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1M1N19103 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2019

MST1C01 – Analytical Tools for Statistics – I

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

PART A

Answer any four(2 weightages each)

1. Define the continuity of multivariable function. Give an example
2. Define local maxima and minima of multivariable function.
3. Find the real and imaginary part of the complex function: $w= z^2+3z$; where $z=3+5i$.
4. State Cauchy's residue theorem.
5. What is an analytic function? Is $f(z) = e^z$ is analytic? Justify
6. Define residues and poles. Give one example.
7. What is the Laplace transform of $f(t) = \cos(at)$

(2 x 4=8 weightages)

PART B

Answer anyfour(3 weightages each)

8. Show that the function $f(x,y) = \begin{cases} \frac{x-y}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$ is continuous at origin.
9. Find Jacobian of transformation of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$
10. Obtain the power series expansion of the function $f(z) = \frac{1}{z+i}$ at the point $z = a$.
11. State and prove Morera's theorem
12. State and prove Cauchy - Riemann conditions
13. Show that, for the function f with first three derivatives,

    $f',\ f'',\ f'''\ L(f''') = s^3 L(f) - sf(0) - sf'(0) - f''(0)$
14. Solve the initial value problem using Laplace transformation

    $y'' - 2y + 3 = t$, $y'(0)=1$, $y(0)=1$

(3x 4=12 weightages)

PART C

Answer any two(5 weightages each)

15. State and prove Taylor's theorem for multivariable function.

16. (a) Varify Cauchy-Riemann equations: $f(z)=z^3+z+1$

(b) $f(z) = u+iv$ is an analytic function with $u=2x(1-y)$. Determine the function completely.

17. State and prove Cauchy's integral theorem.

18. Find Laplace transforms of following functions

(a) $tSin\theta t$

(b) $Cos\pi t . e^{-32t}$

(c) $te^t + cosht$

(5x2=10 weightages)

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1M1N19104 (Pages :2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2019

MST1C02 – Analytical Tools for Statistics – II

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any four questions. Weightage 2 for each question)

1. Distinguish between linear dependence and independence
2. Define Inner product and orthogonality.
3. Explain Inverse of a partitioned matrix
4. Distinguish between image space and kernel of a linear mapping.
5. Show that the eigen values of all symmetric matrices are real.
6. Define algebraic multiplicity and geometric multiplicity
7. State Rank Nullity Theorem.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Define direct sum and compliment of a subspace. Write the necessary and sufficient condition for sum of two subspaces to be direct sum.
9. Intersection of two subspaces is also a subspace.
10. Let T be a linear operator on R^2 represented by the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$. Find the characteristic values of T.
11. Let V be a vector spaces of functions with basis $S = \{e^{3t}, te^{3t}, t^2e^{3t}\}$ and let D: V→V be the differential operator. Find the matrix of linear transformation of D relative to basis S.

12. State and prove Cayley-Hamilton theorem
13. Define G inverse and Moore-Penrose inverse.
14. Explain extreme of quadratic forms and simultaneous diagonalisation of matrices.

(4 x 3= 12 weightage)

Part C

Long Essay Type questions

(Answer any two questions. Weightage 5 for each question)

15. a) Show that the vectors $(1,2,1)$, $(2,1,3)$ and $(1,0,1)$ form a basis of R^3.

b) Let V be a finite dimensional vector space and let $\{v_1, v_2, \ldots, v_n\}$ be any basis. Prove that if a set has more than n vectors, then it is linearly dependent.

c) Compute an orthonormal basis for the basis $S = \{u1, u2, u3\}$ where $u_1 = (1,-2,-1), u_2 = (-1,1,-1)$ and $u_3 = (1,-2,1)$.

16. a) Distinguish between Hermitian and skew Hermitian matrices,

b) Show that the diagonal elements of a Hermitian matrix must necessarily be real.

c) Define Idempotent matrix, Nilpotent matrix

d) Show that every matrix A such that $A^2 = A$ is similar to a diagonal matrix.

17. a) Define Jordan canonical form of a matrix

b) Find the Jordan canonical form for $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

18. a) Show that Moore-Penrose inverse is unique.

b) Find the solution of the homogeneous system

$$x + y - 2z + 2s - t = 0,$$

$$x + y - z + 2s + 2t = 0,$$

$$x + 4y - 7z + s + t = 0,$$

$$x - 2y + 3z - s + 2t = 0.$$

c) Classify the quadratic form, $x^2 + 2y^2 + z^2 + 2xy + 6xz + 2yz$.

(2 x 5= 10 weightage)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2019

MST1C03 – Distribution Theory

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any fourquestions.Weightage 2 for each question)

1. Define Moment generating function. How will you obtain mean and variance from it?
2. Obtain the distribution of the sum of iid geometric random variables.
3. Define power series distribution. Also obtain its moment generating function.
4. Define Pareto distribution. Obtain its geometric mean.
5. Derive the joint distribution of all the order statistics of a random sample of size n from a population with pdf $f(x)=e^{-x}$, $x>0$ and 0 elsewhere.
6. Obtain the pdf of sample range based on a random sample of size n from a uniform distribution over (0,1).
7. If (X,Y) have a joint pdf given by $f(x,y) = 2$, $0<x<y<1$ then find $P(0<X<0.5, 0<Y<0.3)$.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Obtain the p.g.f. of binomial random variable. Deduce the first four factorial moments using the p.g.f. and hence obtain the first four central moments.
9. Derive the p.d.f. of a compound Poisson distribution with a gamma compounding density.

