1M1N17328 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2017

MT1C01– Algebra – I

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

Answer all the questions

(Each Question has weightage one)

1 Find the order of (8,4,10) in the group $z_{12} \times z_{60} \times z_{24}$.

2 Compute the factor group $z_4 \times z_6/<(0,1)>$.

3 Define normal and subnormal series of a group G and give an example.

4 Define a free group generated by a set A.

5 Show that every finite p-group is solvable.

6 Find the elements in S_n/A_n .

7 Let $\phi : G \rightarrow G'$ be a group homomorphism. If N is a normal subgroup of G then show that ϕ [N] is a normal subgroup of ϕ [G].

8 Define center and commutator subgroup of a group.

9 Let $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ and $g(x) = x^2 - 2x + 3$ be in $z_5[x]$. Find the quotient and reminder when f(x) is divided by g(x).

10 State Eisenstein Criterion.

11 Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$, and let N be the subring of F consisting of all functions f such that f(2)=0. Is N is an ideal in F ?.

12 Define refinement of a subnormal series. Find a refinement of $\{0\} < 72z < 8z < z$.

13 Find the center of S_3.

14 Find the order and number of Sylow 2-subgroups of S_3.

(14 x 1=14 weightage)

Part B

Answer Any Seven Questions
(Each Question has weightage two)

15 If m divides the order of a finite abelian group G, then show that G has a subgroup of order m.

16 Show that the group $z_m \times z_n$ is cyclic and is isomorphic to z_{mn} if and only if m and n are relatively prime.

17 Let H be a normal subgroup of G. Then show that the cosets of H form a group G/ H under binary operation (aH)(bH)=(ab)H

18 Show that a nonzero polynomial $f(x) \in F[x]$ of degree n can have atmost n zeros in the field.

19 Let G be a group. Then show that set of all commutators $aba^{-1}b^{-1}$ for $a,b \in G$ generates subgroup of G.

20 Let X be a G-set. Then show that G_x is a subgroup of G for each $x \in X$.

21 If N is a normal subgroup of G, and if H is any subgroup of G, then show that $H \vee N = HN = NH$

22 Show that any two composition series of a group G are isomorphic.

23 Show that the center of a finite nontrivial p-group is nontrivial.

24 Show that no group of order 30 is simple.

(7×2=14 weightag

Part C

Answer Any Two Questions
(Each Question has weightage Four)

25 Let X be G-set and let $x \varepsilon X$. Then show that $|Gx|=(G:G_x)$. If $|G|$ is finite, then $|Gx|$ is a divisor of G.

26 Derive the class equation for a group G. Find the conjugate classes of S_3 and verify th class equation.

27 Let p be a prime. Let G be a finite group and let p divide $|G|$. Then show that G has an element of order p.

28 a) Check whether the polynomial $\Phi_p(x) = \frac{x^p-1}{x-1}$ is reducible over $\mathbb{Q}$ for any prime p.

b) If F is a field , then show that every nonconstant polynomial $f(x) \in F[x]$ can be factored in F[x] into a product of irreducible polynomials, the irreducible polynomials being unique except for order and for unit factors in F.

(4×2=8 weighta

1N17329 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2017

MT1C02– Linear Algebra

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part- A

Answer *all* questions.

Each question has *one* weightage.

1. Is the collection of all sequences of complex numbers that converges to a real number, a vector space over C?
2. Show that if v and w are linearly independent vectors in V, then so are $v+w$ and $v-w$.
3. Prove that null space of a linear transformation is a subspace.
4. Find the coordinate matrix of $(1,-4)$ relative to the ordered basis $\{(1,-1),(2,4)\}$ of $\Re^2$.
5. Let $T:\Re^2 \to \Re^3$ defined by $T(x,y)=(x,y+1,x+y)$. Is T linear?
6. Let $T:R^3 \to R^3$ be the linear map defined by $T(x,y,z)=(y,0,z)$. Find the rank of T.
7. What is the kernel of $T:R^3 \to R^3$ defined by $T(x,y,z)=(x+z,x+y+2z,2x+y+3z)$?
8. If A and B are $n\times n$ complex matrices, show that $AB-BA=I$ is impossible.
9. Define annihilator of a subset S of a vector space V.
10. What do you mean by a diagonalizable operator?
11. Prove that if E is a projection on R along N, then $I-E$ is the projection on N along R.
12. Let V be a vector space over C with an inner product $(|)$, show that $\operatorname{Im}(\alpha \mid \beta)=\operatorname{Re}(\alpha \mid i\beta)$.
13. Let $(|)$ be the standard inner product on $\Re^2$. If $\alpha=(1,2)$ and $\beta=(-1,1)$, find γ such that $(\alpha|\gamma)=-1$ and $(\beta|\gamma)=3$.
14. Define orthonormal sets in an inner product space.

