2B5N20301

38

Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2020

BMAT5B05 – Vector Calculus

(2018 Admission onwards)

Time: 3 hours

Max. Marks: 120

Part A

Answer ALL questions (1 - 12). Each question carries 1 mark.

1. Evaluate $\lim_{(x,y)\to(1,1)} \frac{3x^2-xy-2y^2}{x-y}$.
2. Find the critical point of $x^2+3xy+3y^2-6x+3y-6$.
3. If a vector function $\vec{r}(t)$ has constant magnitude, prove that $\vec{r}\cdot\frac{d\vec{r}}{dt}=0$.
4. Find the rate of change of $f(x,y)=x^3+2y^2+4x+3y+5$ in the direction of $\hat{j}$.
5. Find the divergence of the vector field $(x^2+y-z)\hat{i}+(2x+y^3-1)\hat{j}+(x+y+z^2)k$.
6. Write the normal form of Green's theorem on a plane.
7. State the Fubini's theorem (strong form).
8. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\, dxdydz$.
9. If $u=x-2y$ and $v=2x+5y$, then write the relation between $dxdy$ and $dudv$.
10. Write the double integral of $x+y$ over the triangular region bounded by the co-ordinate axes and the line $x+y=1$.
11. Write the parametrisation of the cylinder $(x-2)^2+y^2=4$.
12. State the divergence theorem.

Part B

Answer ANY TEN from the FOURTEEN questions (13 - 26). Each question carries 4 marks.

13. Test the existence of $\lim_{(x,y)\to(0,0)} \frac{3x^2+5y^2}{4x-y}$
14. If a vector function $\vec{r}(t)$ has constant direction, prove that $\vec{r}\times\frac{d\vec{r}}{dt}=\vec{0}$.
15. If $f(x,y)=y^2+xy+3x+1$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(1,2)$.
16. Find linearisation of x^3+y^2 at $(1,1)$.
17. Find $\frac{dw}{dt}$ if $w=xy^2+x^2+3y$, $x=t^2$ and $y=\sin t$ at $t=1$.
18. Locate the critical points and find the local extreme values of $f(x,y)=xy-x^2-y^2-2x-2y+4$.

19. Find the area of the ellipse $9x^2+16y^2=144$ using the method of double integral.

20. Evaluate $\int_0^{\sqrt{2}}\int_0^{3y}\int_{x^2+3y^2}^{8-x^2-y^2} dz\,dx\,dy$.

21. Rewrite $\int_0^1\int_{-1}^0\int_0^{y^2} dz\,dy\,dx$ as an equivalent triple integral in the order $dy\,dz\,dx$.

22. Show that $\mathbf{F}=(e^x\cos y)\hat{i}+(xz-e^x\sin y)\hat{j}+(xy+z)\hat{k}$ is a conservative vector field.

23. Show that the differential form in the integral $\int_{(1,1,2)}^{(3,5,0)}(yz\,dx+xz\,dy+xy\,dz)$ is exact. Evaluate the integral.

24. Find the line integral of $x+3y-2z$ along the line segment from $(1,3,2)$ to $(2,5,5)$.

25. Write the formula for surface integral. Explain all terms used in it.

26. For a scalar field $f(x,y,z)$, prove that $Curl\ grad\ f=\vec{0}$.

Part C

Answer ANY SIX from the NINE questions (27 - 35). Each question carries 7 marks.

27. Find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ at $u=1$ and $v=-2$ when $z=\ln q$ and $q=\sqrt{v+3}\tan^{-1}u$.

28. Find equations of tangent plane and normal at $(2,3,2)$ on the surface $x^3-y^3+z^4+3=0$.

29. Find the average value of $xy(x+y)$ over the region bounded by $x^2=y$ and $y=x$.

30. Write an equivalent polar integral of $\int_{-1}^0\int_{-\sqrt{1-x^2}}^0 \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$ and evaluate it.

31. Find the volume of the region bounded by the surfaces $z=x^2+3y^2$ and $z=8-x^2-y^2$.

32. Use $u=\frac{2x-y}{2}$ and $v=\frac{y}{2}$ to evaluate the integral $\int_0^4\int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1}\frac{2x-y}{2}\,dx\,dy$.

33. Find the flux of the vector field $(x+y)\hat{i}-(x^2+y^2)\hat{j}$ outward across the triangle with vertices $(1,0)$, $(0,1)$ and $(-1,0)$.

