

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
 Second Semester BSc Degree Examination, March/April 2020
 BMT2C02 – Mathematics – 2
 (2019 Admission onwards)

Time: 2 hours

Max. Marks : 60

Section A

A maximum of 20 marks can be earned from this section.
 Each question carries 2 marks.

- Find the Cartesian equation of $r = \frac{4}{2\cos\theta - \sin\theta}$.
- Evaluate $\int \frac{4dx}{(e^x + e^{-x})^2}$.
- Prove that $\operatorname{sech}^{-1} x = \cosh^{-1} \frac{1}{x}$.
- Show that $\sum_{i=1}^{\infty} \frac{i+1}{i}$ diverges.
- If $f(x) = \frac{1}{1-x}$, $|x| < 1$, find series for $f'(x)$.
- Define absolute convergence and conditional convergence of a series.
- Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$.
- Define vector space.
- Show that the determinant of every orthogonal matrix is ± 1 .
- Define eigen value and eigen vector of a matrix.
- Find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ with the help of Cayley Hamilton theorem.
- Define Linear span of a nonempty subset S of a vector space V.

Section B

A maximum of 30 marks can be earned from this section.
 Each question carries 5 marks.

- Graph the curve $r^2 = \sin 2\theta$.
- Find the length of the graph of the function $f(x) = \frac{x^4}{8} + \frac{1}{4x^2}$ on $[1, 3]$.
- Show that $\int_a^{\infty} \frac{1}{x^p} dx$ (p is a constant and $a > 0$) converges if $p > 1$ and diverges if $p \leq 1$.
- Use Newton's method to find a solution of $x^3 - 8x^2 + 2x + 1 = 0$.
- Check whether the set $\{1, 1+x, (1+x)^2, x^2\}$, a subset of P_2 , is linearly independent or not.
- Show that $\{(0,1,1), (1,1,0), (1,0,1)\}$ is a basis of \mathbb{R}^3 .
- Solve the system of equations $2x - y + z = 7$; $3x + y - 5z = 13$; $x + y + z = 5$ using Gaussian elimination Method.

Section C

Answer any One question. Each question carries 10 marks

20. Diagonalize the matrix $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

21..

- a. Find the length of the curve $r = 1 - \cos\theta, 0 \leq \theta \leq 2\pi$.
- b. Find the area shared by the circles $r = a\sqrt{2}$ and $r = 2a \cos\theta$.

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
 Second Semester BSc Degree Examination, March/April 2020
 BMT2B02 – Calculus – 2
 (2019 Admission onwards)

Time: 2 ½ hours

Max. Marks : 80

Section A (Short Answer type)
Ceiling (Maximum marks)– 25Marks
Each question carries 2 marks

1. Find the area of the region between the graph of $y = x^2 + 2$ and $y = x - 1$ and the vertical lines $x = -1$ and $x = 2$
2. Draw the graph of the natural logarithmic function $y = \ln x$.
3. Find the derivative of $\ln(2x^2 + 1)$.
4. Solve $e^{2-3x} = 6$.
5. Find $\int \frac{\sqrt{\ln x}}{x} dx$.
6. Evaluate $\int_0^3 2^x dx$.
7. Find the derivative of $y = \cos^{-1} 3x$.
8. Evaluate $\sin(\sin^{-1} 0.7)$.
9. Prove the identity $\cosh^2 x - \sinh^2 x = 1$
10. Define Improper Integrals.
11. State Monotone Convergence Theorem for Sequences.
12. Define the term partial sum and when a series $\sum_{n=1}^{\infty} a_n$ is said to be convergent.
13. Find the radius of convergence and interval of convergence of $\sum_{n=0}^{\infty} n! x^n$.
14. Find the Maclaurin series of $f(x) = \sin x$.
15. What is a Taylor series?

(Ceiling =25Marks)

Section B (Paragraph type)
Ceiling (Maximum marks) – 35Marks
Each question carries 5 marks

16. Find the length of the graph $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ on the interval $[1, 3]$.
17. Find the volume of the solid obtained by revolving the region bounded by the graphs of $y = x^3$, $y = 8$, and $x = 0$ about the y axis.
18. Using l'Hopital rule, evaluate $\lim_{x \rightarrow 0} \frac{x^3}{x - \tan x}$.
19. Find $\int \frac{1}{x\sqrt{x^2-16}} dx$.

20. Find the values of p for which $\int_1^{\infty} \frac{1}{x^p} dx$ is convergent.
21. a) State the integral Test for the convergence or divergence of the series $\sum_{n=1}^{\infty} a_n$.
 b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converge or diverge.
22. Prove that every absolutely convergent series is convergent.
23. Determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent or divergent.

(Ceiling =35Marks)

Section C (Essay type)

Answer any two questions

Each question carries 10 marks

- 24.
- a) Find the area of the surface obtained by revolving the graph of the function $f(x) = \sqrt{x}$ on the interval $[0, 2]$ about the x -axis.
- b) Show that $\sinh^{-1}x = \ln(x + \sqrt{x^2 + 1})$.
25. Using l'Hopital rule
- a) Evaluate $\lim_{x \rightarrow 0^+} \left(\frac{1}{x}\right)^{\sin x}$.
- b) Evaluate $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.
- 26.
- a) State the limit Comparison Test.
- b) Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2 + n}{\sqrt{4n^2 + 3}}$ converges or diverges.
- 27.
- a) State and prove the alternating series Test.
- b) Show that the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is convergent.

(2 x 10 =20 Marks)