30

1B4M20197 (Pages : 2) Reg. No:......................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester BSc Degree Examination, March/April 2020

BMAT4B04 – Theory of Equations, Matrices & Vector Calculus

(2018 Admission onwards)

Time: 3 hours Max. Marks : 80

PART- A

Answer *all* Questions. Each carries *one* mark

1. If $1+i$ is a root of $4x^4-8x^3+7x^2+2x-2=0$ without solving the equation completely, state the other root.
2. If α_1 α_2, α_3, ...are the roots of $f(x)=0$, then what is the equation whose roots are α_1 -2 ,α_2-2, α_3-2, ...
3. If α, β, γ are the roots of $x^3-px^2+qx-r=0$, find the value of $\Sigma\frac{1}{\alpha\beta}$
4. Define a reciprocal equation.
5. What is the rank of the identity matrix of order n?
6. Find the characteristic root of $\begin{bmatrix}1 & 3\\3 & 1\end{bmatrix}$
7. A system of m homogeneous linear equations AX=0 in n unknowns has only trivial solution if ....
8. For what value of a the system of equations $ax+2y+2=0, x+2y=3, 2x+3y=5$ are consistent?
9. Parametrize the line segment joining the points $(-3,2,-3)$ and $(1,-1,4)$
10. Find the angle between the planes $2x+2y+2z=3, 2x-2y-z=5$.
11. Find the unit tangent vector at the point (2,0,0) to the curve

$$r(t)=2\,cost\,\boldsymbol{i}+2\,sint\,\boldsymbol{j}+t\,\boldsymbol{k}$$

12. Write the equation relating rectangular and cylindrical coordinates.

(12 x1=12 marks)

PART B:Short answer Type.

Answer any *nine* questions. Each question carries *two* marks

13. Solve $x^3-6x^2+13x-10=0$,whose roots are in arithmetic progression.
14. Solve $x^3-4x^2-20x+48=0$ given that two of its roots α, β are connected by the relation $\alpha+2\beta=0$.
15. If α, β, γ are the roots of $2x^3+3x^2-x-1=0$, find the equation whose roots are α-1, β-1, γ-1.
16. Find the rank of the matrix $\begin{bmatrix}1 & 21 & 2\\1 & 32 & 2\end{bmatrix}$

17. If A = $\begin{bmatrix}1 & 5 & 2\\3 & 2 & 1\end{bmatrix}$ and B = $\begin{bmatrix}2 & 3\\1 & 4\\2 & 1\end{bmatrix}$, then rank of AB is:

18. State Sylvester's law of nullity.

19. If A is a non-singular matrix then prove that $rank(AA^T) = rank(A)$

20. Find the value of x for which $rank(A) = 3$ where $A = \begin{bmatrix}2 & 4 & 4\\3 & 1 & 2\\1 & 0 & x\end{bmatrix}$

21. Find the velocity and acceleration vectors of $\boldsymbol{r}(t) = (t^2+1)\boldsymbol{i} + (2t-1)\boldsymbol{j},\ t = \frac{1}{2}$

22. Evaluate $\int_0^{\pi}[cost\ \boldsymbol{i} + \boldsymbol{j} - 2t\ \boldsymbol{k}]\ dt$

23. Find the normalvectors for$\boldsymbol{r}(t) = (a\ cost)\boldsymbol{i} + (a\ sint)\boldsymbol{j}$

24. Find the length of one turn of the helix $\boldsymbol{r}(t) = (cost)\boldsymbol{i} + (sint)\boldsymbol{j}$+ t **k**

(9 x 2 = 18 mark

PART-C: Short Essay Type.

