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2M1N20074 (Pages : 2) Reg. No:.............................

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**First Semester M.Sc Mathematics Degree Examination, November 2020**

**MMT1C01– Algebra – I**

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

## Part A

**(Answer all questions each question carries 1 weightage)**

1. Define an isometry of the Euclidean plane $R^2$. Give an example of an isometry.
2. Find the order of (8, 10) in the group $Z_{12}\ X\ Z_{18}$.
3. Find all proper nontrivial subgroups of $Z_2\ X\ Z_2$.
4. A Sylow 3-subgroup of a group of order 12 has order________.
5. Find the reduced form and the inverse of the reduced form of $a^2a^{-3}b^3a^4c^4c^2a^{-1}$.
6. What is group presentation?
7. Determine whether the polynomial $4x^{10} - 9x^3 + 24x - 18$ is irreducible over Q.
8. Let G = {e, a, b} be a cyclic group of order 3 with identity element e.
   Write the element (2e + 3a + 0b) + (4e + 2a + 3b) in the group algebra $Z_5(G)$
   in the form re + sa + tb for r, s, t $\epsilon$ $Z_5$.

**(8 x 1 = 8 weightage)**

## Part B

**Answer any two from each unit (Each question carries 2 weightage)**

### UNIT I

9. Find all abelian groups, up to isomorphism of order 720.
10. Prove that factor group of a cyclic group is cyclic.
11. Let X be a G – set. Prove that $G_x = \{g \in G/\ gx = x\}$ is a subgroup of G for each x $\epsilon$ X.

## UNIT II

12. Let $\phi : Z_{18} \to Z_{12}$ be the homomorphism such that $\phi(1) = 10$

    a. Find the kernel K of $\phi$.

    b. List the cosets in $Z_{18}/K$, showing the elements in each coset.

13. Give the isomorphic refinements of the two series:

    $\{0\} < 10Z < Z$ and $\{0\} < 25Z < Z$

14. For a prime number p, Prove that every group G of order $p^2$ is abelian.

## UNIT III

15. Consider the evaluation homomorphism $\phi_2 : Z_7[x] \to Z_7[x]$.

    Compute $\phi_2[(x^3 + 2)(4x^2 + 3)(x^7 + 3x^2 + 1)]$.

16. Let G be a finite group of the multiplicative group $(F^*, .)$. Prove that G is cyclic.

17. Show that an intersection of ideals of a ring R is again an ideal of R.

**(6 x 2 =12 weightage )**

## Part C

**Answer any two (Each question carries 5 weightage)**

18. a. If m is a square free integer, then prove that every abelian group of order m is cyclic.

    b. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.

19. a. State and Prove First Sylow Theorem.

    b. Prove that every group of prime power order is solvable.

20. a. State and Prove Second Isomorphism Theorem.

    b. State and Prove Eisenstein Criterion.

21. Show that every non-constant polynomial $f(x) \in F[x]$ can be factored into a product of irreducible polynomials in F[x] in unique way, where F is a field.

**(2 x 5 =10 weightage)**

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**First Semester M.Sc Mathematics Degree Examination, November 2020**

**MMT1C02– Linear Algebra**

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

**Part- A**

**Answer *all* questions. Each question has *one* weightage.**

1. Is the set $\{(1,0,-1),(2,-1,2),(3,-2,1)\}$ forms a basis for $\Re^3$ ?
2. Is the union of subspaces of a vector space, a subspace? Justify.
3. Describe explicitly a linear transformation $T: R^3 \to R^3$ whose range is the subspace spanned by $(1,0,-1)$ and $(1,2,2)$.
4. Find linear operators T and U on $R^2$ such that $TU = 0$, but $UT \neq 0$.
5. What do you mean by the dual space of a vector space V? If V is finite dimensional, what you can say about the dimension of the dual space of V?
6. Define the transpose of a linear transformation.
7. Show that similar matrices have the same characteristic polynomial.
8. Show that an orthogonal set of non-zero vectors in an inner product space is linearly independent.

**(8 × 1 = 8 Weightage)**

**Part- B**

**Answer any *two* from each unit. Each question has *two* weightage**

**Unit - I**

9. If $V$ is any n-dimensional vector space, then show that no subset of $V$ that contains less than n vectors can span $V$.

