

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester BSc Degree Examination, November 2016

MAT3B03 - Calculus and Analytic Geometry

(2015 Admission onwards)

Max. Time: 3 hours

Max. Marks: 80

**Part – A**

Answer all the twelve questions, Each question carries 1 mark.

1. Give a simpler expression for  $e^{\ln(x^2+y^2)}$
2. What is the domain of the Natural Logarithms,  $\ln(X)$  ?
3. What is the derivative of  $a^x$
4. What is the  $n^{\text{th}}$  term test for the divergence of a series ?
5. Find  $\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\sec x}{1 + \tan x} \right)$
6. State whether the Series  $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \dots$  is Conditionally Convergent or absolutely Convergent ?
7. What is the eccentricity of the conic  $2x^2 + y^2 = 4$
8. Identify the Conic  $3x^2 - 7xy + \sqrt{17}y^2 = 1$
9. What is the Formula to find the area of the surface generated by revolving the Curve  $x = f(t)$ ,  $y = g(t)$ ,  $a \leq t \leq b$  about  $x$  - axis
10. What is the Cartesian equivalent of the polar equation  $r \sin(\theta) = 0$
11. Write the Taylor - Series of  $F(x) = e^x$  at  $x = 0$
12. What is the eccentricity of parabola?

(12 x 1 = 12marks)

**Part – B**

Answer any nine questions

Each question carries 2 marks

13. Express  $\ln\sqrt{13.5}$  in terms of  $\ln 2$  and  $\ln 3$
14. Find  $\frac{dy}{dx}$  if  $\ln y = e^y \sin x$
15. Find  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$
16. State Sandwich theorem for sequences and use it to show that the sequence  $\left\{ \frac{\cos n}{n} \right\} \rightarrow 0$
17. Find the sum of the series  $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$
18. Find the sum of the telescoping series  $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$
19. Find  $\frac{d^2y}{dx^2}$  if  $x = a \sec t$ ,  $y = b \tan t$
20. The parabola  $y^2 = 8x$  is shifted down 2 units and right 1 unit, without changing the direction of axes. Find the equation, Foci and vertices of the new parabola.
21. Identify the Particle's path by finding a Cartesian equation and graph if for the parametrized curve  $x = \sin(2\pi(1-t))$ ,  $Y = \cos(2\pi(1-t))$ ,  $0 \leq t \leq 1$
22. Show that the Series  $\sum_{n=1}^{\infty} \frac{n+1}{n}$  diverges
23. Check the convergence of the series  $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$
24. Find the Taylor series of  $f(x) = \cos x$  at  $x=0$ .

(9 x 2 = 18 marks)

**Part – C**Answer any six questions  
Each question carries 5 marks

Drop a ball from 'a' meters above a flat surface. Each time the ball hits the surface after falling a distance h, it rebounds a distance rh, where r is positive but less than 1. Find the total distance the ravel up and down.

Find  $\lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right)$

Find  $\frac{dy}{dx}$  if  $y = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}$

Check the convergence of the series  $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n! 3^n}$ Show that the Maclaurin series for  $\sin x$  converges to  $\sin x$  for all  $x$ .

The ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  is shifted 4 units to the left and 1 units down .Find the new equation, foci, centre, vertices and eccentricity.

Find the Centre, Foci, vertices and axis of the conic  $2x^2 - y^2 + 6y = 3$ Show that  $\frac{d^2y}{dx^2} = \frac{-1}{4a} \cos^4(t/2)$  for the cycloid  $x = a(t - \sin t)$ ,  $y = a(1 - \cos t)$ 

Find the Cartesian equation Corresponding to the Polar equation

$$r \sin\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}$$

(6x5 = 30 marks)