34. Find the area of the surface cut from the bottom of the paraboloid $x^2+y^2-z=0$ by the plane $z=4$.

35. Parametrise the sphere $x^2+y^2+z^2=9$ and hence find its surface area.

Part D

Answer ANY TWO from the THREE questions (36 - 38). Each question carries 13 marks.

36. Using the method of Lagrange multipliers minimise $f(x,y,z)=x^2+y^2+z^2$ subject to the constraints $x^2-xy+y^2-z^2-1=0$ and $4x^2+y^2-1=0$.

37. a) Test for exactness of the differential form $\cos x\tan z\,dx-\sin y\tan z\,dy+(\sin x+\cos y)\sec^2 z\,dz$. 4 Marks

    b) Evaluate the line integral of $\cos x\tan z\,\hat{i}-\sin y\tan z\,\hat{j}+(\sin x+\cos y)\sec^2 z\,\hat{k}$ along the arc of an ellipse having eccentricity 0.5 joining the points $(0,0,0)$ to $(1,1,1)$. 9 Marks
    Hint : You can use a suitable theorem for an easy evaluation of the line integral.

38. Verify Stoke's theorem for $\vec{F}=y\hat{i}-x\hat{j}$ for the hemisphere $x^2+y^2+z^2=4$, $z\geq 0$ bounded by the circle $C\ :\ x^2+y^2=4,\ z=0$.

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2B5N20302 (Pages : 2) Reg. No:.................................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2020

BMAT5B06 – Abstract Algebra

(2018 Admission onwards)

Time: 3 hours Max. Marks: 120

Section A

Answer all the twelve questions

Each question carries 1 mark.

1. Is the binary operation* on Z^+ defined by $a * b = a^b$, commutative? Justify.
2. Find orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$
3. Give an example for a cycle.
4. Give an example for an even permutation.
5. Find all cosets of the subgroup $H = \{0,3\}$ of a group Z_6.
6. Find the $Ker\ \varphi$ if $\varphi: Z_2 \times Z_2 \quad \to Z_2, given\ by\ \varphi(a) = 0.$
7. Define a ring homomorphism
8. Find a subgroup of Z_{30} generated by 25.
9. Give an example for a normal subgroup H of any group G.
10. Give an example of a non commutative division ring.
11. Find the remainder when -32 is divided by 5.
12. Give an example for an integral domain that is not afield.

(12×1=12 marks)

Section B

Answer any* ten *out of fourteen questions.

Each question carries 4 marks.

13. Show that $(Q^+,*)$ is a group, where * is defined by $a * b = \frac{ab}{2}$.
14. Find all subgroups of S_3, draw the subgroup diagram.
15. State and Prove Lagranges theorem.
16. Show that every group of prime order is cyclic.
17. Show that every cyclic group is abelian? Is the converse is true? Justify.
18. Let S_n be the symmetric group of n letters. Show that $\varphi: S_n \to Z_2$ defined by $\varphi(\sigma) = \begin{cases} 0\ if\ \ \sigma\ is\ even\ permutation \\ 1\ if\ \sigma\ is\ odd\ permutation \end{cases}$ is a homomorphism.
19. Is $\varphi: Z \to 2Z$ defined by $\varphi(x) = 2x, for\ x \in Z$, a ring homomorphism, Justify?
20. Show that $charecteristic\ of\ R\ is$ zero if $n.1 \neq 0\ for\ all\ n \in Z^+$, and $charecteristic\ of\ R$ is n if $n.1 = 0, for\ some\ n \in Z^+$, where n is the smallest such positive integer.

21. Let H be a subgroup of a group G. Show that the relation $\sim_L$ on G defined by $a \sim_L b$ if and only if $a^{-1}b \in H$ is an equivalence relation on G.

22. State and prove the necessary and sufficient condition that the group homomorphism $\varphi: G \rightarrow G'$ is one-to-one.

23. Exhibit all left cosets of the subgroup $\{\rho_0, \mu_2\}$ of the dihedral group D_4.

24. State and prove the necessary and sufficient condition to hold cancellation law in a ring R.

25. Define units in a ring R. Describe all units in the ring $\mathbb{Z} \times \mathbb{Z}$.

26. If p is a prime, show that Z_p has no divisors of 0.

(10×4=40 marks)

Section C

Answer any six out of nine questions
Each question carries 7 Marks

27. Let G be a group and $a \in G$, Show that $H = \{a^n : n \in Z\}$ is the smallest subgroup of G containing a.

28. Show that every infinite cyclic group is isomorphic to the group $(\mathbb{Z}, +)$.

29. Find all subgroups of D_4. Draw its subgroup diagram.

30. Let A be a nonempty set, and let S_A be the collection of all permutations of A. Then show that S_A is a group under permutation multiplication.

