53

109 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2018

MSTA2B06 –Theory of Estimation

(2017 Admission onwards)

ıe: 3 hours Max. Weightage: 36

Part A

(Answer ALL questions. Weightage 1 for each questions.)

efine complete statistics and give example?

ʼhat do you mean by a sufficient statistics?

efine Exponential family of distributions?

ive an example of an unbiased estimator which is not consistent?

ıefine Fisher information.

ıefine best linear unbiased estimator.

ʋhat do you mean by a CAN estimator? Give an example.

ːxplain the method of moments.

State Cramer-Huzurbazar theorem.

)efine unbiased confidence interval.

)istinguish between Prior and Posterior distributions.

)efine Pivotal quantity? Give an example.

(12 x 1 = 12 weightage)

Part B

Answer any EIGHT questions. Weightage 2 for each question.

:f $X_1, X_2,\ldots X_n$ is a random sample from $N(10,\sigma^2)$ find a sufficient estimator of σ^2.

Explain the criteria to find minimal sufficient statistics. Find the minimal sufficient θ *in* $\cup(-\theta,\theta)$.

Show that the sample mean $\overline{X}$ is complete sufficient μ *in* $N(\mu,\sigma^2)$. when σ^2 is known.

16. Obtain UMVUE of σ^2 in $N(\mu,\sigma^2)$ based on a sample of size n.

17. State and prove a sufficient condition for consistency of an estimator.

18. Find the UMVUE of θ^2 based on a random sample of size n from Bernoulli(θ).

19. Find the maximum likelihood estimator of θ based on random sample of size n from $f(x,\theta)=1, \theta<x<\theta+1, \theta \in R$

20. State and prove the invariance property of maximum likelihood estimator.

21. Find the moment estimators of a and b of U(a,b).

22. Explain the procedure to construct large sample confidence interval for population mean μ, stating the assumptions clearly?

23. If the Prior distribution of θ in Poisson(θ) is an exponential distribution with mean 2, obtain Bayes' estimate of θ under squared error loss function.

24. Define non informative Prior and conjugate Prior and give examples in each case?

(8 x 2 = 16 weightage)

Part C

(Answer any TWO questions. Weightage 4 for each question.)

25. (a) State and prove Cramer-Rao lower bound and derive the condition for attaining this bound.

(b) Obtain minimum variance bound estimator of 2λ, if it exists, where λ is the mean of the Poisson distribution.

26. (a) State and prove Rao-Blackwell theorem.

(b) Obtain UMVUE of μ^2 in $N(\mu,\sigma^2=5)$

27. Show that under some regularity conditions maximum likelihood estimator is a CAN estimator?

28. Based on samples of sizes n_1 and n_2 from two independent normal populations $N(\mu_1,\sigma^2)$ and $N(\mu_2,\sigma^2)$ construct 95% confidence interval for $\mu_1-3\mu_2$, when

(a) σ^2 are unknown and (b) σ^2 are known.

(2 x 4 = 8 weightage

1M2M18110 (Pages : 3) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2018

MSTA2B07 – Probability Theory

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage

Part A

(Answer **all** questions. Each question carries **1 weightage.**)

1) Describe the axiomatic approach to probability.

2) Define random variable. Let $(\Omega, \mathcal{A}, P)$ be a probability space and $X : \Omega \rightarrow \mathcal{R}$ be such that $X(\omega) = c$, a constant $\forall\ \omega \in \Omega$. Show that X is a random variable on $(\Omega, \mathcal{A}, P)$.

3) Define pairwise independence and mutual independence of events.

4) Define convergence in law. Give an example.

5) Prove or disprove: "if $X_n \rightarrow^L c$, where c is a constant then $X_n \rightarrow^p c$".

6) Describe WLLN.

7) Define characteristic function. Is $\phi(t) = 1, \forall\ t \in \mathcal{R}$, a characteristic function?

8) Prove that characteristic function of a random variable always exists.

9) What is moment problem? Explain.

10) Define conditional expectation.

11) What is a martingale? Give an example.

12) What is stopping time?