**Part D**Answer any two questions  
Each question carries 10 marks

a) Show that  $\lim_{x \rightarrow 0^+} (1+x)^{1/x} = e$

b) Find the area of the surface swept out by revolving the circle  $x = \cos t$ ,  $y = 1 + \sin t$ ,  $0 \leq t \leq \pi$  about the  $x$ -axis. Also find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$

a) Investigate the convergence of  $\sum_{n=1}^{\infty} \frac{4^n \cdot n! \cdot n!}{(2n)!}$

b) Show that the P-series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges if  $p > 1$  and diverges if  $p \leq 1$ 

(a) Find  $\lim_{x \rightarrow \infty} x^{1/x}$

(b) Solve For  $x$  :  $3^{\log_3(x^2)} = 5 \cdot e^{\ln x} - 3 \cdot 10^{\log_{10}(2)}$

A) Find the Cartesian equation for the hyperbola centered at the origin that has focus at (4,0) and line  $x=2$  as the corresponding directrix

b) Find the polar equation corresponding to the Cartesian equation  $x^2 + 4x + y^2 + 2y = 0$

(2x10 = 20 marks)

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Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester BSc Degree Examination, November 2016

MAT3C03 - Mathematics

(2015 Admission onwards)

Max. Time: 3 hours

Max. Marks: 80

**Part A (Objective Type Questions)**

Answer all questions

1. Order and degree of the differential equation  $y'' + 2(y')^2 + 3 = 0$  is .....and .....
2. Solution of the differential equation  $y' = x/y$ .
3. Write the general form of Bernoulli's differential equation.
4. What is the rank of  $3 \times 3$  unit matrix?
5. The system of equations  $AX = B$  is consistent if.....
6. Give an eigen value of the matrix  $A^T$ , given 3 is an eigen value of the matrix A.
7. Find the eigen values of the matrix  $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ .
8. Find a unit vector in the direction of the vector  $i + 3j + 2k$ .
9. Write the parametric equation of  $(x-1)^2 + y^2 = 9$
10. A vector  $V$  is solinoidal if .....
11. Find  $\text{grad}F$  where  $F(x,y,z) = x^2 + y^2 + z^2$
12. What you mean by singular solution of a differential equation.

(12 × 1 = 12 marks)

**Part B(Short Answer Type Questions)**

Answer any nine Questions

13. Solve the differential equation  $y' = (1+x)(1+y^2)$ .
14. Show that the differential equation  $(3x^2y + e^y) dx + (x^3 + x e^y - 2y)dy$  is exact and hence solve it.
15. Solve  $y' + y \tan x = \cos^3 x$ .
16. Find the orthogonal trajectories of the family of parabolas  $y = kx^2$ .

17. Find the eigen values of the matrix  $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ .

18. Find the rank of matrix  $\begin{bmatrix} 2 & 4 & 1 \\ 3 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix}$ .

19. Test for the consistency of the system of equations  $x+y = 3$ ,  $x+2y=4$ ,  $x + 4y = 6$ .

20. Prove that eigen values of a diagonal matrix is the diagonal elements.

21. Find the angle between the planes  $x+y+z = 1$ ,  $x+2y+3z = 6$ .

22. Find the length of the semi cubical parabola  $r(t) = t i + t^{3/2} j$   $t=0$  to  $t=1$ .

23. Find the directional derivative of  $f(x,y,z) = 2xy + z^2$  in the direction of  $i + 2j + 2k$  at the point  $(1, -1, 3)$ .

24. Find  $\text{div} f$ , where  $f(x,y,z) = (3x^2y - y^3z^2) i - 3z k$  at the point  $(1,1,2)$

(9 × 2 = 18 marks)

**Part C (Short Essay Questions)**

Answer any six questions

25. Solve  $(xy^3 + y)dx + 2(x^2y^2 + x + y^4) dy = 0$ .

26. Solve  $y' + x \sin x = x^3 \cos^2 y$ .

27. Reduce to the normal form and hence find the rank of the matrix  $\begin{pmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \end{pmatrix}$ .

