

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
 Second Semester B.Sc Statistics Degree Examination, March 2017
 ST2B02 – Bivariate Random Variable & Probability Distribution
 (2016 Admission onwards)

Time: 3 hours

Max. Marks : 80

Part A(Answer **all** questions, each question carries 1 mark)

$E(X - K)^2$ is the variance of X when

- (a) $K < E(X)$ (b) $K = E(X)$ (c) $K > E(X)$ (d) $K^2 = E(X)$

$E(X \cdot Y) = E(X) E(Y)$ if

- (a) X&Y are independent (b) X&Y are identical
 (c) X&Y are dependent (d) none of the above

$F_{x,y}(x, \infty) = \dots\dots\dots$

- (a) 1 (b) 0 (c) $F_x(x)$ (d) $F_y(y)$

Correlation coefficient lies in the interval :

- (a) (0, 0) (b) (-1, 1) (c) (-1, 0) (d) (0, 1)

A distribution for which mean = variance is:

- (a) Poisson (b) Uniform (c) Binomial (d) Exponential

The points of inflexion of the normal curve are:

- (a) $\mu \pm \sigma$ (b) $\mu \pm 3\sigma$ (c) $\mu \pm 1$ (d) $\mu \pm 3$

If X and Y are independent then correlation coefficient between X and Y is =

If $E(X) = 4$, $E(Y) = 5$ then $E(2XY) = \dots\dots\dots$ if X&Y are independent

The mean of a Gamma distribution with parameter m and p is

The mean of a Poisson distribution is 3. Then $P(X = 1) = \dots\dots\dots$

Moment generating function of binomial distribution is $(1/3 + 2/3 e^t)^5$, find the pmf.

The joint density function of two random variables X and Y is $f(x, y) = 3xy$, $0 \leq x \leq 1$, $0 \leq y \leq 3$. Find $E(XY)$?

(12 x 1 = 12 Marks)**Part B**

(Answer any 7 questions, each question carries 2 marks)

Define expectation of bivariate random variables.

Define marginal p.d.f of a r.v

Prove that $\text{cov}(X, Y) = E(XY) - E(X)E(Y)$

Prove or disprove $E(X + Y) = E(X) + E(Y)$ for any two random variables X and Y

Define log normal distribution

Derive the variance of an exponential distribution

19. If $X \rightarrow N(25, 3)$ and $Y \rightarrow N(20, 4)$ and X and Y are independent then find the distribution of $X+Y$?
20. Define continuous rectangular distribution in the interval $(0, 2)$ and hence obtain its moment generating function?
21. If X and Y are random variables with joint probability density function $f(x, y) = \frac{x+2y}{18}$ where $(x, y) = (1, 1) (1, 2) (2, 1) (2, 2) = 0$ Otherwise, are the variables independent ?

(7 x 2 = 14 Marks)

Part C

(Answer any 6 questions, each question carries 5 marks)

22. Derive Poisson distribution as a limiting form of Binomial distribution.
23. Show that correlation coefficient is a dimensionless measure independent of origin and unit of measure?
24. Obtain the Median of Normal distribution.
25. If X and Y are independent random variables, show that $V(X + Y) = V(X - Y)$.
26. Establish the recurrence relation for moments of a Binomial distribution and find the first four central moments?
27. Find mean, variance and moment generating function of Rectangular distribution over the interval (a, b) .
28. Define Geometric distribution and explain the lack of memory property of G.D
29. The joint probability density function of (X, Y) is $f(x, y) = 3xy, 0 \leq x \leq 1, 0 \leq y \leq 1$ Find $E(Y|X = x)$?

(6 x 5 = 30 Marks)

Part D

(Answer any 3 questions , each question carries 8 marks)

30. a) If X and Y are independent Binomial variates obtain the conditional distribution of X given $X+Y$?
b) Find the r^{th} central moment of normal distribution?
31. The Joint probability density function of (X, Y) is $f(x, y) = \frac{x+y}{21} \quad x=1,2,3; y=1,2$
Obtain $\text{cov}(x,y)$, correlation coefficient & regression functions.
32. a) Define moments. Establish the relation between row moments and central moments?
b) Among a large group of students 5% are under 150cm and 40% are between 150 cm and 162cm in height. Assuming normal distribution , find mean and standard deviation of height?
33. Derive the moment generating function of Poisson distribution and hence obtain the first four central moments of the distribution?
34. a) Define normal distribution and write its properties?
b) Stating the conditions prove that binomial distribution tends to normal distribution.

(3 x 8 = 24 Marks)

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 Second Semester B.Sc Statistics Degree Examination, March 2017

ST2C02 –Probability Distribution

(2016 Admission onwards)

Time: 3 hours

Max. Marks : 80

Part A

Answer all questions, each carries one mark

Choose the correct answer

If $f(x, y) = \frac{x+y}{21}$, $x=1,2,3$; $y=1, 2$ then $f(y)$:

- a) $\frac{1+y}{21}$ b) $\frac{2+y}{21}$ c) $\frac{3+y}{21}$ d) $\frac{6+3y}{21}$

If X is a r.v, then $V(2-3x)$ is

- a) $2 - \text{Var}(3x)$ b) $\text{Var}(2) - \text{Var}(3x)$ c) $2 - 3 \text{Var}(x)$ d) $9 \text{V}(x)$

Binomial distribution is negatively skewed when

- a) $P=0$ b) $p > 1/2$ c) $p < 1/2$ d) $p = 1/3$

In a Poisson distribution, if the values of λ is an integer, then the distribution will be