10. Show that $\Re$ over Q is not finite dimensional.

11. If $T: V \to V$ a linear operator, then prove that

$Range(T) \cap Nullspace(T) = \{0\} \Leftrightarrow$ If $T(T(\alpha)) = 0$, then $T(\alpha) = 0$

## Unit -II

12. If $B = \{\alpha_1, \alpha_2, \alpha_3\}$ is the basis for $C^3$ given by $\alpha_1 = (1,0,-1), \alpha_2 = (1,1,1), \alpha_3 = (2,2,0)$.

Find the dual basis of B.

13. Let $T$ be the linear operator on $R^3$, the matrix of which in the standard ordered basis is

$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$. Find the basis for the range of $T$ and a basis for the null space of $T$.

14. Let $T$ be a linear operator on an n-dimensional vector space $V$. Show that the characteristic and minimal polynomials for $T$ have the same roots except for multiplicities.

## Unit – III

15. Define a projection of $V$ and give one example. Prove that if $R$ and $N$ are subspaces of $V$ such that $V = R \oplus N$, there is only one projection operator $E$ on $V$ which has range $R$ and null space $N$.

16. Define inner product. Describe explicitly all inner products on $\Re^1$ and on $C^1$

17. Explain the Gram- Schmidt orthogonalization process.

**(6× 2 = 12 Weightage)**

## Part- C

**Answer any *two* from the following four questions. Each question has *Five* weightage.**

18. (a) Let $W_1$ and $W_2$ be the subspace of $\Re^5$ spanned by $\{(1,0,1,0,1),(1,1,1,0,0)\}$ and $\{(1,0,1,1,1),(1,0,1,0,0),(1,1,1,0,1)\}$ respectively. Find $W_1 \cap W_2$. Also find a basis for $W_1 \cap W_2$.

(b) Define a linear transformation. If $V$ is an $n-$ dimensional vector space and $W$ be an $m$ dimensional vector space over the field $F$, show that $L(V, W)$ is finite dimensional and has dimension $mn$.

19. (a) Let $V$ be an $n$-dimensional vector space over $F$ and let $W$ be an $m$-dimensional vector space over $F$. Show that there is a one to one correspondence between the set of all linear transformations from $V$ into $W$ and the set of all $m \times n$ matrices over the field $F$.

(b) If $V$ is a finite dimensional vector space, then show that $V$ and its double dual $V^{**}$ are isomorphic.

20. (a) Let $T$ be a linear operator on the $n$-dimensional vector space $V$, and suppose that $T$ has $n$ distinct characteristic values. Prove that $T$ is diagonalizable.

(b) Let $V$ be a finite dimensional vector space over the field $F$ and let $T$ be a linear operator on $V$. Then, show that $T$ is triangulable if and only if the minimal polynomial for $T$ is a product of linear polynomials over $F$.

21. (a) For any $\alpha \in \Re^2$, with standard inner product, show that $\alpha = (\alpha \mid e_1)e_1 + (\alpha \mid e_2)e_2$, where $\{e_1, e_2\}$ is the standard basis for $\Re^2$.

(b) Let $W$ be a subspace of an inner product space $V$ and let $\beta$ be a vector in $V$. Show that a vector $\alpha$ in $W$ is a best approximation to $\beta$ by vectors in $W$ if and only if $\beta - \alpha$ is orthogonal to every vector in W.

(c) Define the orthogonal complement of any set $S$ in an inner product space $V$. Show that it is always a subspace of $V$.

