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2M19109 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March /April 2019

MSTA2B06 –Theory of Estimation

(2018 Admission onwards)

ne: 3 hours Max. Weightage: 36

Part A

(Answer ALL questions. Weightage 1 for each questions.)

Show that Poisson family $P(\lambda)$, $\lambda > 0$ belongs to one parameter exponential family.

Define sufficient statistic.

Find a sufficient statistic for σ^2 in N(0 , σ^2) based on a sample of size n.

Give an example of a consistent estimator which is not sufficient.

If there exist two unbiased estimators for θ, show that there exits infinitely many unbiased estimators for θ.

Explain uniformly minimum variance unbiased estimator.

What do you mean by one parameter Cramer family?

. If (0,1) is sample of size 2 from Bernoulli (θ) where $\theta = 0.1$ or 0.6, find the MLE of θ.

. State Cramer-Huzurbazer theorem.

0. Define shortest expected length confidence interval.

1. Define Bayesian credible interval for a parameter.

2. Give a pivotal quantity for the parameter μ in N(μ , 1).

(12 x 1 = 12 weithtage)

Part B

Answer any EIGHT questions. Weightage 2 for each question.

13. State factorization theorem for sufficiency. Prove it in the discrete case.

14. Let $X_1, X_2, \ldots X_n$ be random sample from exponential distribution with mean $\frac{1}{\theta}$. Show $T = \sum X_i$ is complete sufficient for θ.

15. Show that $(\sum X_i, \sum X_i^2)$ is jointly sufficient for (μ, σ^2) *in* $N(\mu, \sigma^2)$

16. Obtain UMVUE of p in Binomial(n,p) based on random sample of size n.

17. If T is a consistent estimator for θ_1, show that e^T is a consistent for e^{θ}.

8. Find the UMVUE of μ^2 based on random sample of size n from N(μ , 1)?

9. Find the maximum likelihood estimator for θ based on a random sample of size n from $f(x;\theta) = \frac{1}{\theta}, \theta < x < 2\theta,\ \theta > 0$

0. Let $X_1, X_2 \ldots X_n$ be random sample from N(μ , 1), where θ is an integer. Obtain maximum likelihood estimate of μ ?

1. Find moment estimators of m and p of Gamma(m, p)?

2. Derive a 100(1 - α)% confidence interval for θ in $f(x;\theta) = \theta x^{\theta-1},\ 0 < x < 1, \theta > 0$.

3. If the Prior distribution of θ in *Binomial*(m, θ) is Beta(3,2), obtain Bayes estimate of θ under squared error loss.

4. State Bayes' theorem and explain how it is used as a tool for estimation of parameters.

(8 x 2 = 16 weightage)

Part C

(Answer any TWO questions. Weightage 4 for each question.)

5. (a) State and prove Basu's theorem.

(b) Obtain UMVUE of $1 - e^{-\theta}$ based on a random sample of size n from *Poisson* (θ).

6. (a) State and prove Rao-Blackwell theorem.

(b) Obtain UMVUE of $2\mu + \sigma^2$ in $N(\mu, \sigma^2)$

7. (a) Show that the maximum likelihood estimator is consistent and asymptotically normaly distributed under some regularity conditions.

(b) Explain the percentile method of estimation to construct CAN estimator and illustrate it in the case of Pareto distribution with pdf

$f(x;\alpha,\beta) = \beta(x-\alpha)^{-(\beta+1)},\ x > \alpha, \beta > 0, \alpha \in R$

8. (a) Explain the construction of large sample confidence interval and illustrate it to obtain confidence interval for success probability of a binomial population.

(b) Explain the construction of confidence interval for ratio of population variances based on samples from independent normal populations $N(\mu_1, \sigma_1^2)$ *and* $N(\mu_2, \sigma_2^2)$.

(2 x 4 = 8 weightage)

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I19110 (Pages : 3) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March /April 2019

MSTA2B07– Probability Theory

(2018 Admission onwards)

3 hours Max. Weightage: 36

Part A

(Answer **all** questions. Each question carries **1 weightage.**)

ine distribution function. Prove that it is non-decreasing.

(and Y are two independent random variables with joint distribution function $_Y(x,y)$ and marginal distribution functions $F_X(x)$, $F_Y(y)$. Prove that $_Y(x,y) = F_X(x)F_Y(y),\ \forall x, y \in \mathcal{R}$.

at is probability space induced by a random variable X?

ine convergence in r^{th} mean. Give an example to quadratic mean convergence.

w that r^{th} mean convergence implies convergence in probability.

cuss strong law of large numbers.

ine characteristic function. Check whether the following functions are character-
: functions:

) $g_1(t) = \sin t,\ t \in \mathcal{R}$

) $g_2(t) = \cos t,\ t \in \mathcal{R}$

:e Helly-Bray theorem.

d the distribution function of a random variable whose characteristic function is $= \cos 2t, t \in \mathcal{R}$

at is a martingale? Give an example.