(14 x 1 = 14 weightage)

Part- B

Answer any *seven* from the following ten questions.

Each question has *two* weightage

15. Show that a linear transformation maps a linearly dependent set in to a linearly dependent set. Is it true that any linearly independent set is mapped to another linearly independent set under a linear transformation?
16. Show that any vector space of dimension n over F is isomorphic to F^n.
17. Find a basis and the dimension of the linear subspace of $\Re^n$ given by $\{(x_1,x_2,\ldots,x_n)\in\Re^n / x_1+x_2+\ldots+x_n=0\}$.

18. If $T : V \to V$ a linear operator, then prove that

$Range(T) \cap Nullspace(T) = \{0\} \Leftrightarrow$ If $T(T(\alpha)) = 0$, then $T(\alpha) = 0$

19. If W is a subspace of a finite dimensional vector space V, then show that

$dim\ W + dim\ W^0 = dim\ V$

20. If $B = \{\alpha_1, \alpha_2, \alpha_3\}$ be the basis for C^3 given by $\alpha_1 = (1,0,-1), \alpha_2 = (1,1,1), \alpha_3 = (2,2,0)$.

Find the dual basis of B.

21. If T is a diagonalizable linear operator on a finite dimensional vector space V, then show that the characteristic polynomial for T is $f = (x - c_1)^{d_1} \ldots (x - c_k)^{d_k}$.

22. Let W be a subspace of an inner product space V and $\beta \in V$. Show that $\alpha \in W$ is a best approximation to β by vectors in W if and only if $\beta - \alpha$ is orthogonal to every vector in W.

23. Show that $(\alpha | \beta) = x_1 y_1 - x_2 y_1 - x_1 y_2 + 4 x_2 y_2$, where $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$, defines an inner product on $\Re^2$.

24. State and prove Bessel's inequality. **(7 x 2 = 14 weightage)**

Part- C

Answer any *two* from the following four questions.
Each question has *four* weightage.

25. (a) Prove that two vectors in a vector space V are linearly dependent if and only if one of them is a scalar multiple of the other

(b) Suppose that $\{x_1, \ldots, x_m\}$ are linearly independent vectors in a vector space V, but $\{y\} \cup \{x_1, \ldots, x_m\}$ is linearly dependent. Then show that y can be written as a unique linear combination of $\{x_1, \ldots, x_m\}$.

26. State and prove *rank – nullity* theorem. If T is a linear operator on an n – dimensional vector space V whose range and null space are identical, then show that n is even.

27. (a) Show that minimal polynomial and characteristic polynomial for a linear operator have the same roots except for multiplicities.

(b) Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Then, show that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \cdots (x - c_k)$ where $c_1, \ldots, c_k$ are distinct elements of F.

28. If V is a real or complex vector space with an inner product $(|)$, then for any α, β in V, prove that (a) $(\alpha | \beta) = 0,\ \forall \beta \in V \Rightarrow \alpha = 0$.

(b) $|(\alpha | \beta)| \le \|\alpha\|\ \|\beta\|$, and (c) $\|\alpha + \beta\|^2 + \|\alpha - \beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$

(2 x 4 = 8 weightage)

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M1N17330 (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2017

MT1C03– Real Analysis – I

(2017 Admission onwards)

Max. Time: 3 hours Max. Weightage : 36

Part A

Answer all Questions

Each question has 1 weightage.