**(2 × 5 = 10 Weightage)**

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# FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**First Semester M.Sc Mathematics Degree Examination, November 2020**

**MMT1C03 – Real Analysis – I**

(2020 Admission onwards)

Time: 3 hours Max. weightage : 30

Part A: Answer all questions (Each question carries 1 weightage)

1. Give an example of a set which is not perfect. Justify.
2. Is the set of all irrational real numbers countable ? Justify.
3. Let $E^0$ denote the set of all interior points of a set $E$. Then prove that $E$ is open if and only if $E^0 = E$.
4. Suppose $f$ is a real function defined on $\mathbb{R}^1$ which satisfies $\lim_{h\to 0}[f(x+h)-f(x-h)] = 0$. Does this imply that $f$ is continuous? Justify.
5. Let $f$ be defined for all real $x$, and suppose that $|f(x) - f(y)| \leq (x-y)^2$ for all real $x$ and $y$. Prove that $f$ is constant.
6. Suppose $\alpha$ increases on $[a, b]$, $a \leq x_0 \leq b$, $\alpha$ is continuous at $x_0$, $f(x_0) = 1$, and $f(x) = 0$ if $x \neq x_0$. Prove that $f \in \mathcal{R}(\alpha)$ and that $\int f d\alpha = 0$.
7. Suppose $f$ is a bounded real function on $[a, b]$ and $f^2 \in \mathcal{R}$ on $[a, b]$ . Does it follow that $f \in \mathcal{R}$ on $[a, b]$? justify
8. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(8 ×1= 8 )

Part B- Answer any two from each unit ( Each question carries 2 weightage)

Unit I

9. Show that closed subsets of compact sets are compact.
10. Suppose $f$ is a continuous mapping of a compact metric space $X$ into a metric space $Y$. Then prove that $f(X)$ is compact.
11. Let $f$ be a monotonic on $(a, b)$. Then prove that the set of points of $(a, b)$ at which $f$ is discontinuous is at most countable.

Unit II

12. Suppose $f$ is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a, b]$, $g$ is defined on an interval $I$ which contains the range of $f$ , and $g$ is differentiable at the point $f(x)$. If $h(t) = g(f(t))$ $(a \leq t \leq b)$ then show that $h$ is differentiable at $x$ and $h'(x) = g'(f(x))f'(x)$.

13. Suppose $f$ is continuous mapping of $[a,b]$ into $\mathbb{R}^k$ and $f$ is differentiable in $(a,b)$ Then prove that there exists $x \in (a,b)$ such that $|f(b) - f(a)| \leq (b-a)|f'(x)|$.

14. If $f$ is monotonic on $[a,b]$, and if $\alpha$ is continuous on $[a,b]$ prove that $f \in \mathcal{R}(\alpha)$ on $[a,b]$

Unit III

15. Show that a sequence $\{f_n\}$ converges to $f$ with respect to the metric of $C(X)$ if and only if $f_n \to f$ uniformly on $X$.

16. Even if $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set $E$, prove that there need not exist a subsequence which converges pointwise on $E$.

17. If $K$ is a compact metric space, if $f_n \in \mathcal{C}(K)$ for $n = 1, 2, \cdots$ and if $\{f_n\}$ converges uniformly on $K$, then prove that $\{f_n\}$ is equicontinuous on $K$.

(6 ×2= 12 )

Part C - Answer any two ( Each question carries 5 weightage)

18. (a) Show that every $k$-cell is compact.
    (b) Prove that there exist real numbers which are not algebraic.

19. (a) Show that mean value theorem fails for complex valued functions.
    (b) Suppose $f \in \mathcal{R}(\alpha)$ on $[a,b], m \leq f \leq M, \phi$ is continuous on $[m,M]$, and $h(x) = \phi(f(x))$ on $[a,b]$. Then prove that $h \in \mathcal{R}(\alpha)$ on $[a,b]$.

20. (a) Let $f \in \mathcal{R}$ on $[a,b]$. For $a \leq x \leq b$ , put $F(x) = \int_a^x f(t)dt$. Then show that $F$ is continuous on $[a,b]$. Furthermore, if $f$ is continuous at a point $x_0$ of $[a,b]$, then prove that $F$ is differentiable at $x_0$, and $F'(x_0) = f(x_0)$.
    (b) Suppose $F$ and $G$ are differentiable functions on $[a,b], F' = f \in \mathcal{R}$, and $G' = g \in \mathcal{R}$. Then prove that $\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$.

21. (a) Suppose $f_n \to f$ uniformly on a set $E$ in a metric space. Let $x$ be a limit point of $E$, and suppose that $\lim_{t\to x} f_n(t) = A_n$ $(n = 1, 2, \cdots)$. Then prove that $\{A_n\}$ converges and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.
    (b) Prove or disprove that every member of an equicontinuous family is uniformly continuous.