Part B

(Answer **any eight** questions. Each question carries **2 weightage.**)

13) Define independence of random variables. If X and Y are independent random variables and g, a continuous real valued function. Prove that $g(X)$ and $g(Y)$ are independent.

14) State and prove Borel-Cantelli lemma.

15) Define conditional probability measure. Prove that conditional probability measure satisfies all the axioms of probability.

16) If $X_n \to^p X$, prove that there exist a subsequence of $\{X_n\}$ which converges to X almost sure.

17) Prove that for a $\{X_n\}$ of random variables WLLN holds if and only if the following three conditions hold:

(i) $\lim_{n\to\infty} \sum_{k=1}^{n} P(|X_k| > n) = \lim_{n\to\infty} \sum_{k=1}^{n} P(X_k \neq X_k^n) \to 0$

(ii) $\frac{1}{n} \sum_{k=1}^{n} E(X_k^n) \to 0$

(iii) $\frac{1}{n^2} \sum_{k=1}^{n} V(X_k^n) \to 0$,

where X_k^n is X_k truncated at n.

18) Let $\{X_n\}$ be a sequence of random variables with uniformly bounded variance, every pair of which is negatively correlated. Prove that $\{X_n\}$ obeys WLLN.

19) If X is a random variable with pdf

$$f(x) = \frac{1 - \cos x}{\pi x^2}, \quad -\infty < x < \infty,$$

what is the characteristic function of X?

20) For any characteristic function ϕ, prove that

(i) $Re(1 - \phi(u)) \geq \frac{1}{4} Re(1 - \phi(2u))$

(ii) $|\phi(u) - \phi(u+h)|^2 \leq 2(1 - Re\phi(h))$

21) State and prove Liapunov's CLT.

22) Let X be a random variable on $(\Omega, \mathcal{F}, P)$ such that E(X) exists, let $\mathcal{D} \subset \mathcal{F}$ be a sigma field. Let Y be another random variable on $(\Omega, \mathcal{F}, P)$ such that E(Y) exists. Let $a, b \in \mathcal{R}$, prove that $E(aX + bY|\mathcal{D}) = aE(X|\mathcal{D}) + bE(Y|\mathcal{D})$

$\Omega = (0, 1)$, $\mathcal{B}$, the sigma field of subsets of (0, 1) and λ, the Lebesgue measure on

Let $Y : \Omega \to \mathcal{R}$ such that,

$$Y(\omega) = \begin{cases} 2, & \text{if } 0 < \omega < 1/3; \\ 5, & \text{if } 1/3 \leq \omega < 1. \end{cases}$$

$\mathcal{D} = \sigma(Y)$, the sigma field induced by the random variable Y. If $X(\omega) = \omega$ $\in (0, 1)$ find $E(X|\mathcal{D})$.

$\{X_n, D_n : n \geq 1\}$ be a submartingale. Prove that X_n has a decomposition $= X'_n + X''_n$ a.s, where $\{X'_n, D'_n\}$ be a martingale, $\{X_n''\}$ is a non-decreasing uence of a.s non-negative random variables such that X_n'' is D_{n-1} measurable, ≥ 2.

Part C

(Answer **any two** questions. Each question carries **4 weightage**.)

te and prove Jordan-decomposition theorem on distribution functions.

fine convergence in probability. Let $X_n \to^P X$ and $Y_n \to^P Y$, prove that

i) $aX_n \to^P aX,\ a \in \mathcal{R}$

i) $X_n + Y_n \to^P X + Y$

i) $X_nY_n \to^P XY$.

t F be a distribution function with characteristic function ϕ. If a, b $(a < b)$ are ints of continuity of F, prove that

$$F(b) - F(a) = \lim_{u \to \infty} \frac{1}{2\pi} \int_{-u}^{u} \frac{(e^{-iua} - e^{-iub})}{iu} \phi(u)du$$

t $\{S_n\}$ be a martingale with $E(S_n^2) < c < \infty,\ \forall\, n \geq 1$. Then prove that there exist random variable S such that S_n converges almost surely and in mean square to S. oreover $E(S_n) = E(S),\ \forall\ n$.