Answer any *six* questions. Each question carries *five* marks

25. If α, β, γ are the roots of $x^3 + qx + r = 0$, form the equation whose roots are $\alpha^3, \beta^3, \gamma^3$

26. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

27. Solve the equation $x^3 - 9x + 12 = 0$ by Cardan's method.

28. Solve the system of equations $x + 2y + 3z = 0,\ 2x + y + 3z = 0,\ 3x + 2y + z = 0$

29. For the matrix A find non singular matrices P and Q such that PAQ is in the normal form where $A = \begin{bmatrix}1 & 2 & 3 & -2\\2 & -2 & 1 & 3\\3 & 0 & 4 & 1\end{bmatrix}$

30. Find the distance of the point $P = (1,1,5)$ to the line $x = 1 + t, y = -2t, z = 2t$

31. Find the characteristic equation of $A = \begin{bmatrix}1 & 1 & 2\\0 & 2 & 2\\-1 & 1 & 3\end{bmatrix}$ and verify Cayley-Hamilton theorem.

32. Find the distance from (0, 0, 12) to the line $x = 4t, y = -2t, z = 2t.$

33. Find the curvature of $\boldsymbol{r}(t) = (a\ cost)\boldsymbol{i} + (a\ sint)\boldsymbol{j} + bt\ \boldsymbol{k}, a, b \geq 0, a^2 + b^2 \neq 0$

(6 x 5 = 30 marks)

PART- D: Essay Type Questions.

Answer any *two* questions. Each question carries *ten* marks

34. Solve $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$using Ferrari's method.

35. Solve the system of equations $x + 2y - 3z - 4w = 6, x + 3y + z - 2w = 4,$
$2x + 5y - 2z - 5w = 10.$

36. For the curve $\boldsymbol{r}(t) = (2 + 3t + 3t^2)\boldsymbol{i} + (4t + 4t^2)\boldsymbol{j} - 6\ cost\ \boldsymbol{k}$ write the acceleration a in the form **a**= a_T**T**+ a_N**N** at $t = 0.$

(2 x 10 = 20 marks)

31

1B4M20198(A) (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester BSc Chemistry Degree Examination, March/April 2020

BMAT4C04(CH) – Mathematics

(2018 Admission onwards)

Time: 3 hours Max. Marks : 80

Part A

Answer All Questions. Each question has One mark.

1. Find the Laplace transform of $f(t) = t^2 - 2t$.
2. Define the periodic function.
3. Determine whether the function $f(x) = |x|$ is even or odd.
4. Let $a = [2, -1, 0]$and $b = [-4, 2, 5]$. Find $a \times b$.
5. Find the parametric representation of the circle $x^2 + y^2 = 4, z = 0$.
6. Find the gradient field of $f(x, y) = x^2 + xy$.
7. Write the Two-dimensional Laplace equation.
8. Find the domain of the function $f(x, y) = \sqrt{y - x^2}$
9. State the Mixed Derivative Theorem.
10. Give an example of an infinite group which is not cyclic.
11. Identify all the generators of the group $< Z, + >$.
12. The order of identity element in any group G is ....

(12 x 1 = 12 Marks)

Part B

Answer Any Seven Questions. Each question hasTwo marks.

13. Find the inverse Laplace transform of $F(s) = \frac{7}{(s-1)^3}$.
14. Find a_0 in the Fourier series expansion of $f(x) = \begin{cases} -k & ; if -\pi < x < 0 \\ k & ; if\, 0 < x < \pi \end{cases}$
15. Find divF and curl **F**for the vector field $\mathbf{F}(x, y, z) = x^2\mathbf{i} - 2\mathbf{j} + yz\mathbf{k}$
16. Find the velocity and acceleration of $r(t) = [4t, -3t, 0]$.
17. Find the components of the vector **v** with initial point (3, 2, 0) and terminal point (5, -2, 0). Find$|\mathbf{v}|$.
18. Describe the level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
19. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = x \cos xy$.
20. On $\mathbb{Z}^+$, $*$ is defined by $a * b = a - b$. Determine whether $*$ is a binary operation.
21. Find the order of the subgroup of Z_{12} generated by 8.

(7 x 2 = 14 Marks)

Part C

Answer Any Six Questions. Each question hasFive marks.