1. Write down a set with exactly five limit points.
2. Show that Intervals are connected subsets of *R*.
3. True or false? Justify: Subsets of compact sets are compact.
4. Show that every neighborhood is an open set.
5. Define the continuity of a function. If *f* and **g** are continuous at a point **p** show that $f+g$ is continuous at **p**.
6. Let *f* be a continuous mapping of a metric space X to a metric space Y and E be a subset of X. Show that $f(\bar{E}) \subset \overline{f(E)}$
7. Give an example of a continuous and unbounded function on (0,1) and a bounded continuous function on $\mathcal{R}$
8. If *f* is differentiable at a point **x**, show that it is continuous at **x**.
9. Give an example to show that mean Value Theorem is not true for complex valued functions.
10. Suppose *f* is differentiable in (a,b) and $f'(x) \geq 0$ for all x in (a,b). Show that *f* is monotonically increasing.
11. Let *f* be defined on $\mathcal{R}$ and suppose that $|f(x) - f(y)| \leq (x - y)^2$ for all x, y $\in \mathcal{R}$. Show that *f* is a constant.
12. If *f* is Reimann integrable over [a,b], then show that $|f|$ is Reimann integrable over [a,b]..
13. Let γ be defined on [0,1] by $\gamma(t) = (\cos 2\pi t, \sin 2\pi t)$. Show that γ is rectifiable and find its length.
14. Give an example of convergent sequence of functions which is not uniformly convergent.

(14 x 1 = 14 weightage)

Part B

Answer any seven Questions
Each question has 2 weightage.

15. Show that every k-cell is compact.

16. Define a perfect set. Show that any nonempty perfect set is uncountable.

17. Let f be a continuous mapping of a compact metric space X to a metric space Y. Show that f is uniformly continuous.

18. Show that a monotonic function cannot have a discontinuity of second kind.

19. State and prove Chain rule for differentiation..

20. If f is continuous mapping of $[a, b]$ into $\mathcal{R}^k$ and if f is differentiable in (a,b), show that there is some x in (a, b) such that $|f(b) - f(a)| \leq (b-a)|f'(x)|$

21. Suppose f is Monotonic and α increasing and continuous on $[a, b]$. Show that $f \in \mathcal{R}(\alpha)$ on $[a, b]$

22. State and prove fundamental theorem of calculus.Deduce integration by parts.

23. Prove or disprove: If $\{f_n\}$ is a sequence which converge to f on $[a, b]$. Then $\int_a^b f_n$ converge to $\int_a^b f$.

24. Show that the uniform limit of a sequence of continuous functions is continuous.

(7 x 2 = 14 weightage)

Part C

Answer any two questions
Each question carries 4 weightage.

25. a) Let $E \subset \mathcal{R}^k$. Show that the following are equivalent.

    i. E is closed and bounded

    ii. E is compact

    iii. Every infinite subset of E has a limit point.

    b) Prove or disprove: the closure and interior of connected sets are connected.

26. a) Discuss the continuity of the function defined by,

$$f(x) = \begin{cases} 1 & if\ x\ is\ rational \\ 0, & if\ x\ is\ irrational. \end{cases}$$

    b) State and prove L'Hospital's rule

27. a) Let f be differentiable on [a, b]. Show that f' can not have discontinuity of first kind

    b) If P_1 and P_2 are partitions of [a,b] , P_2 is a refinement of P_1 and f a bounded function on [a, b] .Then show that $U(P_2, f, \alpha) \leq U(P_1, f, \alpha)$

28. a) Show that there exist a real continuous function which is nowhere differentiable.

    b) Define an equicontinuous family of functions. Give an infinite equicontinuous family of functions.

(2 x 4 = 8 weightage)

N17331 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2017

MT1C04– Number Theory

(2017 Admission onwards)

Time: 3 hours Max. Weightage : 36

Part A

(Answer all questions.)