31. Show that a non empty set H of a group G is a subgroup of G if and only if $ab^{-1} \in H$ for all a, b $\in$ H.

32. Show that $M_2(R)$, the set of all 2×2 matrices with real entries is a ring under matrix addition and matrix multiplication.

33. Show that in a ring Z_n, the devisors of zero are precisely those non zero elements that are not relatively prime to n.

34. Show that $if \gcd(r, s) = 1 \; then \; Z_{rs} \cong Z_r \times Z_s$

35. Define zero divisors in a ring $(\mathbb{R}, +, .)$. Find all zero divisors in the ring $(\mathbb{Z}_{12}, +_{12}, \times_{12})$.

(6×7=42 marks)

Section D

Answer any two out of three questions
Each question carries 13 Marks

36. Let G be a cyclic group with n elements generated by a. Let $b \in G$ and $b = a^s$ then show that b generates a cyclic subgroup H of G containing $\frac{n}{d}$ elements, where $d = \gcd(n, s)$

37. Let $\varphi : G \rightarrow G'$ be a group homomorphism and let $H = Ker\, \varphi$ Let $a \in G$. Then prove that the set $\varphi^{-1}[\{\varphi(a)\}] = \{x \in G : \varphi(x) = \varphi(a)\}$ is the left coset aH of H and is also right coset Ha of H.

38. (a) Show that every field is an integral domain.

(b) Is the converse true always? Justify your claim. If not, is it true in any case? Justify.

(2×13=26 marks)

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2B5N20303 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2020

BMAT5B07– Basic Mathematical Analysis

(2018 Admission onwards)

Time: 3 hours Max. Marks: 120

SECTION A

Answer all the twelve questions.

Each question carries 1 mark .

1. Find the $\bigcap_{n=1}^{\infty} I_n$, where $I_n = (\,0\,,\,\frac{1}{n}\,)$
2. Define a countable set.
3. State completeness property of $\mathbb{R}$.
4. Give an example of a Cauchy sequence.
5. State Nested interval property of $\mathbb{R}$.
6. Give an example of a subset of $\mathbb{R}$,which is bounded below but not bounded above.
7. State Archimedean property .
8. State Squeeze theorem for limit of sequences.
9. If $c > 0$, then lim $c^{1/n}$ = ------------.
10. Find lim $\frac{3n+2}{n+1}$.
11. Is every open interval an open set?
12. Find $Im\left(\frac{3+4i}{7-i}\right)$.

(12x1=12 marks)

SECTION B

Answer any ten out of fourteen questions.

Each question carries 4 marks.

13. For any three sets A,B and C , prove that A\ (B∩C) = (A\ B) ∪ (A\ C).
14. Prove that the inequality $2^n > 2n + 1$ holds for all $n \geq 3,\ n \in N$.
15. Prove: If $a \in \mathbb{R},\ b \in \mathbb{R}$ *is* such that $ab > 0$ then, either

    (i) $a>0$ and $b>0$, or
    (ii) $a<0$ and $b<0$.
16. Prove or disprove : Arbitrary union of closed sets is closed .
17. Define Cantor set.
18. Prove that the set of all integers $\mathbb{Z}$ is denumerable.
19. State and prove the arithmetic-geometric mean inequality .
20. Determine the set of all $x \in \mathbb{R}$ such that $|x - 1| < |x|$.
21. Prove that if $a > 0$, then lim $\frac{1}{1+na} = 0$.

22. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}$. Find $inf\ S$ and $\sup S$.

23. If $t > 0$, there exists $n_t \in N$ such that $0 < \frac{1}{n_t} < t$.

24. Prove that if a sequence(x_n)converges to x , then the sequence $(|x_n|)$ converges to $|x|$.

25. If Z_1 and Z_2 are any two complex numbers , then prove that $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$.

26. Find the principal argument of $1 - i\sqrt{3}$

(10x4=40 marks)

SECTION C

Answer any six out of nine questions.
Each question carries 7 marks.

27. Let $g(x) = x^2, and\ f(x) = x + 2\ for\ x \in \mathbb{R}$. Then

 (a) Find $h = g \circ f$.

 (b) Find the direct image $h(E)$, where $E = \{x \in \mathbb{R}: 0 \leq x \leq 1\}$.

