

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Sixth Semester B.Sc Degree Examination, March/April 2020

BSTA6B10 – Mathematical Methods in Statistics II

(2017 Admission onwards)

Time: 3 hours

Max. Marks:120

Part A

(Answer all questions; each question carries 1 mark)

Multiple Choice Questions (Questions 1-6)

- The function $f(z) = e^x(\cos x + i \sin y)$ is
 - Analytic in the finite plane
 - analytic $z = \pi$
 - not analytic at $z = \frac{\pi}{2}$
 - None of these
- If $C: |z - 2| = 3$ (positively oriented) $\int_C \frac{1}{z-1} dz$ is
 - 2π
 - $-2\pi i$
 - $-\pi$
 - $2\pi i$
- The value of $\int_C \bar{z}^2 dz$ where C is the upper half of the circle $|z|=1$
 - $-2\pi i$
 - 0
 - $-\frac{1}{2}$
 - $-\pi i$
- The value of $\int_C z^2 dz$ where $C: |z-2|=1$ is
 - 4
 - 2
 - 1
 - 0
- $f(z) = \sum_{n=1}^{\infty} \frac{\pi}{2^n} z^n$ has radius of convergence :
 - 0
 - 1
 - 2
 - None of these

Fill in the blanks (Questions 7-12)

- The analytic function $f(z)$ with real part $u = x^3 - 3xy^2$ is
- Converse of the Cauchy's integral theorem is known as
- A singular point of the function $\frac{\sin z}{z}$ is
- The singularity of the function $\frac{\sin z}{z}$ at $Z=0$ is
- If $z = a$ is a pole of the function then $\lim_{n \rightarrow a} f(z)$

(10x1=10 Marks)

Part B

(Answer any eight questions; each question carries 5 marks)

11. Obtain harmonic conjugate of $u(x, y) = x^2 - y^2$.
12. Define analytic function also verify Cauchy- Riemann equation for Z^2 .
13. Find the Laurent series expansion for $f(z) = \frac{1}{(z-1)(z-3)}$ in the powers of z and specify the region in which the expansion is valid.
14. Derive the necessary and sufficient condition for a function to be analytic.
15. Show whether $\frac{1}{z}$ and $\frac{1}{(z-2)^2}$ is analytic at $z = 0$.
16. Define a) Pole b) removable singularity 3) essential singularity. Also give one example for each.
17. State Fourier integral theorem and write any one form of Fourier integral formula.
18. Obtain Taylor series expansion of $\frac{1}{z}$ about $z = 1$ and $z = 2$.
19. Distinguish between the terms residue at pole and residue at infinity with examples.
20. Prove that a function $f(z) = u(x, y) + iv(x, y)$ defined in a region D is analytic in it if and only if $u(x, y)$ and $v(x, y)$ are harmonic conjugate functions.
21. Evaluate $\int_{c_1} z^2 dz$, where c_1 is the line joining the points 0 and $1 + i$
22. State and prove Cauchy's residue theorem.

(8x5=40 Marks)

Part C

(Answer any four questions; each question carries 10 marks)

23. Find the Taylor's series to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| < 2$.
24. State the nature of singularity of $f(z)$ at $z = 0$ in the following cases
 - i) $\frac{e^z-1}{z}$
 - ii) $\frac{\sin z}{z}$
25. Evaluate $\int_C \frac{1}{z^2-1} dz$, along C , the positively oriented circle $c: |z|=2$.
26. State and prove Cauchy's inequality.
27. If $f(z)$ is a regular function of z , prove that
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2.$$
28. State and prove Cauchy's integral formula.