- a) unimodal b) bimodal c) trimodal d) none

The range of beta variate is

- a) $(-\infty, 0)$ b) $(1, \infty)$ c) $(-\infty, \infty)$ d) $(0, 1)$

If $x \sim p(\lambda)$ and $p(x=2) = p(x=3)$, then the value of λ is

- a) 2 b) 3 c) 4 d) 6

Fill in the blanks

$E(x|y=2) = \frac{26}{12}$, $E(x^2|y=2) = \frac{64}{12}$, then $V(x|y=2) =$ _____

If $y=5x+10$ and x is $N(10, 25)$. Then mean of $y =$ _____

The mean of the discrete uniform distribution is _____

If C is a constant, then $V(C) =$ _____

The 4th central moment of normal distribution is _____

The characteristic function is defined as _____

(12×1 = 12 Marks)

Part B

Answer any seven questions, each carries two marks.

13. Define conditional variance of $x|y = y$
14. Define Karl Pearson's correlation coefficient.
15. Derive the m.g.f of Poisson distribution.
16. Write any four properties of normal distribution
17. If $f(x) = q^{x-1} p$, $x = 1, 2, \dots$. Find mean of X .
18. State Chebysheff's inequality.
19. Define Cauchy's distribution.
20. Define convergence in probability.
21. Derive the mean of gamma distribution.

(7×2 = 14 Marks)

Part C

Answer any six questions, each carries five marks.

22. A random variable x has a probability function $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3, \dots$, find the m.g.f and hence find the mean and mode of X .
23. The joint density function $f(x, y) = 2 - x - y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $f(x, y) = 0$ elsewhere. Obtain the regression function of x on y and y on x .
24. Derive the m.g.f of binomial distribution and hence obtain mean and variance.
25. For a binomial distribution $\beta_1 = \frac{1}{15}$ and $\beta_2 = \frac{89}{30}$, obtain mean and variance.
26. Derive Poisson distribution as a limiting case of binomial distribution.
27. Find the mode of the Normal distribution.
28. Define exponential distribution. Obtain the m.g.f and hence find mean and variance.
29. Examine whether the weak law of large numbers holds good for the sequence $\{x_n\}$ of independent r. v where $P\left[x_n = \frac{1}{\sqrt{n}}\right] = \frac{2}{3}$, $P\left[x_n = -\frac{1}{\sqrt{n}}\right] = \frac{1}{3}$

(6×5 = 30 Marks)

Part D

Answer any three questions, each carries 8 marks.

30. If $P(x, y) = xye^{-(x+y)}$, $x \geq 0$, $y \geq 0$ find
 - i) $P(x < 1)$
 - ii) $P(y < 2)$
 - iii) $P(x < \frac{1}{2}, y < \frac{1}{2})$
 - iv) verify the independence
31. For a normal distribution 31% of the items are under 45 and 8% are over 64. Obtain $P(45 \leq x \leq 55)$
32. State and prove Lindberg - Levy form of CLT
33. Prove that
 - i) $E(x+y) = E(x) + E(y)$
 - ii) $E(xy) = E(x)E(y)$ if x and y are independent
 - iii) $E\{E(x|y)\} = E(x)$
34. State and prove Renovsky formula for finding central moments of Binomial Distribution. Hence obtain μ_2 , μ_3 and μ_4 .

(3×8 = 24 Marks)

PART-B

Answer any *seven* questions. Each question carries *two* marks.

13. Define k/q_{xy}
14. Define future lifetime
15. Prove the identity $\delta \bar{a}_x + \bar{A}_x = 1$.
16. Prove that $\mu(x) = \frac{-1}{l_x} \frac{dl_x}{dx}$
17. If $s(x) = 1 - \frac{x}{100}$, $0 \leq x \leq 100$, calculate $\mu(x)$
18. Define continuous whole life annuity.
19. Define ${}_{t|}q_x$.
20. Prove that $f_{T(x)}(t) = {}_t p_x \mu(x+t)$
21. Using the ELT15(males) life table, calculate the probability of 42 year old dying between ages 62 and 75

(7 x 2 = 14 Marks)

PART-C

Answer any *six* questions. Each question carries *five* marks.

22. Explain n-year deferred whole life insurance.
23. Write a note on Analytical Laws of Mortality.
24. Prove that $\ddot{a}_{x:n|\square} = \sum_{k=0}^{n-1} v^k$
25. Briefly explain present values of joint life and last survivor assurances.
26. Calculate ${}_3 P_{45.5}$ and ${}_{6.75} P_{52.5}$ using the UDD assumption (AM92)
27. Prove that $n/\bar{a}_x = {}_n E_x \bar{a}_{x+n}$
28. Explain whole life immediate annuity.
29. Explain n-year temporary life annuity due.

(6 x 5 = 30 Marks)

PART-D

Answer any *three* questions. Each question carries *eight* marks.

30. Derive the relationship between Insurance payable at the moment of death and the value of the year of death.
31. Explain the following :
 - a) Present values of joint life and last survivor assurance.
 - b) Present values of joint life and last survivor annuities.
32. Derive the commutation function for whole life annuity due, n-year temporary annuity due and n-year deferred annuity due.
33. Explain n-year Endowment Assurance contract. Find its Mean and Variance.
34. Calculate ${}_3 P_{62.5}$ based on the PFA92C20 table in the table using
 - i. The UDD assumption.
 - ii. The CFM assumption.

(3 x 8 = 24 Marks)