22. Find the Fourier cosine series of the function $f(x) = \pi - x, 0 < x < \pi$.

23. Find the Laplace transform of $f(t) = \sin^2 4t$.

24. Find the tangent to the ellipse $\frac{x^2}{4} + y^2 = 1$ at $p: (\sqrt{2}, \frac{1}{\sqrt{2}})$.

25. Find the length of the circular helix $r(t) = [2 \cos t, 2 \sin t, 6t]$ from $(2, 0, 0)$ to $(2, 0, 24\pi)$.

26. Show that the function $f(x, y) = \frac{2x^2 y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.

27. The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$.

28. Prove that the binary structure $< Z, + >$ is isomorphic to $< 2\mathbb{Z}, + >$.

29. Let S be the set of all real numbers except -1. Define $*$ on S by $a * b = a + b + ab$. Show that $< S, * >$ is a group.

(6 x 5 = 30 Marks)

Part D

Answer Any Three Questions. Each question hasEight marks.

30. Find the Fourier series expansion of $f(x) = x + \pi, \ -\pi < x < \pi$ and
$$f(x + 2\pi) = f(x).$$

31. Find the inverse Laplace transform of $F(s) = \frac{3s-137}{s^2+2s+401}$.

32. a) Find a unit normal vector **n** of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point $p: (1, 0, 2)$.

    b) Prove that $\text{div}(\text{curl}\mathbf{F}) = 0$.

33. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s at the point $(\frac{1}{2}, 1)$ if $w = xy + yz + zx$, $x = r + s, y = r - s, z = rs$.

34. a)Construct the group table of Z_6.

    b) Prove that a group has only one identity element.

    c) Show that if $(a * b)^2 = a^2 * b^2$ for a and b in a group G, then $a * b = b * a$.

(3 x 8 = 24 Marks)

32

1B4M20198(D) (Pages : 2) Reg. No:.........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester BSc Statistics Degree Examination, March/April 2020

BMAT4C04(ST) – Mathematics

(2018 Admission onwards)

Time: 3 hours Max. Marks : 80

PART A: Answer *ALL* the questions. Each question carries *one* mark.

1. $L(\sin \pi t) =$ ________
2. $L^{-}\left(\frac{1}{s^2+9}\right) =$ ________
3. State the first shifting theorem for Laplace transform.
4. If $f(x,y,z) = xy - yz$,then $\nabla f =$ ________
5. Three vectors form a linearly independent set of vectors if and only if their scalar triple product is ________
6. The primitive period of $\sin 3x$ is
7. Give an example of a function which is neither even nor odd.
8. The two dimensional Poisson equation is ________
9. The domain of the function $f(x,y) = \sqrt{x^2 - y}$ is ________
10. $\frac{4+i}{2-3i} =$ ________
11. $e^{i\pi} =$ ________
12. $Arg(-i) =$ ________

$(12 \times 1 = 12 marks)$

PART B: Answer *any SEVEN* questions. Each question carries *two* marks.

13. Prove that the Laplace transform is a linear operation.
14. Find the Laplace transform of $a + bt + ct^2$.
15. Find the angle between the vectors $(1,2,0) and (3,-2,1)$.
16. If $f(x)$is a periodic function of x of period p,then show that $f(ax), a \neq o$, is a periodic function of x of period$\frac{p}{a}$.
17. Find the Fourier coefficient$a_n$for the 2π periodic function $f(x)$ given by

$$f(x) = \begin{cases} -k \; if -\pi < x < 0 \\ \;\; k \; if \; 0 < x < \pi \end{cases}$$

18. Find the directional derivative of $f(x, y) = x^2 + y^2$ at the point $P(1,1)$ in the direction of $a = 2i - 4j$.

19. Evaluate $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$

20. If $f(z) = z^2 + 3z$,then find the real and imaginary parts of $f(z)$

21. Write the complex number $-1 - i$ in the exponential form.

$(7 \times 2 = 14 marks)$

PART C: Answer *any SIX* questions. Each question carries *five* marks.