 (c) Find the inverse image $h^{-1}(G)$, where $G = \{x \in \mathbb{R}: 0 \leq x \leq 4\}$.

28. Prove that $\sqrt{2}$ is irrational.

29. State and prove the Bernoulli's Inequality.

30. If x and y are any real numbers with $x < y$, then prove that there exists a rational number $r \in Q$ such that $x < r < y$.

31. Prove that [0,1] is not countable.

32. If (x_n) and (y_n) are sequence of real numbers with $\lim\ x_n = x$ and $\lim\ y_n = y$, prove that

 (a)$\lim\ (x_n + y_n) = x + y$.

 (b) $\lim\ (x_n y_n) = xy$.

33. If $X = (x_n)$ is a sequence of real numbers , then there is a subsequence of X that is monotone.

34. Find all values of $(-1)^{\frac{1}{3}}$.

35. Prove that the equation $|z - 1| = |z + i|$ represents a line through the origin whose slope is -1.

(6x7=42 marks)

SECTION D

Answer any two out of three questions.
Each question carries 13 marks.

36. (a)State and Prove the Characterization theorem for intervals.

 (b)Prove that a bounded sequence of real numbers has a convergent subsequence.

37. State and prove Monotone convergence theorem.

38. Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence.

(2x13=26 marks)

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2B5N20304 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2020

BMAT5B08 - Differential Equations

(2018 Admission onwards)

Time: 3 hours Max. Marks: 120

PART A

Answer *ALL* the questions

(Each question carries 1 mark)

1. True or false? $y = 3t + t^2$ is a solution of the differential equation $ty' - y = t^2$.
2. The value of b for which the equation $(xy^2 + bx^2 y)dx + (x + y)x^2 dy = 0$ is exact, is......
3. An integrating factor for the equation $\frac{dy}{dt} - y = e^{2t}$ is...........
4. Write the standard form of a second order linear non-homogenous differential equation.
5. Wronskian of $y_1 = e^{-2t}, y_2 = te^{-2t}$ is............
6. State Abel's theorem.
7. General solution of $y'' + 4y = 0$ is..............
8. Laplace transform of $\{6\sin 2t - 5\cos 2t\}$ is........
9. $L^{-1}\left[\frac{2\pi}{S+\pi}\right] =$ ....
10. Laplace transform of the unit step function is……
11. Give an example for a periodic function having no primitive period.
12. When do the Fourier expansion of a function contain only the cosine terms?

(12x 1=12Marks)

PART B

Answer any TEN questions

(Each question carries *4* marks)

13. Determine the values of r for which the differential equation $y'' + y' - 6y = 0$ has solutions of the form $y = e^{rt}$.
14. Solve the differential equation $\frac{dy}{dx} - 2y = 4 - x$
15. State some differences between linear and non-linear differential equations.
16. Check the exactness and solve the differential equation $(2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$
17. Determine the longest interval in which the solution of the initial value problem $(t-1)y'' - 3ty' + 4y = \sin t,\ y(-2) = 2, y'(-2) = 1$, is certain to exist.
18. Show that the Wronskian of the fundamental solutions of $y'' - 6y' + 9y = 0$ is actually nonzero.
19. Find the general solution of the IVP $16y'' - 8y' + 145y = 0, y(0) = -2, y'(0) = 1$
20. Find the solution of the IVP $9y'' - 12y' + 4y = 0,\ y(0) = 2, y'(0) = -1$

21. Find a particular solution of $y'' - 3y' - 4y = 3e^t$.

22. Using the definition Laplace transform find $L[e^{-at}]$.

23. Find $L\left[\frac{\sin at}{t}\right]$.

24. Find $L^{-1}\left[\frac{s}{(S-1)^2 - 4}\right]$

25. Show that the product of two odd functions is an even function.

26. Show that $u = e^x \cos y$ is a solution of the two dimensional Laplace equation.

(10 x 4=40Marks)

PART C

Answer any SIX questions

(Each Question carries 7 marks)

27. Solve $(3xy + y^2) + (x^2 + xy)y' = 0$.

28. Find an integrating factor and solve the equation $(3x^2 y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$.

29. Solve the IVP $y' = x + y, y(0) = -1$, by Picard's iteration method (do three steps). Also find the exact solution.

30. Verify that the functions $y_1 = \cos 2t,\ y_2 = \sin 2t$ are solutions of the differential equation $y'' + 4y = 0$. Do they constitute a fundamental set of solutions?

