1B3N17112 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2017

ST3B03 - Statistical Estimation

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

Part A (Answer **all** questions. Each carries 1 mark)

Fill in the blanks (Questions 1-6)

1. Bernoulli's law of large numbers is a particular case of ....
2. The standard deviation of the sampling distribution of a statistic is known as ....
3. Mode of a chi-square distribution with 'n' degrees of freedom is..........
4. Student's t distribution with '1' degree of freedom reduces to ....
5. An estimator T of a parameter θ is said to be unbiased if ....
6. An unbiased estimator whose variance tends to zero as the sample size increases is........

Choose the Correct Answer (Questions 7 – 12)

7. If $X_1, X_2, \ldots, X_n$ are ' n' independent observations from $N(\mu, \sigma^2)$ then the distribution of $\sum_{i=1}^{n} \frac{(X_i-\mu)^2}{\sigma^2}$ is
   (a) t distribution with n degrees of freedom
   (b) χ^2 distribution with (n-1) degrees of freedom
   (c) χ^2 distribution with n degrees of freedom
   (d) None of these
8. The mean of F distribution with (n_1, n_2) degrees of freedom is
   $(a)\ \frac{n_1}{n_1-2}$ $(b)\ \frac{n_2}{n_2+2}$ $(c)\ \frac{n_2}{n_2-2}$ $(d)\ \frac{n_1 n_2}{n_1+n_2}$
9. If Z and V are independent random variables and $Z \sim N(0,1)\, and\ V \sim \chi^2_n$, then $\frac{Z\sqrt{n}}{\sqrt{V}}$ follows
   (a) Standard normal distribution
   (b) χ^2 with n degrees of freedom
   (c) Student's t distribution with n degrees of freedom
   (d) F distribution with (n_1, n_2) degrees of freedom
10. If T_1 and T_2 are two estimators such that $Var(T_1) < Var(T_2)\ then$:
   (a) T_1 and T_2 are equally efficient
   (b) T_1 is more efficient than T_2
   (c) T_2 is more efficient than T_1
   (d) none of these
11. The 99% confidence interval for the mean of a normal distribution $N(\mu, \sigma^2)$ when σ known is:
   (a) $\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}},\ \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)$
   (b) $\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n-1}},\ \bar{X} + 1.96\frac{\sigma}{\sqrt{n-1}}\right)$
   (c) $\left(\bar{X} - 2.58\frac{\sigma}{\sqrt{n}},\ \bar{X} + 2.58\frac{\sigma}{\sqrt{n}}\right)$
   (d) $\left(\bar{X} - 2.58\frac{s}{\sqrt{n-1}},\ \bar{X} + 2.58\frac{s}{\sqrt{n-1}}\right)$
12. The formula for the confidence interval for the ratio of variances of the two normal population involves
   (a) χ^2 distribution (b) F distribution (c) t-distribution (d) none of these

(12 ×1 = 12 marks)

Part B

(Answer any seven questions. Each carries 2 marks)

13. What are the assumptions of central limit theorem?
14. Define convergence in probability.
15. Distinguish between parameter and statistic.
16. Define χ^2 distribution with 'n' degrees of freedom.
17. What is F statistic?
18. Distinguish between estimate and estimator
19. State Cramer- Rao inequality.
20. Give an example of an estimator which is consistent but not unbiased.
21. 1.5, 2.3, 1.4, 3.6, 2.5 is a random sample from a population. Obtain the maximum likelihood estimate of θ if the p. d. f of the population is $f(x) = \frac{1}{2} e^{-|x-\theta|}$

(7 × 2 = 14 marks)

Part C

(Answer any six questions. Each carries 5 marks)

22. If X follows a binomial distribution with $n = 100$ and $p = \frac{1}{2}$, obtain using Chebychev's inequality a lower limit for $P[|X - 50| < 7.5]$.
23. State and prove weak law of large numbers.
24. Derive the moment generating function of χ^2 distribution with n degrees of freedom.
25. State the relations between the Normal, χ^2, t and F distributions.
26. What are the desirable properties of a good estimator?
27. If $X_1, X_2, \dots X_n$ is a random sample from a Normal population with mean μ and variance 1, show that $T = \frac{1}{n} \sum {X_i}^2$ is an unbiased estimator of $\mu^2 + 1$
28. Explain the method of moments estimation. Obtain the moment estimator of the parameter θ based on a random sample of size 'n' from a population with p.d.f
$$f(x;\theta) = \theta e^{-\theta x}, x > 0, \theta > 0$$
29. Explain the concept of 'Interval estimation'. Obtain the interval estimator of mean of a normal population $N(\mu, \sigma^2)$ with confidence coefficient $(1 - \alpha)$, when σ^2 is known.