:e Lindeberg-Feller form of CLT.

ine stopping time.

Part B

(Answer any eight questions. Each question carries 2 weightage.)

13) Prove that a distribution function can have atmost a countable number of discontinuities.

14) Define tails sigma field and tail events. State and prove Kolmogrov's 0-1 law.

15) Define independence of events. State and prove Borel 0-1 law.

16) Define convergence in probability and convergence in distribution. Bring out the mutual implication of these modes of convergence.

17) Let $X_1, X_2, \ldots, X_n$ be a sequence of independent random variables such that $X_i \sim U(0, \theta),\ \forall\ i = 1, 2, \ldots, n$. Define $Z_n = max(X_1, X_2, \ldots, X_n)$. Justifying the results you use, prove that $Z_n \rightarrow^{a.s} \theta$.

18) Let $\{X_n\}$ be a sequences of independent and identically distributed random variables with $E(X_i) = \mu$, prove that $\frac{S_n}{n} \rightarrow^{p} \mu$.

19) If $\phi(t)$ is a characteristic function, prove that $\phi(t)$ is uniformly continuous.

20) Find the distribution function of a random variable whose characteristic function is $\phi(t) = e^{-|t|}, t \in \mathcal{R}$

21) State Liapunov's form of CLT. Deduce Lindeberg-Levy CLT from Liapunov's CLT .

22) Let $\Omega = (0, 1)$, $\mathcal{B}$, the Borel σ-field of subsets of Ω and P, the Lebesgue measure on $\mathcal{B}$. Let $\mathcal{D}$ be the sigma field generated by the class $\{(0, \frac{1}{3}], (\frac{1}{3}, \frac{2}{3}], (\frac{2}{3}, 1)\}$. Define X on Ω by $X(\omega) = \omega^2$. Find $E(X|\mathcal{D})$.

23) Define conditional expectation. What are its properties?

24) Define submartingale. If $\{Z_n, n \geq 1\}$ is a non-negative submartingale, prove that $P(\max(Z_1, Z_2, \ldots, Z_n) > a) \leq E(Z_n|a)\ for\ a > 0$.

Part C

(Answer any two questions. Each question carries 4 weightage.)

5) State Jordan-decomposition theorem. Identify the following distribution as discrete, continuous or mixture. If it is a mixture decompose it.

$$F(x) = \begin{cases} 0, & x < 0; \\ (1-p) + p(1-e^{-\lambda x}), & 0 \leq x \leq T; \\ 1, & x > T. \end{cases}$$

26) Let $\{X_n\}$ and $\{Y_n\}$ be sequences of a random variables on a probability space $(\Omega, \mathcal{A}, P)$ such that $X_n \rightarrow^L X$ and $Y_n \rightarrow^L c$, where c is a constant. Prove that

(i) $X_n + Y_n \rightarrow^L X + c$

(ii) $X_n Y_n \rightarrow^L cX$

(iii) $\frac{X_n}{Y_n} \rightarrow^L \frac{X}{c}$ where $c \neq 0$.

27) State and prove uniqueness theorem on characteristic functions.

28) State and prove martingale convergence theorem.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March /April 2019

MSTA2B08– Regression Methods

(2018 Admission onwards)

Time: 3 hours Max. Weightage: 3

PART A

Answer all questions each carry one weight

1. What you mean by estimability? Explain with an example.
2. Explain Gauss Markov Linear Model.
3. What you mean by design matrix?
4. Distinguish between error and residuals.
5. What are outliers? Explain with examples.
6. What you mean by serial correlation?
7. Define orthogonal polynomial. How it is useful in multiple regression model?
8. Define Mallow's C_p statistics.
9. Distinguish between Parametric and Non-parametric regression.
10. Explain Poisson regression model.
11. What is GLM?
12. Define model deviance.

PART B

Answer any eight. Each carries two weights.

13. State and Prove Gauss- Markov theorem.
14. Describe analysis of variance in simple linear regression model.
15. Develop confidence interval for the parameter σ^2 for a simple linear regression model.
16. Obtain the least square estimates of the slope and intercept of the simple linear regression model.
17. Describe residual analysis in GLM.
18. Given that Y_1, Y_2, Y_3 are random variables with means $\beta_1+\beta_2$, $\beta_1+\beta_3$, and $\beta_3+\beta_2$ and a common variance then show that $l_1\beta_1+l_2\beta_2+l_3\beta_3$ is estimable if $l_1=l_2+l_3$.
19. Explain the weighted least squares.
20. Explain piecewise polynomial fitting.
21. Define non-parametric regression estimator and verify whether it is unbiased.
22. In the linear regression model, propose an unbiased estimator of error variance σ^2. Prove your claim.
23. Explain logistic regression model.
24. Explain different estimation methods of GLM.