31. Use the method of reduction of order to find a second solution of the differential equation $t^2 y'' - t(t+2)y' + (t+2)y = 0,\ t > 0,\ y_1(t) = t$.

32. Find a particular solution y_p of $4y'' + 4y' + y = 3xe^x$

33. Using convolution property, find $L^{-1}\left[\frac{1}{(S^2 + a^2)^2}\right]$

34. Find the Fourier series for the function $f(x) = \begin{cases} -1, & when -2 \le x < 0 \\ 1, & when 0 \le x < 2 \end{cases}$ and $f(x+4) = f(x)$.

35. Obtain the half range cosine series of $f(x) = x$ when $0 < x < 2$

(6 x 7=42Marks)

PART D

Answer any TWO questions

(Each Question carries *13* marks)

36. Find the general solution of the differential equation $y'' - 4y' + 13y = 12e^{2t} \sin 3t$

37. Solve $y'' + 3y' + 2y = \delta(t-5) + u_{10}(t), y(0) = 0, y'(0) = 1/2$.

38. Find the Fourier series for the function $f(x) = \begin{cases} -k, & when -\pi < x < 0 \\ k, & when 0 < x < \pi \end{cases}$ and $f(x+2) = f(x)$ and hence deduce the Madhav-Gregory series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots\ldots$

(2 x 13 = 26 Marks)

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2B5N20305 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics(Open Course) Degree Examination, November 2020

BMAT5D01– Mathematics for Social Science

(2018 Admission onwards)

Time: 2 hours Max. Marks: 40

Section A

Answer* all *the six questions

Each question carries* 1 *mark

1. Complete the square of the expression $x^2 - 5x$.
2. Find the slope of the line passing through the points (3,11) and (6,2).
3. Find $\lim_{x\to 2}\sqrt{6x^2+1}$
4. Find the successive derivatives of the function $y = (5x-9)^3$.
5. Solve $\log_a 4 = \frac{2}{3}$.
6. Define concave upward and downward.

(6 x 1 = 6 marks)

Section B

Answer any* five *out of seven questions

Each question carries* 2 *marks.

7. Define the product and quotient rule of differentiation.
8. Prove that a polynomial function is continuous for any real value a of $x$given $f(x) = k_0x^n + k_1x^{n-1} + k_2x^{n-2} + \cdots + k_{n-1}x + k_n$
9. Find the critical value and determine whether the critical value is relative maximum or minimum for $f(x) = 3x^2 - 42x + 34$.
10. Find the vertical and horizontal asymptote of $y = \frac{x+2}{x-5}$.
11. Use the properties of logarithms, write $ln\sqrt{\frac{x^5}{y^3}}$.
12. If the total cost function $C(x) = 0.5\,x^2 + 1.5x + 8$, find the average cost at $x = 4$.
13. Find the anti derivative F(x) of $\int(4x^{-1} + 5x^{-2})dx$, given F(1) = 3.

(5 x 2 = 10 marks)

Section C

Answer any **three** *out of five questions*

Each question carries 4 marks.

14. A particle moved by an accelerator a distance w miles in t seconds given by the function $w(t) = 25t^2 + 45t$. Find (a) The average velocity between t= 1 and t = 3. (b) the average velocity for a small change in time starting at t = 1 (c) the instantaneous velocity at t = 1.
15. Find the vertex and axis of the parabola $y = x^2 - 8x + 19$ and then draw the parabola.
16. Determine the integral $\int 24x^3(x^4 + 9)dx$.
17. Find the cross partial derivatives of $f(x, y) = 5x^3y^2 - 10x^2y^4$.
18. Find the effective rate of interest for P = Rs. 500 at $r = 12\%$ when compounded (a) semi-annually (b) continuously.

(3 x 4 = 12 marks)

Section D

Answer any **two** *out of three questions*

Each question carries 6 marks.

19. A projectile shot straight in to the air has height in feet $S(t) = 288t - 16t^2$ after t seconds. Find (a) the velocity $V(t)$ at $t = 3$, (b) the acceleration $A(t)$ at $t = 5$,

    (c) the time t the object will hit the ground, and

    (d) the velocity with which it hits the ground.
20. Find the level of output at which profit π is maximized, given that the total revenue $R = 6400Q - 20Q^2$ and total cost $C = Q^3 - 5Q^2 + 400Q + 52,000$ assume Q > 0.
21. Find the equation for the line passing through $(-2, 5)$ and parallel to the line having the equation $y = 3x + 7$.

(2 x 6 = 12 marks)