(2 ×5 = 10 )

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**First Semester M.Sc Mathematics Degree Examination, November 2020**

**MMT1C04– Discrete Mathematics**

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

**Part A (Short Answer Questions) (1-8)**
**Answer all questions.**
**Each question carries 1 weightage**

1. Let ( X, + , . , ' ) be a Boolean algebra. Prove that x +x.y = x , for all x, y∈ X.
2. State Stone representation theorem for finite Boolean algebra .
3. Prepare table values of the function $f(x_1,x_2,x_3) = x_1x_2 + x_2'x_3$.
4. Let G be a simple graph . Prove that $\sum_{v\in V(G)} d(v) = 2e(G)$.
5. Prove that a simple graph G with n vertices, n≥2, is complete if and only if $\kappa(G) = n-1$.
6. Prove that for any simple graph G, $\delta(G) \le 5$.
7. Find a grammar that generates the language $L = \{a^{nbn+1} : n \ge 0\}$ on $\Sigma = \{a, b\}$.
8. Find a dfa that accepts all strings on {0,1} , starting with prefix 01.

**( 8 x 1 = 8 weightage)**

**Part B**

**Answer any two questions from each unit. Each question has 2 weightage.**

**UNIT – I**

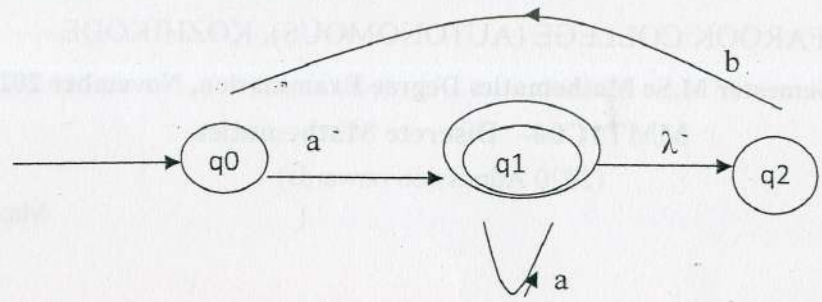
9. Let X = RU{*} where * is some point not on the real line. Define ≤ on X as {(x,y) ∈ R×R: x≤ y in the usual order }U{(*,*)}. Prove that ≤ is a partial order on X.
10. Let (X,+,.,') be a finite Boolean algebra. Prove that every element of X can be uniquely expressed as sum of atoms.
11. Write the following Boolean function in the disjunctive normal form $F(x_1, x_2, x_3) = (x_1+x_2+x_3)(x_1'+x_2+x_3')(x_1+x_2'+x_3')(x_1'+x_2'+x_3')(x_1+x_2+x_3')$.

**UNIT – II**

12. State and prove Whitneys theorem on 2-connected graphs.
13. Prove that graph G is planar if and only if , each of its blocks is planar.
14. Prove that an edge is a cut edge if and only if it belongs to no cycle.

## UNIT – III

15. Find a dfa equivalent to the following nfa




16. Are the grammars $G_1 = (\{S\}, \{a,b\}, S, \{ S \to SS, S \to aSb, S \to \lambda, S \to bSa\})$ and $G_2 = (\{S\}, \{a,b\}, S, \{ S \to SS, S \to SSS, S \to aSb, S \to bSa, S \to \lambda \})$ are equivalent.

17. Find a regular expression for the language $L = \{w \in \{0,1\}^*$: w has at least one pair of consecutive zeros$\}$.

**(6 x 2 = 12 weightage)**

## Part C

**Answer any two from the following four questions (18-21)**
**Each question carries 5 weightage.**

18. (a) ) Let (X, +, . ,') be a finite Boolean algebra , Prove that every element of X can be uniquely expressed as sum of atoms.

(b) Prove that the set of all symmetric Boolean functions of n Boolean variables $x_1, x_2 \ldots, x_n$ is a sub algebra of the Boolean algebra of all Boolean functions of these variables . Also prove it is isomorphic to the power set Boolean algebra of the set $\{0,1,\ldots,n\}$.

