20

1B2M20108 (Pages : 2) Reg. No:.......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Mathematics Degree Examination, March/April 2020

BST2C02 – Probability Theory

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

Part A

Each question carries 2 Marks.

Maximum Marks that can be scored in this Part is 20

1. Define random experiment.
2. When do you say(a) Two events are independent and(b) Two events are mutually exclusive?
3. Give the multiplication rule for the simultaneous occurrence of two events A and B.
4. What is the probability that a leap year will have 53 Sundays?
5. Distinguish between discrete and continuous random variables. Give examples.
6. Given that f(x) $= kx^4 e^{-x}\,; x \geq 0$ is a p.d.f. Find k.
7. A r.v X has the following p.m.f

$$f(x)=\begin{cases} k & \text{if } x=-1 \\ 2k & \text{if } x=0 \\ 3k & \text{if } x=1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Determine the value of k.

   (b) Find $P(X<0)$ and $P(X \geq 0.5)$

8. Let X be a random variable with the following probability distribution. Find E(X) and $E(X^2)$ and using the laws of expectation , evaluate $E(2X+1)^2$

x	-3	6	9
P(X=x)	1/6	1/2	1/3

9. State the properties of the Characteristic function.
10. Find the m.g.f of X with p.d.f $f(x)=\frac{1}{\theta}$, $0<x<\theta$ and 0 elsewhere. Also find V(X).
11. Define marginal and conditional distributions of a continuous bivariate random variable (X,Y).
12. Prove that, if X and Y are independent random variables, then $E(XY)=E(X)\,E(Y)$.

Part B

Each question carries 5 Marks.

Maximum Marks that can be scored in this Part is 30

13. Prove that, if A and B are independent events, then (i)A and $\overline{B}$ are independent (ii) $\overline{A}$ and B are independent (iii) $\overline{A}$ and $\overline{B}$ are independent.

14. A problem is given to three students. Their chances of solving the problem are respectively $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$. What is the probability that the problem is solved?

15. A continuous r.v X has the probability density function

$$f(x) = \begin{cases} Ae^{-\frac{x}{5}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(1) Find A

(2) Show that for any two positive numbers s and t, $P(X > s+t \mid X > s) = P(X > t)$.

16. Define distribution function of a random variable. What are its properties?

17. If E(X)= 2 and V(X)= 3 , find(1) $E(X^2)$, (2) E(2X+2), (3) $E[X(X+1)]$ and (4) $E[X(X-1)]$.

18. Let $M_X(t)$ be the moment generating function of a random variable with r^{th} raw moment μ_r'. Show that $\mu_r' = \left[\frac{d^r}{dt^r} M_X(t)\right]_{t=0}$, $r = 1, 2, 3, \ldots$

19. Explain (i) joint probability density function, (ii) joint distribution function (iii) marginal distributions and (iv) conditional distributions of a two continuous random variables X and Y. When do you say that X and Y are independent?

Part C

Answer any one question and carries 10 Marks.

20. State and prove Bayes' theorem. Two urns I and II contain respectively 3 white and 2 black balls and 2 white and 4 black balls. One ball is transferred from urn I to urn II and then a ball is drawn from the latter. It happens to be white. What is the probability that the transferred ball was white?

21. If the joint density function of (X,Y) is given by $f(x, y) = x + y \quad 0 \le x \le 1, 0 \le y \le 1$. Find (i) $Cov(X, Y)$ and (ii) $E(X \mid Y = y)$.

21

:2M20107 (Pages : 2) Reg. No:.............................

Name:.............................

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Degree Examination, March/April 2020

BST2B02 – Bivariate Random Variables & Probability Distributions

(2019 Admission onwards)

ime: 2 ½ hours Max. Marks : 80

SECTION-A

Each question carries 2 Marks.

Maximum Marks that can be scored in this section is 25.

1. The mean and variance of the binomial distribution is 6 and 2 respectively. Find the value of probability of success
2. Find the expectation of the number on a die when thrown
3. Define convergence in probability
4. Distinguish between joint and marginal density functions
5. Show that $E(x^2) \geq [E(x)]^2$
6. Obtain the m.g.f of geometric distribution
7. Obtain second raw moment of discrete uniform distribution
8. Find the mean of a random variable X with first moment about 4 is given as 7
9. State Bernoulli's law of large numbers
10. Find c, if $f(x,y) = c(2x+3y), x = 0, 1$ and $y = 1,2$ is a joint pmf.
11. Define characteristic function
12. State any four properties of expectation
13. State uniqueness theorem of m.g.f
14. For a random variable X, $P(X=3) = 2P(X=4) = P(X=5)$. Find $V(x)$
15. First three raw moments of X are -1, 55 and -62.5. Obtain coefficient of skewness based on moments

SECTION-B

Each question carries 5 Marks.

Maximum Marks that can be scored in this section is 35.

