27

2M20113 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester MSc Degree Examination, March/April 2020

MST2C09 – Testing of Statistical Hypothesis

(2019 Admission onwards)

me : 3 hours Max. Weightage : 30

PART A (Short Answer type)

(Answer any 4 questions. Weightage 2 for each question)

1. a) Define unbiased test.

   b) Distinguish between randomized and non-randomized test function.
2. Explain locally most powerful test and locally most powerful unbiased test.
3. Explain Bayesian testing of hypothesis.
4. If T is boundedly complete sufficient statistic for $\theta \in \Theta$, then show that every similar test would have Neyman structure.
5. Explain Median test.
6. Obtain the expression for OC function.
7. Show that SPRT terminate with probability one.

(4 x2 = 8 weightage)

PART B (Short Essay type questions)

(Answer any 4 questions. Weightage 3 for each question)

8. A sample of size 1 is taken from a population distribution P(λ). To test $H_0: \lambda = 1$ against $H_1: \lambda = 2$, consider the nonrandomized test $\varphi(x) = 1 \; if \; x > 3, and = 0, if \; x \leq 3$. Find the probabilities of type I and type II errors and the power of the test.
9. State and prove Neyman – Pearson lemma
10. Let $T(x)$ be maximal invariant. Then show that φ is invariant if and only if φ is a function of T.
11. Compare the Chi square test of goodness of fit with Kolmogorov Smirnov test.
12. Explain Kendall's correlation and give the properties.
13. Show that for a SPRT with stopping bounds A and B (A>B) and strengths(α,β), $A \leq \frac{1-\beta}{\alpha}$ and $B \geq \frac{\beta}{1-\alpha}$.
14. Define ASN function. Let $X \sim P(\lambda)$, Consider $H_0: \lambda = \lambda_0$ against $H_1: \lambda = \lambda_1 (\lambda > 0)$. Derive SPRT and find ASN of the test.

(4 x 3 = 12 weightage)

PART C (Long Essay type questions)

(Answer any 2 questions. Weightage 5 for each question)

15. (a) Prove that one parameter exponential family has MLR property.

(b) Let $X_1, X_2, \ldots, X_n$ be a sample of size n from $U(0,\theta), \theta > 0$. Find the UMP test of size α for testing $H_0: \theta \leq \theta_0$ against $H_1: \theta > \theta_0$.

16. Let a random sample of size n drawn from a normal population with mean μ and variance σ^2 and are unknown. Obtain likelihood ratio test of $H_0: \sigma^2 = {\sigma_0}^2$ against $H_1: \sigma^2 \neq {\sigma_0}^2$.

17. Describe the sign test and Wilcoxen signed rank test for the median of a population. Show how these can be adapted to the case of paired samples.

18. (a) Develop the SPRT for testing $H_0: p = p_0 \; vs \; H_1: p = p_1 (p_1 > p_0)$ for observations taking from Binomial Distribution.

(b) State and prove Wald's fundamental identity.

(2 x 5 = 10 weightage)

28

1M2M20112 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2020

MST2C08 – Sampling Theory

(2019 Admission onwards)

Time : 3 hours Max. Weightage : 30

PART A

Short Answer Type Questions

(Answer any four questions. Weightage 2 for each question)

1. Derive an unbiased estimator for the population variance in SRSWR.
2. What is sampling error? (b) Explain unrestricted random sampling.
3. (a)Explain about Random number table.(b)Explain Narain's scheme of sample selection.
4. What are the principles of Sampling? Explain
5. (a)What you mean by General selection procedure in PPS sampling?(b)Prove that in ratio estimation $B(\hat{R}) = -Cov(\hat{R}, \overline{x})/\overline{X}$
6. (a)Explain Stratified sampling.(b)Explain Systematic sampling.
7. (a)Describe Lahiri's method of selection under PPS Sampling.(b)What are the principles of stratification.

(4x2=8 weightage)

PART B

Short Essay Type / Problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Cluster sampling will be efficient only when the variation between clusters is as small as possible; Prove.
9. (a)Explain Durbin's Π PS sampling.(b)What are the advantages of stratified sampling?
10. Show that in SRSWOR Sample mean $\overline{y}$ is the BLUE of $\overline{Y}$
11. What are non sampling errors? Explain its sources.
12. Write about sampling frame. Explain various defects associated with it.
13. Prove that in SRS the bias of regression estimator $\overline{ylr}$is approximately – Cov $(\overline{x}, b)$.
14. If the population consists of liner trend, then prove that

$$Var\ \overline{(Yst)} \leq Var\ \overline{(Ysys)} \leq Var\ \overline{(Yran)}.$$

(4x3=12 weightage)

PART C
Long Essay Type Questions
(Answer any two questions. Weightage 5 for each question)

15. (a) Derive Yates-Grundy form of estimated variance of Hurvitz-Thomson estimator of population mean upon a PPS sample without replacement.

    (b)For an SRSWOR with population size N and sample size n, show that the probability of a specified unit being selected at any given draw is 1/N.

16. (a) Explain Periodic and Linear trends in Systematic Sampling.

    (b) Prove that in PPS sampling without replacement, Desraj ordered estimator is unbiased for population total. Derive its sampling variance.

17. (a)Prove that $V(ran) \geq V(pro) \geq V(opt)$ (b) Explain Hartly-Ross estimator also obtain an unbiased estimator for population total.

18. (a)Carry out a comparison between the mean per unit and ratio estimator with regression estimator. (b)Obtain the mean and its variance in equal cluster sampling. (c) Differentiate between Multistage and multi phase sampling.