1. Prove that $\varphi(n)$ is even for $n \geq 3$.
2. Prove that the Mobius function is multiplicative but not completely multiplicative.
3. Prove that $\prod_{t/n} t = n^{\frac{d(n)}{2}}$, where $d(n)$ denotes the number of divisors of n.
4. $\forall n \geq 1$, prove that $\sigma_\alpha^{-1}(n) = \sum_{d/n} d^\alpha \mu(d)\mu\left(\frac{n}{d}\right)$.
5. For any arithmetical functions α and β and for any real or complex valued function F on $(0,\infty)$ such that $F(x) = 0$ for $0 < x < 1$, prove that $\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$.
6. Prove that $\forall x \geq 1, \sum_{n \leq x} \Lambda(n) \left[\frac{x}{n}\right] = \log([x]!)$.
7. $\forall x \geq 2$,prove that $\vartheta(x) = \pi(x)\log x - \int_2^x \frac{\pi(t)}{t} dt$.
8. Find the quadratic residues and non residues modulo 17.
9. Determine whether 888 is a quadratic residue modulo 1999.
10. Prove that if P is an odd integer, then prove that $(-1|P) = (-1)^{\frac{P-1}{2}}$.
11. Describe about shift cryptosystem and find a formula for the number of different shift transformations with an N-letter alphabet.
12. Find the cipher text corresponding to the text 'FRIDAYMORNING' in the affine cryptosystem with 26 letter alphabet system and enciphering key (7,3).
13. Write a short note on frequency analysis in cryptography.
14. Find the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$ $(mod\ 26)$.

(14×1= 14 weightage)

Part B

(Answer any 7 questions)

15. Prove that $\forall\, n \geq 1, \varphi(n) = n\prod_{p/n}\left(1 - \frac{1}{p}\right)$.

16. If f is a multiplicative arithmetical function and $f^{-1}(n) = \mu(n)f(n), \forall n \geq 1$, then prove that f is completely multiplicative.

17. State and prove Selberg Identity.

18. Prove that for all $\geq 2, \sum_{n\leq x} \frac{\log n}{n} = \frac{1}{2} log^2 x + A + o\left(\frac{\log x}{x}\right)$, where A is a constant.

19. For $x > 0$, prove that $0 \leq \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\log 2}$ and hence deduce $\lim_{x\to\infty} \frac{\psi(x)}{x} = \lim_{x\to\infty} \frac{\vartheta(x)}{x}$.

20. Prove that $\lim_{x\to\infty}\left(\frac{M(x)}{x} - \frac{H(x)}{x\log x}\right) = 0$.

21. State and prove Gauss' lemma.

22. Determine the odd primes p for which 3 is a quadratic residue and those for which 3 is a quadratic non residue.

23. Solve the system of simultaneous congruences

    $x + 3y \equiv 1(mod\ 26)$

    $7x + y \equiv 1(mod\ 26)$.

24. Compare private key cryptosystem with Public key cryptosystem.

(7×2= 14 weightage)

Part C

(Answer any 2 questions)

25. Prove that if both g and $f * g$ are multiplicative then f is also multiplicative and hence show that the set of all multiplicative functions is a subgroup of the group of all arithmetical functions f with $f(1) \neq 0$.

26. If a and b are positive real numbers such that $ab = x$, then for any arithmetical functions f and g, prove that $\sum_{\substack{q,d \\ qd\leq x}} f(d)g(q) = \sum_{n\leq a} f(n)\, G\left(\frac{x}{n}\right) + \sum_{n\leq b} g(n)\, F\left(\frac{x}{n}\right) - F(a)G(b)$

    where $F(x) = \sum_{n\leq x} f(n)$ and $G(x) = \sum_{n\leq x} g(n)$.

27. Let $\{a(n)\}$ be a non negative sequence such that $\sum_{n\leq x} a(n)\left[\frac{x}{n}\right] = x\log x + O(x)$, for all $x \geq 1$. Show that for all $x \geq 1$ (i) $\sum_{n\leq x} \frac{a(n)}{n} = \log x + O(1)$ (ii) There is a constant $B > 0$ such that $\sum_{n\leq x} a(n) \leq Bx$.

28. For every odd prime p, prove that $(2|p) = (-1)^{\frac{p-1}{2}} = \begin{cases} 1 & if\ p \equiv \pm 1(mod\ 8) \\ -1 & if\ p \equiv \pm 3(mod\ 8) \end{cases}$

(2× 4= 8 weightage)

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1M1N17332 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester M.Sc Degree Examination, November 2017

MT1C05– Discrete Mathematics

(2017 Admission onwards)

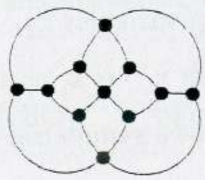
Max. Time: 3 hours

Max. Weightage : 36

PART A

Answer all. 1 weightage each.

