each question has 4 weights.

25. Grade C for part (a) and Grade C for part (b). Grade as per the steps.

26. Grade B for proof, (Proposition 14, Page 62) & Grade D for the example. Grade as per the steps.

27. Proposition 3, Page 79. Grade as per the steps.
Grade C for "Measurable $\Rightarrow$ the Condition" ad Grade C for the other part.

28. Grade C for first part & Grade C for secod part; Theorem 3. Page 100. Grade as per the steps.

$2 \times 4 = 8$ Weights.

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Mathematics Degree Examination, March 2017

MT2C08- Topology I

(2016 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A(Short Answer Type Questions)

Answer all the questions

Each question has weightage 1.

1. Define a cofinite topology.
2. Let $\tau_1$and $\tau_2$be two topologies on a set X. Show by an example that $\tau_1 \cup \tau_2$ need not be a topology on$X$.
3. Define a sub-base for a topology on a set. Give an example.
4. Prove or disprove.
   Second countability is a hereditary property.
5. Find the derived set of integers in the real line$R$ with usual topology.
6. Let $X = \{1,2,3\}$and$\tau = \{X, \phi, \{1,2\}\}$. Describe the closure operator on this space.
7. Is the real line $R$with usual topology separable? Justify your answer.
8. Define a divisible property. Give an example for the same.
9. Prove or disprove.
   The intersection of two connected sets is connected.
10. Give an example of a topological space which is connected but not locally connected.
11. Define the weak topology determined by the family of functions.
12. Define a regular space. Give an example.
13. Give an example of a T_0 space which is not T_1.
14. State Urysohn's Lemma.

($14 \times 1 = 14$ weightage)

Part B(Paragraph Type Questions)

Answer any seven questions

Each question has weightage 2.

15. Show that metrizability is a hereditary property.
16. Prove that open balls in a metric space are open sets.
17. Prove that a subset A of a topological space X is dense in X if and only if for every non empty open subset B of Xintersects A.
18. Prove that every closed surjective map is a quotient map.
19. Prove that a compact subset in a Hausdorff space is closed.
20. Prove that a second countable space is Lindeloff.
21. Show that closure of a connected set is connected.
22. Prove that a continuous bijection from a compact space onto a T_2 space is a homeomorphism.
23. Prove that every open subset of the real line in the usual topology can be expressed as the countable union of mutually disjoint open intervals.
24. Distinguish between locally connectedness and path connectedness.

Part C(Essay Type Questions)

Answer any two questions
Each question has weightage 4.

25. (a) Prove that a discrete space is second countable if and only if the underlying set is countable.
    (b) Let X be a topological space and B be a family of open sets in X. Prove that B is a base for a topology on X if and only if for all $B_1, B_2 \epsilon B, B_1 \cap B_2$ is a union of some members of B.
26. (a) Define derived set and closure of a set in a topological space. Obtain the relation connecting a set, its closure and the derived set.
    (b) Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
27. (a) Prove that every closed bounded interval of real line is compact.
    (b) Show that every continuous real valued function on a compact space is bounded and attains its extrema.
28. (a) Prove that all metric spaces are T_4.
    (b) Give an example of a T_2 space which is not T_3.

($2 \times 4 = 8$ weightage)