(6 × 5=30 marks)

Part D

(Answer any three questions. Each carries 8 marks)

30. State and prove Chebychev's inequality.
31. If $X_1, X_2, X_3,$ *and* X_4 are independent observations from N(0, 1) population, state giving reasons, the sampling distributions of (i) $\frac{\sqrt{2}\, X_2}{\sqrt{X_1^2 + X_2^2}}$, (ii) $\frac{3X_4^2}{X_1^2 + X_2^2 + X_3^2}$, (iii) $\frac{(X_1 - X_2)^2}{2}$ (iv) $\frac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$.
32. Show that $T_n = \frac{n\bar{X}}{n+1}$ is a consistent estimator of $\boldsymbol{\lambda}$ where $\bar{X}$ is the mean of a random sample of size 'n', taken from a Poisson distribution with mean $\boldsymbol{\lambda}$.
33. What do you mean by maximum likelihood method of estimation? Find the m. l. estimates of a and b if the p. d. f. of the population is f(x)= $\frac{1}{b-a}$, $a \le x \le b$
34. Two random samples of sizes 10 and 12 from normal populations having the same variance gave $\bar{X}_1 = 20$, $\bar{X}_2 = 24$, ${S_1}^2 = 25$ *and* ${S_2}^2 = 36$. Find 90% confidence limits for ($\mu_1 - \mu_2$).

(3 × 8 =24 marks)

1B3N17113 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2017

ST3C03 - Statistical Inference

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART A

(Answer ALL the questions. Each carries 1 mark.)

Fill in the blanks (Questions 1-6)

1. Standard deviation of a statistic is known as ______________
2. The variance of a chi-square distribution with 'n' degrees of freedom is__________
3. Square of a t-variable with n degrees of freedom follows ______________distribution.
4. An estimator θ^* of a parameter θ is said to be unbiased if__________
5. If β is the probability of type II error, then $(1-\beta)$ is called _______ of the test
6. Critical region provides us the criterion for the rejection of__________ hypothesis.

Choose the correct answer (Questions 7-12)

7. The range of variation of a chi-square variable is
   (a) 0 to ∞ (b) $-\infty$ to ∞ (c) 0 to 1 (d) -a to a
8. Square of a standard normal variate follows
   (a) Normal distribution (b) t - distribution
   (c) χ^2 - distribution (d) F - distribution
9. The Maximum Likelihood Estimates are
   (a) Unbiased and consistent (b) Unbiased and efficient
   (c) Consistent and invariant (d) Invariant and unbiased
10. If t is a consistent estimation of θ, then
    (a) t^2 is unbiased for θ^2 (b) t^2 is consistent for θ^2
    (c) t^2 is unbiased for θ (d) none of these
11. To test the equality of the means of two normal populations with known variances, we use
    (a) t - test (b) Z - test (c) F - test (d) $\chi 2$ - test
12. The probability of rejecting the null hypothesis when it is true is known as
    (a) power of the test (b) significance level
    (c) type II error (d) critical region **(12 x 1 =12 Marks)**

PART B

(Answer any SEVEN questions. Each carries 2 marks.)

13. Distinguish between parameter and statistic, with suitable examples.
14. Give examples for a statistic which is (a) both unbiased and consistent (b) unbiased but not consistent.
15. Define the t - variate and write down its the probability density function
16. Define consistency of an estimate.
17. Give the properties of Maximum Likelihood Estimate
18. Obtain the moment estimate of the parameter λ for the Poisson distribution.
19. Define null hypothesis and alternative hypothesis.
20. Explain the procedure for testing the equality of the variances of two normal populations.
21. Define interval estimation.

(7 x 2 = 14 Marks)

PART C

(Answer any SIX questions. Each carries 5 marks.)

22. Derive the sampling distribution of the sample variance if samples are taken from $N(\mu, \sigma^2)$.
23. Derive 95% confidence interval for the mean of a normal population.
24. Explain the χ^2 test for goodness of fit.
25. If $X_1, X_2, \ldots, X_n$ are random observations on a Bernoulli variate X taking the values 1 with probability p and the value 0 with probability (1-p), show that $\frac{\Sigma x}{n}\left(1-\frac{\Sigma x}{n}\right)$ is a consistent estimator of p(1-p).
26. Derive the inter relation between Normal, χ^2, t and F distributions.
27. Let p be the probability that a coin will fall head in a single toss in order to test H_0: $p=\frac{1}{2}$ against H_1: $p=\frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and the power of the test.
28. Explain the paired t - test.
29. Verify whether the sample mean $\bar{X}$ is sufficient for the population mean μ for the Normal population $N(\mu, \sigma^2)$.