1. Let X be a set and let $P(X)$ be its power set. Is the relation inclusion a total order on $P(X)$? Justify your answer.
2. Prove that in a lattice, every nonempty, finite subset has a supremum and infimum.
3. Prove that every nonzero element of a finite Boolean algebra contains at least one atom.
4. Prepare table values of the function $f(x_1, x_2, x_3) = x_1x_2 + x_2'x_3$.
5. Let G be a graph with n vertices and m edges. Assume that each vertex of G is of degree k or $k+1$. Show that the number of vertices of degree k in G is $(k+1)n - 2m$.
6. In a group of six people, prove that there must be three people who are mutually acquainted or three people who are mutually non-acquainted.
7. If $G_1[G_2]$ denote the lexicographic product of two simple graphs G_1 and G_2, then prove that $m(G_1[G_2]) = n(G_1)m(G_2) + n(G_2)^2m(G_1)$.
8. Let G be a graph in which the degree of every vertex is at least 2. Then show that G contains a cycle.
9. Define connectivity and edge connectivity. Give an example.
10. Draw the dual of the given graph (Herschel graph)



11. Let u be string on the alphabet Σ. Prove that $|u^n| = n|u|$ for all $n = 1,2, \ldots$.
12. Define regular language and give an example of it.
13. Let $\Sigma = \{a, b\}$. Construct a dfa that accepts the language $\{aba, ba, ab\}$.
14. Show that if L is regular, then so is $L - \{\lambda\}$.

(14 x 1 = 14 weightage)

PART B

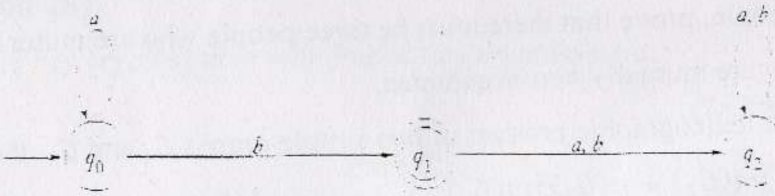
Answer any seven. 2 Weightage each

15. Let X be a finite set and $\leq$ be a partial order on X. R is abinary relation on X defined by xRy iff y covers x. Prove that $\leq$ is the smallest order relation containing R.
16. Prove that every finite Boolean algebra is isomorphic to a power set Boolean algebra.
17. Write the Boolean function

$$f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1' + x_2 + x_3')(x_1 + x_2' + x_3')(x_1' + x_2' + x_3')(x_1 + x_2$$

in the disjunctive normal form.

18. Prove that, in a connected graph with at least three vertices, any two longest paths have a vertex in common.
19. Show that a simple cubic connected graph G has a cut vertex if, and only if, it has a cut edge.
20. Prove that every connected graph contains a spanning tree.
21. State and prove Euler's formula for a plane graph.
22. Find a grammar that generates the language $L = \{a^n b^{n+1} : n \geq 0\}$.
23. Find the language accepted by the dfa



24. Find regular expression for the language

$L = \{w \in \{0,1\}^* : w$ has at least one pair of consecutive zeros$\}$

(7 x 2 = 14 weightage)

PART C

Answer any Two. 4 Weightage each

25. (a) Prove that every Boolean function of n variables $x_1, x_2, \dots, x_n$ can be uniquely expressed as the form of $x_1^{\varepsilon_1} x_2^{\varepsilon_2} \dots x_n^{\varepsilon_n}$ where each $x_i^{\varepsilon_i}$ is x_i or x_i'

(b) Prove that the characteristic numbers of a symmetric Boolean function completely determine it.

26. (a) Show that for any loopless connected graph, $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

(b) Prove that the connectivity and edge connectivity of a simple cubic graph are equal.

27. Prove that a connected graph G with at least two vertices is a tree if, and only if, its degree sequence $(d_1, d_2 \dots, d_n)$ satisfies the condition $\sum_{i=1}^{n} d_i = 2(n-1)$ with $d_i > 0$.
28. (a) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a dfa and let G_M be its associated transition graph. Then prove that for every $q_i, q_j \in Q$, and $w \in \Sigma^+, \delta^*(q_i, w) = q_j$ if, and only if, there is in G_M a walk with label w from q_i to q_j.

(b) Find a dfa that recognises the set of all strings on $\{a, b\}$ starting with prefix ab.

(2 x 4 = 8 weightage)