22. Find $L(\sin 2t \cos 2t)$.

23. Calculate the length of the catenary $r(t) = ti + \cosh t\, j$ from $t = 0$ to $t = 1$.

24. Find the work done by a force $p = [2,6,6]$ acting on a body if the body is displaced from a point $A(3,4,0)$ to $B(5,8,0)$.

25. Find the unit normal to the surface $z = \sqrt{x^2 + y^2}$ at the point(6,8, 10).

26. If $z = \tan^{-1}\frac{y}{x}$,prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

27. Find the solution of the partial differential equation $\frac{\partial^2 u}{\partial x \partial y} = -\frac{\partial u}{\partial x}$

28. Show that $f(x, y) = \frac{4x^6 y^2}{x^{12}+y^4}$ has no limit as (x,y) approaches (0,0).

29. Prove that the equation$|z - 1 + 3i|$=2 represents a circle of radius 2.

$(6 \times 5 = 30 marks)$

PART D: Answer *any THREE* questions. Each question carries *eight* marks.

30. Find the inverse Laplace transform of $\frac{1-7s}{(s-1)(s+2)(s-3)}$.

31. Find the Fourier series expansion for the function $f(x) = x^2$ in $[-\pi, \pi]$ with $f(x) = f(x + 2\pi) \forall x \in R$.

32. If $\boldsymbol{v} = (2y, 2z, 4x + z)$ and$\boldsymbol{w} = (3z^2, 2x^2 - y^2, y^2)$then calculate div $\boldsymbol{v}$, grad(div$\boldsymbol{w}$)and$div(curl\ \boldsymbol{w})$.

33. a) State the chain rule for composite function of single variable.

b) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and if$w = x + 2y + z^2$,

x$=\frac{r}{s}$, $y = r^2 + \log s$, $z = 2r$

34. Find the square roots of $1 - \sqrt{3}i$ and express them in the rectangular coordinates.

$(8 \times 3 = 24 marks$

1B4M20198(C) (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester BSc Physics Degree Examination, March/April 2020

BMAT4C04(PH) – Mathematics

(2018 Admission onwards)

Time: 3 hours

Max. Marks : 80

Part A: Answer All questions. Each question has ONE mark

1) The Laplace transform of $\cos at$ is $\cdots$.

2) The Inverse Laplace transform of $\frac{5}{s+3}$ is $\cdots$.

3) What is the shifting Property of Laplace transform.

4) If $\mathcal{L}(f(t)) = F(s)$ then $\mathcal{L}(f''(t))$ is $\cdots$

5) Define a periodic function.

6) Give an example of an even function.

7) Let $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} - 3\vec{j} + 2\vec{k}$, find $\vec{a}.\vec{b}$

8) Find a normal vector to the line $x - 2y + 2 = 0$.

9) Compute the scalar triple product of the vectors $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{c} = 2\vec{i} + 3\vec{j} + 4\vec{k}$.

10) Find the tangent to the curve $\vec{r}(t) = t^2\ \vec{j} + t^3\ \vec{k}$ at $(1, 1, 1)$.

11) Domain of the function $f(x, y) = \sqrt{9 - x^2 - y^2}$ is $\cdots$

12) Let $f(x, y) = x^2 + xy^2 + 2xy + y^2$, find $\frac{\partial f}{\partial x}$ at the point $(1, 0)$

(12×1 = 12 Marks)

Part B: Answer Any Seven questions. Each question has Two marks

13) Let n be positive integer, then show that $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$

14) Let $f * g$ denotes the convolution of f and g, show that $f * g = g * f$

15) Find $\mathcal{L}(t\ \ e^{2t})$.

16) Find the Fourier series of $f(x) = -1, \quad -\pi < x < \pi$.