29. Evaluate $\int_c \frac{dz}{z(z-2)^4}$, where $c: |z-2|=1$ oriented in the counter clockwise direction..
30. If $f(z)$ is an analytic function in a domain D , then prove that $f(z)$ must be constant in D if $\operatorname{Re} f(z)$ is constant in D

(4x10=40 Marks)

Part D

(Answer any two questions; each question carries 15 marks)

31. Show that $\int_0^{2\pi} \frac{d\theta}{1+a \sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}$, $-1 < a < 1$.
32. Find the Laurent's series expansion for $\frac{3-2z}{z^2-3z+2}$ and specify the region in which the expansion is valid.
33. State and prove Poisson integral formula.
34. Evaluate $\int_0^\infty \frac{\cos mx}{x^2+a^2} dx$, $m > 0, a > 0$.

(2x15=30 Marks)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
Sixth Semester B.Sc Degree Examination, March/April 2020
BSTA6B11 – Design of Experiments
(2017 Admission onwards)

Time: 3 hours

Max. Marks:120

Part A**Answer All Questions. Each carries 1 mark****Questions 1 to 5 state true or false**

1. $H : \beta_1 = \beta_2 = \beta_3 * \beta_4$ is an example of linear hypothesis.
2. LSD is a double grouped design
3. F and $\frac{1}{F}$ has identical distribution always.
4. Experimental errors are assumed to be identically and independently distributed random variables
5. In factorial experiments, each treatment combination will have one degree of freedom.
6. State degrees of freedom of error sum of squares of a LSD with side t .
7. State all possible interactions of a 2^2 design.
8. Give examples of experimental units.
9. Define model of one way ANCOVA.
10. Define orthogonal contrast. (10 x 1 = 10 marks)

Part B**Answer Any 8 Questions. Each carries 5 marks**

11. Define ANOVA. State assumptions of ANOVA.
12. State Standard Gauss Markov Set up. Define BLUE.
13. State ANOVA table of a CRD.
14. Derive Critical Difference for treatment pairs of a LSD.
15. State Merits and demerits of RBD.
16. Briefly explain Duncans Multiple Range Test.
17. Complete ANOVA table of RBD given below

Source	Sum of Squares	Degrees of freedom	Mean Sum of Squares
Blocks	26.8	4	----
Treatments	-----	3	----
Error	-----	----	2.5
Total	85.3	-----	

18. Explain how Replication is useful in increasing efficiency of designs?

19. Distinguish between Main and Interaction effects in Factorial Design.
20. Explain Missing Plot technique used in design of experiments.
21. Define efficiency and relative efficiency of designs.
22. State necessary and sufficient condition for estimability of a linear parametric function..

(8 x 5 = 40 marks)

Part C

Answer Any 4 Questions. Each carries 10 marks

23. Explain the Principles 1) Randomisation 2) Blocking used in experimental design.
24. The following data gives mileage of tyres (1000 kms) of 4 different brands of tyres tested on 3 different makes of vehicles.

Tyre↓ Vehicle→	I	II	III
A	42	45	43
B	37	39	41
C	47	46	48
D	35	37	38

Analyse the design and state conclusions.

25. Explain Statistical Analysis of CRD.
26. Obtain the least square estimates of two missing observation of a block in RBD.
27. Derive efficiency of LSD compared to CRD.
28. Explain factorial experiments.
29. Explain Yates method of computing totals of treatment combinations in 2^3 designs.
30. Compare ANOVA and ANCOVA.

(4 x 10 = 40 Mark

Part D

Answer Any 2 Questions. Each carries 15 marks

31. State and prove Gauss Markov Theorem.
32. Explain Statistical Analysis of RBD deriving various sums of squares.
33. Estimate the missing observation of LSD given below and carryout ANOVA

A 75	B 92	C70	D64
B101	C86	D75	A 79
C71	D69	A 67	B84
D80	A 80	B*****	C75

34. Analyse the following 2^3 design

Block 1	N 38	P 42	K 31	NP 38	NK 37	PK 30	NPK 47	1 23
Block 2	N 42	P 37	K 33	NP 44	NK 41	PK 32	NPK 53	1 26
Block 3	N 44	P 40	K 39	NP 52	NK 45	PK 37	NPK 60	1 30

(15 x 2 = 30 Mark

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
Sixth Semester B.Sc Degree Examination, March/April 2020
BSTA6B12 – Official Statistics
(2017 Admission onwards)

Time: 3 hours

Max. Marks:120

Part A

Answer all the questions. Each question carries 1 mark.