*****

55

1M2M18111 (Pages : 2) Reg. No:......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2018

MSTA2B08– Regression Methods

(2017 Admission onwards)

Max. Time: 3 hours Max. Weight:

PART A

Answer all questions .Weightage 1 for each question

1. When a parametric function is estimable. Give an example for a non-estimable parametric function.
2. Define R^2 and adjusted R^2Statistics.
3. What is generalized linear model? Explain.
4. What is the need of normal probability plot.
5. Define auto correlation.
6. State the properties of least square estimates.
7. Define a multiple linear regression model. State basic assumptions.
8. Explain variable selection problem.
9. Describe orthogonal polynomials. What are its uses?
10. What do you mean by nonparametric regression.
11. Explain the concept of smoothing.
12. What is meant by multicollinearity?

(12 x 1 = 12 weightag

Part B

Answer any 8 Questions. Weightage 2 for each question.

13. Show that least square estimator of the slop parameter of the two variable regression model is unbiased.
14. Describe analysis of variance in multiple regression model.
15. Explain logistic regression. Discuss the method of estimating the parameters of this model.
16. Define simple linear regression. Obtain least square estimator of the parameter of this model along with its standard error.
17. Develop confidence interval for the parameter σ^2 for a simple linear regression model
18. In the linear regression model propose an unbiased estimator of error variance σ^2. Prove your claim.
19. Describe a test for testing auto correlation.
20. Discuss various variance stabilizing transformations.
21. In a linear set up $(Y, A\beta, \sigma^2)$ derive necessary and sufficient condition for a parametric function $l'\beta$ to be estimable.
22. Describe how do you use residuals for checking the adequacy of the model fitted.
23. Explain link function and linear predictors with respect to GLM
24. Describe the residual analysis in GLM

(8 x 2 = 16 weightage)

Part C

Answer any two questions. Weightage 4 for each question.

25. State and prove Gauss-Markov theorem.
26. Explain the problem of heteroscedasticity and indicate any one method of detecting it. Suggest some remedial measures.
27. Explain how orthogonal polynomials are used for fitting a polynomial of appropriate degree.
28. Briefly outline any non parametric regression procedure.

(2 x 4 = 8 weightag

1M2M18112 (Pages : 2) Reg. No:......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Statistics Degree Examination, March 2018

MSTA2B09– Stochastic Process

(2017 Admission onwards)

Max. Time: 3 hours Max. Weighta

PART-A

Answer all questions. Weightage 1 for each question.

1. How will you classify a stochastic process?
2. Define stochastic process with (i) independent of increments (ii) stationary increments.
3. What is counting process? Give an example.
4. What is non homogenous Poisson Process?
5. Describe hidden Markov chain.
6. Write down the postulates of a pure-birth process.
7. Briefly describe inspection paradox.
8. What is a renewal reward process?
9. What is semi-Markov process?
10. What are multi server queues?
11. What is hitting time?
12. Define a Gaussian process.

(12x1=12 Weighta

PART – B

Answer any 8 questions. Weightage 2 for each question.

13. Describe random walk model. Write the transition probability matrix of one-dimensional random walk.
14. Define recurrent and transient states. Give examples.
15. The transition probability matrix of a Markov chain with three states 0, 1, 2 is given below. Find the period of states 0, 1 and 2.

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \end{bmatrix} \end{array}$$

16. Describe limit theorems on transition probabilities.
17. Show that any Markov chain is completely described by one-step transition probabilities and the initial distribution.
18. Describe birth and death models.
19. Describe Regenerative processes.
20. Explain Yule-Furry process.
21. Describe a linear growth process.
22. Describe Brownian motion.
23. Obtain Pollaczek-Khinchin mean formula.
24. What are network of queues? Illustrate with a suitable example.

(8x2=16 Weightage)

PART – C

Answer any 2 questions. Weightage 4 for each question.

25. What is branching process? State and prove fundamental theorem of branching process.
26. What is Poisson Process? Obtain the distribution of inter-arrival time of Poisson process Explain how Poisson process and binomial distribution are related.
27. State and prove elementary renewal theorem.
28. Describe the $G/M/1$ queueing model.

(2x4=8 Weightag