17) Find the Fourier coefficient a_0 in the Fourier series expansion of the function given by $f(x) = \begin{cases} -1 + x & \text{when} \quad -\pi < x < 0 \\ 1 + x & \text{when} \quad 0 < x < \pi \end{cases}$ and $f(x) = f(x + 2\pi)$.

18) A force $\vec{p} = 3\vec{i} - 6\vec{k}$ acts on a line through a point $A : (0, -1, 4)$ about a point $Q : (4, 6, -1)$. Find the moment vector $\vec{m}$ of p

19) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}| = r$, then show that $\nabla(\frac{1}{r}) = -\frac{\vec{r}}{r^3}$

20) Evaluate $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

21) Find $\frac{\partial z}{\partial x}$, if the equation $yz - lnz = x + y$ defines z as a function of x and y.

(7 × 2 = 14 Marks)

Part C: Answer Any Six questions. Each question has Five marks

22) Using Method of partial fraction, find the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$

23) Find Laplace transform of the function

$$f(t) = \begin{cases} 1 & \text{if} \quad 0 < t < \pi \\ 0 & \text{if} \quad \pi < t < 2\pi \\ \sin t & \text{if} \quad t > 2\pi \end{cases}$$

24) Find Fourier series of the function $f(t) = \begin{cases} 0 & \text{if} \quad -2 < t < 1 \\ 3 & \text{if} \quad -1 < t < 1 \\ 0 & \text{if} \quad 1 < t < 2\pi \end{cases}$ with period 4.

25) Obtain the half range cosine series of $f(x) = x$, when $0 < x < 4$.

26) A particle moves in space so that its radius vector is given by $r = \cos t\,\vec{i} + \sin t\,\vec{j} + t\ \tan t\vec{k}$. Find the velocity and acceleration vectors at $t = \pi/4$.

27) Find the directional derivative at $(1, 3, -2)$ in the direction of $\vec{a} = -\vec{i} + 2\vec{j} + 2\vec{k}$, if $f(x, y, z) = yz + xy + xz$.

28) Show that the function $f(x, y) = \frac{x^4 - y^2}{x^4 + y^2}$ has no limit as $(x, y) \to (0, 0)$.

29) Solve the differential equation $u_{xy} = u_x$, where u is a function of x and y.

$(6 \times 5 = 30$ Marks)

Part D: Answer Any Three questions. Each question has Eight marks

30) Using Laplace transform, solve the initial value problem $y'' + 6y' + 8y = e^{-3t} - e^{-5t}$, $y(0) = 0$, $y'(0) = 0$.

31) Solve the integral equation $y(t) = t + \int_0^t y(\tau)\sin(t - \tau)d\tau$.

32) Find the Fourier series of the function given by $f(x) = \begin{cases} 0 & \text{when} \quad -\pi < x < 0 \\ \frac{\pi x}{4} & \text{when} \quad 0 < x < \pi \end{cases}$ and $f(x) = f(x + 2\pi)$.

33) If $\nabla f = (y + y^2 + z^2)\vec{i} + (x + z + 2xy)\vec{j} + (y + 2xz)\vec{k}$, find f, such that $f(1, 1, 1) = 3$.

34) Let $w = xy + yz + xz$, $x = u + v$, $y = u - v$, and $z = uv$, then evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, in terms of u and v.

$(3 \times 8 = 24$ Marks)

34

1B4M20198(B) (Pages :3) Reg. No:........................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc CS Degree Examination, March/April 2020

BMAT4C04(CS) – Mathematics

(2018 Admission onwards)

Time: 3 hours Max. Marks : 80

Part A

Answer All Questions. Each question has One mark.