(2x5=10 weightage)

29

1M2M20111 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2020

MST2C07 – Estimation Theory

(2019 Admission onwards)

Time : 3 hours Max. Weightage : 30

PART A(Short Answer type)

(Answer any 4 questions. Weightage 2 for each question)

1. Define complete statistic. If T is complete, show that any one to one function of T is also complete
2. Explain how Bayesian approach is different from the classical methodology in the theory of estimation
3. State Cramer-Huzurbazar theorem
4. a) Define BLUE. Give an example of it

   b) Explain Pitman family of distributions. Give an example of a distribution which is a member of this family
5. Find C-R lower bound of the variance of an unbiased estimator of $P(X = 0)$ if X is a Poisson variate with parameter λ
6. Define (i) Confidence region of level $(1-\alpha)$ (ii) Shortest length confidence interval (iii) Unbiased confidence interval
7. Let $X_i, i = 1,2,\dots,n$ be a random sample from a population with probability mass function $P_N(k) = \begin{cases} \frac{1}{N}; k = 1,2,\dots,N \\ 0; otherwise \end{cases}$. Find the MLE of N.

(4 x 2 = 8 weightage)

PART B(Short Essay type questions)

(Answer any 4questions. Weightage3for each question)

8. State and prove factorization criteria on sufficiency
9. Explain the method of moment estimation. Prove or disprove moment estimators are always unbiased
10. a). Distinguish between Bayesian and Fiducial interval.

    b). Let $X_i, i = 1,2,\dots,n$ be a random sample of size drawn from a Poisson distribution with parameterλ. Find $100(1-\alpha)\%$ Bayesian confidence interval for λ with the assumption of prior distribution of λ is $G(\alpha,\beta)$.

11. State and prove Rao-Blackwell theorem

12. Find the moment estimators of the parameters m and p of a Gamma distribution$G(m, p)$

13. Explain the method of ML estimation. Give an example to show that MLE's need not be unique

14. State Basu's theorem. Use Basu's theorem to show that sample mean and sample variance are independently distributed, when sampling from $N(\mu, \sigma^2)$

(4 x 3 = 12 weightage)

PART C (Long Essay type questions)

(Answer any 2 questions. Weightage 5 for each question)

15. Let $X_i, i = 1,2, \ldots, n$ be random sample from $U(0, \theta)$. Find the shortest confidence interval for θ based on $Y = X_{(n)}$, where $X_{(n)} = Max(X_1, X_2, \ldots, X_n)$

16. (a) State and prove Cramer-Rao inequality

    (b) Define UMVUE. Find the UMVUE of $P(X \leq u)$ where $X \sim N(\theta, 1)$

17. Let $X \sim B(n, p)$ and the prior distribution of $p$is $U(0,1)$. Find the Baye's estimator of p under squared error loss function

18. (a) Find the moment estimator of β based on a random sample of size n from Gamma distribution with pdf $F(x, \beta) = \frac{1}{\beta} e^{-x} x^{\beta-1};\ \ x > 0, \beta > 0$

    (b) Define CAN estimator. Let $X \sim P(\lambda)$, find the CAN estimator of $e^{-\lambda}$.

(2 x 5 = 10 weightage)

30

1M2M20110 (Pages : 2) Reg. No.............................

Name.............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester M.Sc Degree Examination, March/April 2020

MST2C06 – Design & Analysis of Experiments

(2019 Admission onwards)

Time : 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any four questions Weightage 2 for each question)

1. Explain the purpose of randomization replication and local control in design of experiments.
2. Differentiate ANOVA and ANCOVA.
3. Define Complete block design (CBD), Incomplete block design (IBD) and Balanced incomplete block design (BIBD).
4. Describe lattice design. Give an example.
5. Describe fractional factorial design.
6. Define split plot and strip plot design.
7. Define Orthogonality and Rotatability.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question

8. How the missing of an observation affect the CRD. Explain the method of estimating a missing value in a RBD.
9. ExplainModel adequacy checking. Explain the method of checking the Normality assumption of a linear model.
10. Establish following parameter relations of a BIBD with parameters, No of treatment v,No of blocks b, block size k,replication r and two treatments together in λ blocks. (i) $bk = vr$, (ii) $b \geq v$ (iii) $\lambda(v-1) = r(k-1)$ and $\lambda < r$

11. Define PBIBD and derive the analysis of PBIBD with two associate classes.

12. What is a 2^n factorial experiment? Describe the effect components of a 2^3 design into seve[r]
mutually orthogonal contrasts.

13. (i) Compare total and partial confounding. (ii) Develop the layout of a 2^4 factorial wit[h]
highest order interaction confounded (iii) Develop the layout of a 2^4 factorial with [4]
replications in which All three factor interactions are partially confounded

14. Describe the method of steepest ascent.

(4 x 3= 12 weightage)

Part C
Long Essay Type questions
(Answer any two questions. Weightage 5 for each question)

15. (i) Explain different models in design of experiments

(ii) Comment on the statement completely randomized design is preferred over other designs.

(iii) Develop ANCOVA for one way classified data with one concomitant variable

16. (i) Describe the construction of BIBD using orthogonal latin squares.

(ii) Develop intra block analysis of a BIBD

(iii)Discuss the efficiency of BIBD relative to RBD for estimating the difference of any two treatment effects.

17. (i) Outline the analysis of a split plot design with r blocks, α main plot treatments and β subplot treatments.

(ii)Define 3^3 Design with an example

(iii) Derive the expression for linear and quadratic effect of a 3^2experiment with factors A and B

18. Write short notes on the following

(i) Analysis with unequal number of observations per cell

(ii) Response surface design.

(iii) Inter and intra block analysis

(iv) Resolution of a design

(2 x 5= 10 weightage)