16. If $E(x) = 4$, $E(x^2) = 25$. Find a lower bound for $P(-2 < X < 10)$ using Tchebyschev inequality

17. For two random variables X and Y, the joint p.d.f

$$f(x, y) = 2 - x - y, 0 \leq x \leq 1, 0 \leq y \leq 1. \text{ Find } \operatorname{cov}(x, y).$$

18. Define raw moments and central moments. State interrelationship between raw moments and central moments

19. Let $f(x, y) = \frac{1}{8}(6 - x - y), 0 \leq x < 2, 2 \leq y < 4$. Find $P(X < 1 / Y < 3)$

20. Show that the Poisson distribution $P(\lambda)$ is bimodal when λ is an integer

21. If X and Y are independent random variables prove that $E(XY) = E(X)E(Y)$. Is th converse true? Justify

22. Let $f(x, y) = c\,x\,y\,e^{-(x^2+y^2)}, x \geq 0, y \geq 0$ be the joint probability density function of (X, Y). Then (i) Determine c and (ii) examine the independence of X and Y

23. Define m.g.f of a random variable. Examine the effect of the shifting of the origin ar change of scale on the m.g.f of a random variable

SECTION-C

(Answer *any two*Questions and each carries 10 marks)

24. (a) Define negative binomial distribution and state its important properties

 (b) Derive Poisson distribution as a limiting case of negative binomial distribution

25. (a) State and prove weak law of large numbers

 (b) For a sequence of random variables (X_k) given that

 $P(X_k = -2^k) = 2^{-(2k+1)} = P(X_k = 2^k)$, $P(X_k = 0) = 1 - 2^{-(2k+1)}$. Examine if the law of large numbers holds for this sequence

26. Let $f(x, y) = 8xy, 0 < x < y < 1$. Find (a) $E(Y / X = x)$ and (b) $V(Y / X = x)$

27. (a) Obtain the m.g.f of X following binomial distribution with parameters n and p Hence state and prove additive property of binomial distribution

 (b) If X and Y are independent binomial random variables with parameters (6, 0.5) and (4, 0.5) respectively. Calculate $P(X + Y \geq 3)$.

22

1B2N20100 (Pages : 2) Reg. No:..........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Second Semester B.Sc Degree Examination, March/April 2020

BST2C06 –Regression Analysis & Probability Theory

(2019 Admission onwards)

Time: 2 hours Max. Marks : 60

SECTION-A

Each question carries 2 Marks.

Maximum Marks that can be scored in this section is 20.

1. Define Pearson correlation coefficient
2. Define p.d.f. State its properties.
3. What is meant by positive correlation? Is the correlation between age and height of primary school students is positive?
4. State and prove addition theorem for two events
5. Define sample space. Give the sample space for the experiment of tossing 2 coins.
6. Define (i) Mutually Exclusive Events (ii) Mutually Exhaustive Events
7. Explain the concept of independence of two events A, B.
8. Define Distribution Function F(x) of a random variable.
9. $P(A) = 0.5, P(B) = 0.3. find\ P(A \cup B)\ if\ A\ and\ B\ are\ independent$
10. For the pdf $f(x), find\ the\ value\ of\ k\ and\ hence\ find\ P(x \geq 2.5)$

X	0	1	2	3
f(*x*)	*k*	*3k*	*3k*	*k*

11. The equation of two regression lines obtained in a correlation analysis are 9x-4y+15=0 and 25x-6y-7=0. Obtain mean of x and mean of y.
12. Two unbiased dice are thrown. What is the probability of getting 'sum is less than 5'?

SECTION-B

Each question carries 5 Marks.

Maximum Marks that can be scored in this section is 30.

13. Explain different approaches to the theory of probability.

14. Distinguish between Correlation and regression.

15. A problem in mathematics is given to three studentsA, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$respectively. What is the probability that the problem will be solved?

16. Define Partial correlation. If $r_{12} = 0.8, r_{13} = 0.7, r_{23} = 0.5, calculate\ r_{13.2}$

17. Given $\bar{X} = 5.5, \bar{Y} = 4.0, \sum X^2 = 385, \sum Y^2 = 192,\ \sum XY$=185 find the correlation between X and Y. Comment on the nature of Correlation.

18. Three dies are tossed simultaneously. What is the probability of getting atleast one 5?

19. Explain regression equations? How can we identify the regression equations?

SECTION-C

(Answer *any one* Question and carries 10 marks)

20. (i)State the properties of Karl Pearson's correlation coefficient.

(ii)A tobacco company statistician wishes to know whether heavy smoking is related to longevity. From a sample of recently deceased smokers, the number of cigarettes is paired with the number of years that they lived.

Cigarettes	25	35	10	40	85	75	60	45	50
Years Lived	63	68	72	62	65	46	51	60	55

Calculate the correlation coefficient and comment on it.

21. A random variable X has the following probability function

x :	1	2	3	4	5	6	7
P(x) :	k	2k	2k	3k	k^2	$2k^2$	$2k^2+k$

i. Find the constant k

ii. Find $P(x \geq 5)$

iii. Find $P(x < 3)$