1. Find the Laplace transform of $f(t) = a + bt + ct^2$.
2. Find the fundamental period of $\sin 3x$.
3. Give an example of a function which is neither even nor odd.
4. Find the components of the vector v with initial point (4, 0, 2) and terminal point (6, -1, 2).
5. Let $a = [2, -1, 0]$ and $b = [-4, 2, 5]$. Find $a \times b$.
6. Find a parametric representation of the circle of radius 3 with centre (4,6).
7. Find the domain of the function $f(x, y, z) = \frac{y}{x^2}$.
8. Find the value of $\frac{\partial f}{\partial x}$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.
9. Write the order of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$.
10. Write the negation of the proposition: "At least 10 inches of rain fell today in Miami"
11. Write in the form if p then q: "To get tenure as a professor, it is sufficient to be world famous"
12. Find the dual of $(p \vee F) \wedge (q \vee T)$

(12 x 1 = 12 Marks)

Part B

Answer Any Seven Questions. Each question has Two marks.

13. Find the Laplace transform of $f(t) = \cos(\omega t + \theta)$, where θ is a constant.
14. Find a_0 in the Fourier series expansion of $f(x) = \begin{cases} -2 & ; if -\pi < x < 0 \\ 2 & ; if\ 0 < x < \pi \end{cases}$
15. Find the angle between the vectors $a = [1, 2, 0]$ and $b = [3, -2, 1]$.
16. Find the velocity and speed of $\mathrm{r(t)} = [\mathrm{t}, \mathrm{t}^2, 0]$.
17. Find div **F** and curl **F** for the vector field $\mathbf{F}(x, y, z) = (x^3 + y^3)\mathbf{i} + 3xy^2\mathbf{j} + 3y^2 z\,\mathbf{k}$

18. Find an equation for the level surface of the function $f(x, y, z) = \ln(x^2 + y + z^2)$ passes through the point $(-1, 2, 1)$.

19. Find $\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

20. Find $\frac{dy}{dx}$ if $y^2 - x^2 - \sin xy = 0$.

21. Show that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

(7 x 2 = 14 Marks)

Part C

Answer Any Six Questions. Each question has Five marks.

22. Find the inverse Laplace transform of $F(s) = \frac{1-7s}{(s-1)(s+2)(s-3)}$.

23. Find the Fourier cosine series of the function $f(x) = x^2, 0 < x < L$.

24. Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P: (\sqrt{2}, \frac{1}{\sqrt{2}})$.

25. Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at $p: (2, 1, 3)$ in the direction of $a = [1, 0, -2]$.

26. Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2} & ; (x, y) \neq (0,0) \\ 0 & ; (x, y) = (0,0) \end{cases}$ is continuous at every point except at the origin.

27. Write the chain rule and find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ in terms of u and v at the point $(\frac{1}{2}, 1)$ if $w = xy + yz + zx,\ \ x = u + v, y = u - v$ and $z = uv$.

28. Express the statement using predicates and quantifiers:

"Every positive integer is the sum of the squares of four integers."

29. Construct a truth table for $p \to (\neg q \vee r)$.

(6 x 5 = 30 Marks)

Part D

Answer Any Three Questions. Each question has Eight marks.

30. a)Find the inverse Laplace Transform of $F(s) = \frac{1}{s^2-4s+5}$.

b). Find the Laplace Transform of $f(t) = \begin{cases} 1 & ; if\ t \leq 1 \\ 2 & ; if\ 1 < t \leq 2 \\ 0 & ; if\ t > 2 \end{cases}$

31. Find the Fourier series expansion of $f(x) = \begin{cases} -\pi & ; if -\pi < x < 0 \\ x & ; if\ 0 < x < \pi \end{cases}$ and

$$f(x + 2\pi) = f(x).$$

32. a). Prove that gradient fields are irrotational.

b). Find the length of the catenary $r(t) = [t, \cosh t]$ from $t = 0$ to $t = 1$.

33. a). Show that the function $f(x, y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x, y) approaches $(0, 0)$.

b). The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$.

34. a) Express each of these statements using quantifiers, logical connectives:

(i) *"All clear explanations are satisfactory"*

(ii) *"Some excuses are unsatisfactory"*

(iii) *"Some excuses are not clear explanations"*

b) Does (iii) follow from (i) and (ii)?

(3 x 8 = 24 Marks)