19. (a) State and prove a necessary and sufficient condition for a graph to be bipartite

(b) Prove that every connected graph contains a spanning tree.

20. (a) Derive the Euler's formula for a connected plane graph.

(b) Prove that $K_5$ is nonplanar .

21. (a) Show that the language $L=\{awa: w \in \{a, b\}^*\}$ is regular.

(b) Let L be the language accepted by a non deterministic finite accepter $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$. Then prove that there exist a dfa $M_D=(Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L=L(M_D)$.

**( 2 x 5 = 10 weightage)**

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

**First Semester M.Sc Mathematics Degree Examination, November 2020**

**MMT1C05– Number Theory**

(2020 Admission onwards)

Time: 3 hours Max. Weightage : 30

**Part A**

**(Answer all questions. Each question carries 1 weightage.)**

1. If $f$ is a multiplicative function, then prove that $f(1) = 1$
2. Show that $\forall\, n \geq 1, \log n = \sum_{\frac{d}{n}} \Lambda(d)$.
3. Prove that $[2x] - 2[x]$ is either 0 or 1.
4. For $x > 0$, prove that $\psi(x) = \sum_{m \leq log_2 x} \sum_{p \leq x^{1/m}} \log p$.
5. Prove that for $x \geq 2$, $\pi(x) = \frac{\vartheta(x)}{\log x} + \int_2^x \frac{\vartheta(t)}{t log^2 t}\, dt$.
6. Prove that $\sum_{r=1}^{p-1} (r|p) = 0$.
7. Show that if $p$ is an odd positive integer then $(-1|p) = (-1)^{\frac{p-1}{2}}$.
8. In the 27- letter alphabet system (with blank = 26), use the affine enciphering transformation with key $a = 13, b = 9$ to encipher the message 'CANCEL LAST ORDER'.

**(8 ×1= 8 weightage)**

**Part B**

**(Answer any two questions from each unit. Each question carries 2 weightage.)**

**UNIT 1**

9. Prove that $\forall\, n \geq 1,\ \sum_{d/n} \varphi(d) = n$
10. If both $g$ and $f * g$ are multiplicative functions, then prove that $f$ is multiplicative.
11. Prove that $\forall\, x \geq 1, \left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$

**UNIT II**

12. State and prove Abel's identity.

13. For $n \geq 1$, the $n^{th}$ prime $P_n$ satisfies the inequality

$\frac{1}{6} n \log n < P_n < 12\left(n \log n + n . \log \frac{12}{e}\right)$.

14. Prove that $\sum_{p \leq x} \frac{1}{p} = \log\log x + A + O\left(\frac{1}{\log x}\right), \forall\, x \geq 2$.

**UNIT III**

15. Prove that the Diophantine equation $y^2 = x^3 + k$ has no solution if $k$ has the form $k = (4n-1)^3 - 4m^2$ where $m$ and $n$ are integers such that no prime $p \equiv -1 (mod\ 4)$ divides $m$.

16. Solve the system of simultaneous congruence $\begin{matrix} 2x + 3y \equiv 1 (mod\ 26) \\ 7x + 8y \equiv 2 (mod\ 26) \end{matrix}$.

17. $(a)$ Explain Hash function.

$(b)$ How do we send a signature in RSA cryptosystem.

**(6 × 2= 12 weightage)**

**Part C**

**(Answer any two questions. Each question carries 5 weightage.)**

18. (a) State and prove Euler summation formula

(b) Prove that $\sum_{n \leq x} \frac{\log n}{n} = \frac{1}{2} log^2 x + A + O\left(\frac{log x}{x}\right)$, where A is a constant.

19. Prove that $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$ if and only if $\lim_{x \to \infty} \frac{\vartheta(x)}{x} = 1$

20. State and prove Shapiro's Tauberian theorem.

21. $(a)$ State and Prove Gauss lemma.

$(b)$ Let $m$ be the number defined in Gauss lemma. Show that

$$m \equiv \sum_{t=1}^{\frac{p-1}{2}} \left[\frac{tn}{p}\right] + (n-1)\left(\frac{p^2-1}{8}\right) (mod\ 2)$$

**(2 × 5= 10 weightage)**