1. What are the major divisions of NSSO?
2. Mention any two uses of census data?
3. What is whole sale price index?
4. What is Fisher's Ideal Index?
5. What is meant by time reversal test?
6. What is chain base index number?
7. What are the uses of vital statistics?
8. Define total fertility rate.
9. Explain Neo natal mortality rate.
10. Name any three methods of construction of abridged life tables?

(10x1=10 Marks)

Part B

Answer any eight questions. Each question carries 5 marks.

11. Explain the statistical system in Kerala?
12. Explain the uses and limitations of index numbers.
13. During a certain period the cost of living index number goes up from 110 to 200 and the salary of the worker is also increased from Rs.325 to Rs.550. Does the worker really gain, and if so, by how much in real terms?
14. Calculate the index number by simple aggregate method for the following data.

Commodities	Price 1980 (Rs.)	Price 1981 (Rs.)
A	162	170
B	240	180
C	250	200
D	120	160
E	165	200

15. Calculate cost of living index number using Family Budget method from the following data:

Items	Weights	Price in Base Year	Price in Current year
Food	4	30	47
House Rent	2	22	15
Clothing	3	14	18
Fuel	1	8	12
Others	1	25	30

16. The following are the index numbers of prices based on 1997 prices. Shift the base from 1997 to 2001.

Year	1997	1998	1999	2000	2001	2002	2003
Index Number	100	140	260	340	400	450	500

17. What purpose does cost of living index numbers serve? Explain.
 18. Explain Marshall Edge worth index numbers.
 19. Explain the registration method of obtaining vital statistics.
 20. Explain infant mortality rate and maternal mortality rate.
 21. Explain the importance and uses of life table.
 22. What are the principle methods of construction of abridged life tables?

(8x5=40 Marks)

Part C

Answer any four questions. Each question carries 10 marks.

23. Explain the important steps involved in population census in India.
 24. What is meant by deflating the index numbers and how can it be done?
 25. Explain Fisher's index number method of constructing an index number. Show that Fisher's ideal satisfy all the reversal tests for a good index number.
 26. Give the advantages and limitations of chain base indices.
 27. Discuss Paasche's price and quantity index numbers. Also give your choice between Laspeyre's and Paasche's formulae along with your comments.
 28. Discuss gross reproduction rate and net reproduction rate. Comment on the values of net reproduction rate.

29. How can one represent and determine various columns of an abridged life-table? In what way, does the construction of an abridged life table differ from a complete life table?
30. Explain (a) Force of mortality, and (b) Age specific fertility rate.

(4 x 10 = 40 Marks)

Part D

Answer any two questions. Each question has 15 marks.

31. Explain the various sources of vital statistics in India.
32. Construct index numbers of price from the following data by applying:
Laspeyre's method, Paasche's method, Bowley's method, Fisher's Ideal method and Marshall-Edgeworth method.

Commodities	1999		2000	
	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

33. Assume that the proportion of female births is 46.2 per cent, then compute general fertility rate and gross reproduction rate from the data given below:

Age group of child bearing females	Number of women ('000)	Total births
15-19	16.0	260
20-24	16.4	2244
25-29	15.8	1894
30-34	15.2	1320
35-39	14.8	916
40-44	15.0	280
45-49	14.5	145

34. In what manner are the central mortality rate and subsequently the death rate under Kings method for preparing an abridged life table used.