(7 x 5 = 30 Marks)

PART D

(Answer any THREE questions. Each carries 8 marks.)

30. i) Explain the method of maximum likelihood estimation.
    ii) Obtain the M.L.E. for α and β for the rectangular population
    $$f(x;\ \alpha,\beta) = \begin{cases} \frac{1}{\beta-\alpha}, \alpha < x < \beta \\ 0, elsewhere \end{cases}$$
31. Define the χ^2 - distribution and derive its m.g.f. Hence, obtain its mean and variance.
32. i) Explain the test procedure for testing the equality of the population proportion based on large samples.
    ii) In a sample of 600 men from city A, 450 are found to be smokers. Out of 900 from city B, 550 are smokers. Does the data indicate that the cities are significantly different with respect to prevalence of smoking?
33. A group of seven week old chickens reared on high protein diet weigh 12, 14, 15, 11, 16, 14 and16 ounces. A second group of five chickens on ordinary diet weigh 8, 10, 14, 10 and ounces. Test whether there is evidence that additional protein has increased the weight of the chickens.
34. Four coins are tossed and the number of heads X is noted. The experiment is repeated 40 times. The result is as follows:

x:	0	1	2	3	4
f:	5	7	17	8	3

Is there any reason to believe that the coins are all balanced?

(3 x 8 = 24 Marks)

1B3N17114 (Pages : 2) Reg. No:…………………………..

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester B.Sc Degree Examination, November 2017

AS3C03 - Life Contingencies and Principles of Insurance

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART-A

Answer *all* questions. Each question carries *one* mark

1. Who makes the offer under an insurance contract?
   (a) Insurer (b) Company (c) Mediator (d) Policy holder
2. Head office of the Life Insurance Corporation of India is situated in …….
   (a) Delhi (b) Kolkatta (c) Mumbai (d) Bangalur
3. The cause that produces loss is known as ………..
   a)Risk b) Hazard c) Peril d) none of these
4. Write down the premium equation of continuous h-payment whole life insurance.
5. The net premium is also called……..
6. Write down the form of fractional power utility function
7. The amount of money that the insurer sets aside to meet future liabilities is called…
8. The contingent payment linked to the amount of loss is called ……
9. …… is the uncertainty of the occurrence of the event that creates the loss
10. ….. is the satisfaction that a consumer obtains from a particular course of action.
11. Fidelity guarantee insurance is also known as ……..
12. The expected value of random prospects with monetary payments is called …..

(12 x 1= 12Marks)

PART-B

Answer any ***seven*** questions. Each question carries ***two*** marks.

13. Define net premium .
14. Define true fractional premium
15. What is meant by a risk neutral investor?
16. State Jensen's inequality
17. Define 'professional indemnity'
18. What is meant by valuation of the policy?
19. Define Aviation insurance.
20. Define 'prospective reserve'.
21. Define pecuniary loss

(7 x 2= 14Marks)

PART-C

Answer any *five* questions. Each question carries *six* marks.

22. Explain 'fire insurance'
23. Explain the essential features of a contract.
24. Explain n-year endowment insurance.
25. Explain motor insurance.
26. Calculate the annual premium for a term assurance with a term of 10 years to a person aged 30, with a sum assured of Rs.100,000, assuming AM92 ultimate mortality and interest of 5% p.a. Assume that the death benefit is payable at the end of the year of death
27. Explain why the insurer holds reserve?
28. Explain utility theory
29. A 10-year term assurance with a sum assured of Rs.200,000 payable at the end of the year of death, is issued to a person aged 30 for a level annual premium of Rs.300. Calculate the prospective reserve at the end of the fifth year, *ie.,* just before the sixth premium has been paid, assuming AM92 Ultimate mortality and 5% *pa* interest.

(5 x 6= 30Marks)

PART-D

Answer any *three* questions. Each question carries *eight* marks.

30. Explain the history of insurance in India.
31. Explain (i) Apportionable premiums. (ii) Optimal insurance
32. A multiple decrement model with two causes of decrement has forces of decrement given by $\mu_x^{(1)}(t) = 1/(100-x-t)$ and $\mu_x^{(2)}(t) = 2/(100-x-t)$, $t<100-x$.
    If $x = 40$, obtain expressions for
    (i) $f_{T,J}(t,j)$ (ii) $f_T(t)$ (iii) $f_J(j)$ (iv) $f_{J/T}(j/t)$
33. Briefly explain different types of non-life insurance contracts
34. If ${}_{k/}q_x = c(0.9)^{k+1}$, $k = 0,1,2,\ldots$, where $i = 4\%$,
    Find (i) c (ii) p_x (iii) V(L)

(3 x 8= 24Marks)