10. Define hypergeometric distribution and give a practical situation where this distribution arises. Derive the expressions of its mean and variance.
11. Define a trinomial distribution and obtain the correlation coefficient between its component random variables. Also obtain the conditional distributions and conditional means.
12. Obtain the density of the difference of two iid exponential random variables.
13. Obtain the conditional distributions of bivariate normal distribution.
14. What is Pearson system of distributions? Obtain the distribution when the roots of the quadratic equation are real and of opposite signs.

(4 x 3= 12 weightage)

Part C

Long Essay Type questions

(Answer any two questions. Weightage 5 for each question)

15. (a) Derive the conditional distribution of $X_{(s)}$ given $X_{(r)}$, s>r, where $X_{(r)}$ and $X_{(s)}$ are the r-th and s-th order statistics.

    (b) Show that min $(X_1, X_2, \ldots, X_n)$ follows exponential distribution if and only if X_i's follow exponentialdistribution.
16. Define noncentral F-distribution and derive its density.
17. Derive the joint distribution of the sample mean and the variance of a random sample taken from a normal population with mean μ and variance σ^2.
18. If $X_1, X_2, \ldots, X_n$ is a random sample from lognormal distribution then obtain the distribution of the geometric mean. Also obtain the mean, median and mode of log-normal distribution.

(2 x 5= 10 weightage)

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Name:..............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Statistics Degree Examination, November 2019

MST1C04 – Probability Theory

(2019 Admission onwards)

Time: 3 hours Max. Weightage : 30

Part A

Short Answer Type questions

(Answer any four questions. Weightage 2 for each question)

1. Define sigma field of subsets of a set. Prove that sigma field is a monotone field.
2. Let $A_n = \begin{cases} [x: 0 \le x \le 1] \text{ if } n \text{ is odd} \\ [x: 1 \le x \le 2] \text{ if } n \text{ is even} \end{cases}$. Find $lim\,Sup\, A_n$ and $lim\,Inf\, A_n$.
3. Give an example of a class of sets closed under finite unions and finite intersections but not under complementation.
4. Show that a characteristic function is uniformly continuous in any finite interval.
5. Prove or disprove the statement "almost sure convergence implies convergence in probability".
6. If $f(X)$ is continuous real valued function and X_n converges to X in probability, then prove that $f(X_n)$ converges to $f(X)$ in probability.
7. Does the Lindeberg condition hold for the sequence X_n of independent random variables with $P\left\{X_n = \pm\frac{1}{2^n}\right\} = \frac{1}{2}$.

(4 x 2= 8 weightage)

Part B

Short Essay Type/ problem solving type questions

(Answer any four questions. Weightage 3 for each question)

8. Let $A_n = \left\{(x,y): 0 \le x \le n, 0 \le y \le \frac{1}{n}\right\}, (x,y) \in \mathbb{R}^2$. Show that A_n is not monotone but $\lim_{n\to\infty} A_n$ exists. Also find $\lim_{n\to\infty} A_n$.
9. If P_1 and P_2 are two probability measures, then show that $P = \alpha P_1 + (1-\alpha)P_2, 0 < \alpha < 1$ is also a probability measure.

10. Show that $\frac{1}{n+1}\sum_{k=0}^{n} e^{ikt/n}$ is a characteristic function of a random variable. To which random variable the sequence of random variables corresponding to the above characteristic functions converge in distribution.

11. Establish Borel 0-1 Law.

12. State and prove Jordan decomposition theorem for distribution functions.

13. Define conjugate pair of distributions. Obtain the conjugate pair of of the pdf,

$$f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty.$$

14. Let $\{X_n\}$ be an independent sequence of random variables such that each X_n only attains the values 0 and 1 with positive probability. Prove that $\{X_n\}$ satisfies the central limit theorem if the series $\sum_{n=1}^{\infty} P(X_n = 0)P(X_n = 1)$ diverges.

(4 x 3= 12 weightage)

Part C

Long Essay Type questions

(Answer any two questions. Weightage 5 for each question)

15. (a)Show that the intersection of an arbitrary number of sigma fields is a sigma field.

(b) Let B be a Borel set in R and $B' = \{a + x: x \in B\}, a \in R$.Show that $B'$is also a Borel set.

16. (a) State and prove inversion theorem on characteristic function.

(b) Find the density function whose characteristic function is $e^{-|t|}$

17. (a) Prove that $E|X+Y|^r \le C_r E|X|^r + C_r E|Y|^r$, where $C_r = \begin{cases} 1, if\ r \le 1, \\ 2^{r-1}, if\ r > 1 \end{cases}$

(b) Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then show that $X_n + Y_n \xrightarrow{P} X+Y$.

18. (a) Let $\{X_n\}$ be a sequence of i.i.d. random variables with characteristic function$\varphi(u)$. Then prove that $S_n/n \xrightarrow{P} a$ if and only if $\varphi'(0) = a$, where$S_n = \sum_{k=1}^{n} X_k$.

(b) State and prove Kolmogorov's three series theorem.

(2 x 5= 10 weightage)