(2 x 15 = 30 Marks)

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
Sixth Semester B.Sc Degree Examination, March/April 2020
BSTA6E(03) – Reliability Theory
(2017 Admission onwards)

Time: 3 hours

Max. Marks:120

Part A**(Answer all questions; each question carries 1 mark)****Multiple Choice Questions (Questions 1-5)**

1. The structure function of *parallel system* is
 (a) $\phi(x) = \max(x_1, x_2, \dots, x_n)$ b) $\phi(x) = \min(x_1, x_2, \dots, x_n)$
 c) $\phi(x) = (x_1 + x_2 + \dots + x_n)$ d) $\phi(x) = (x_1 + x_n)$
2. Reliability of two component series system is
 (a) p^2 b) $1 - p^2$ c) $p(1 - p)$ d) None of the above
3. The dual of 2-out of-4 structure is
 (a) 2-out of-4 structure b) 1 - out of -4 structure
 c) 3 - out of -4 structure d) Parallel structure of 4 components
4. Reliability of a system can be defined as
 (a) $E[\phi(x)]$ b) $1 - E[\phi(x)]$ c) $E[\phi(x)] - 1$ d) $E[\phi^2(x)]$
5. If $\mu = 1$, then the failure rate of $\exp(\mu)$ is
 (a) 0 b) 1 c) E d) e^{-1}

Fill in the blanks (Questions 6-10)

6. Reliability of a 2- component series system is _____
7. 3-out of -3 system is a _____ system.
8. The distribution having mean and variance equal is _____
9. The number of components in a system is called _____ of the system.
10. $E(\phi(x))$ represents _____ of a system. **(10 x 1 = 10 Marks)**

Part B**(Answer any eight questions; each question carries 5 marks)**

11. Define state of a system. Represent a 2-out-of-3 structure.
12. What do you mean by dual of a structure ϕ .
13. Define a bridge structure with an example.
14. Define coherent system. Give an example.
15. Explain any one method of computing exact system reliability.
16. What do you mean by pivotal decomposition of a structural function?

17. What do you mean by relative importance of components?
18. Give examples of a series system and a parallel system.
19. Discuss the role of Poisson distribution in reliability theory.
20. Show that the hazard function uniquely determines the reliability function.
21. Distinguish between Type I and Type II censoring.
22. Define Exponential distribution. State its mean and variance

(8 x 5 = 40 Marks)

Part C

(Answer any four questions; each question carries 10 marks)

23. When do you say that a component is irrelevant to the structure ϕ . Give *one* example.
24. Derive the reliability of k -out-of- n structure.
25. Let $\phi(\underline{x})$ be the structure function of a coherent system of order n . Then show that

$$\prod_{i=1}^n x_i \leq \phi(\underline{x}) \leq \prod_{i=1}^n x_i .$$

26. Show that the dual of a k -out-of- n structure is $n-k+1$ -out-of- n structure.
27. With usual notation show that for a structure function ϕ order n ,
 $\phi(\mathbf{x}) = x_i \phi(\mathbf{1}_i, \mathbf{x}) + (1 - x_i) \phi(\mathbf{0}_i, \mathbf{x})$ for all \mathbf{x} ($i=1, 2, \dots, n$)
28. Define hazard rate. Find hazard rate of exponential distribution.
29. Derive the relationship between cumulative hazard rate and reliability function.
30. State and prove memory less property of exponential distribution.

(4 x 10 = 40 Marks)

Part D

(Answer any two questions; each question carries 15 marks)

31. Explain Inclusion- Exclusion principle for finding system reliability.
32. Let ϕ be a coherent structure. Then show that
 - (i) $\phi(\underline{x} \vee \underline{y}) \geq \phi(\underline{x}) \vee \phi(\underline{y})$
 - (ii) $\phi(\underline{x} \cdot \underline{y}) \leq \phi(\underline{x}) \cdot \phi(\underline{y})$
33. Define IFR and IFR classes of Distributions. Illustrate with example.
34. Let $h(\mathbf{p})$ be the reliability function of a coherent structure. Show that $h(\mathbf{p})$ is strictly increasing in each p_i for $\mathbf{0} \ll \mathbf{p} \ll \mathbf{1}$.

(2 x 15 = 30 Marks)