28. Find the condition that the system of equations  $3x+4y+5z = a$ ,  $4x+5z+6z = b$ ,  $5x+6y+7z = c$  is consistent.

29. Find the eigen vector corresponding to the least eigen value of the matrix

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

30. Use Cayley Hamilton theorem to find  $A^3$  and  $A^{-1}$  if  $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ .

31. If  $a$  is a constant vector and  $r = x i + y j + z k$ . Show that  $\text{curl}(a \times r) = 2a$ .

32.  $F = xy i + x^2 j$ . Evaluate  $\int_c F \cdot dr$ , where  $c$  is the quarter circle from  $(2,0)$  to  $(0,2)$  with centre as origin.

33. Show that the integral  $\int_c 3x^2 dx + 2yz dy + y^2 dz$  is independent of path and hence solve the integral from  $(0,1,2)$  to  $(1,-1,7)$ .

(6 × 5 = 30 marks)

**Part D (Essay Type Questions)**

Answer any Two Questions

34. a) Solve the differential equation  $(x^2 - 2x + 2y^2) dx + 2xy dy = 0$

- b) Show that  $V = (y^2 + 2xz^2 - 1)i + (2xy) j + (2xz^2)k$  is irrotational.

35. Let  $A = \begin{bmatrix} 6 & -2 & -2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ , Find

- a) Eigen values and corresponding eigen vectors of A

- b) Use Cayley Hamilton theorem to find  $A^{-1}$

36. Verify Gauss's divergence theorem for  $F$ , where  $F = (x^2 - yz) i + (y^2 - zx) j + (z^2 - xy) k$  over the rectangular parallelepiped  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$ ,  $0 \leq z \leq 3$ .

(2 × 10 = 20 marks)

**A11 - Basic Numerical Skills**

(2015 Admission onwards)

Max. Time: 3 hours

Max. Marks: 80

**Part I**

Answer **all** questions in this part.  
Each question carries 1 mark

Choose the correct answer from the choice given:

- The distance of the point P(-3, 4) from the origin is:  
(a) 3 (b) 4  
(c) 5 (d) 7
- The equation  $y=2x+5$  has:  
(a) No Solution (b) One Solution  
(c) Three Solutions (d) Infinity many Solutions
- The point of intersection of the 'less than' and 'more than' 0 given corresponds to:  
(a) Mean (b) Median  
(c) Geometric Mean (d) Harmonic Mean
- The best average to analyze speed is:  
(a) Mode (b) Arithmetic Mean  
(c) Geometric Mean (d) Harmonic Mean
- Which of the following statement is true:  
(a)  $0 \in \{\}$  (b)  $0 \subset \{\}$   
(c)  $0 \in \{0\}$  (d)  $0 \subset \{0\}$

Fill in the blanks:-

- The geometric mean between  $a$  and  $b$  is \_\_\_\_\_.
- Measures of dispersion are called averages of the \_\_\_\_\_ order.
- Condition for the matrix A to be symmetric is \_\_\_\_\_.
- \_\_\_\_\_ is the graphical method of studying dispersion.
- The 6<sup>th</sup> term of  $\frac{3}{7}, \frac{3}{8}, \frac{3}{9}, \frac{3}{10}$  is \_\_\_\_\_.

(10 × 1 = 10 marks)

**Part II**

Answer **any** eight questions.  
Each question carries 2 marks.

- Solve  $x + y = 10$  and  $xy = 24$ .
- Define Power set. If set S is infinite set of 'n' elements, how many elements are in the power set?
- $A = \{x : x \text{ is natural number satisfy } 1 < x < 6\}$   
 $B = \{x : x \text{ is natural number satisfy } 6 < x < 10\}$   
Find  $A \cup B$  and  $A \cap B$
- If the 5<sup>th</sup> and 10<sup>th</sup> terms of G.P are 32 and 1024 respectively, find the first term and the common ratio?
- Let  $P = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ ,  $Q = \begin{bmatrix} -1 & 2 \\ 4 & 3 \end{bmatrix}$  and  $R = \begin{bmatrix} 2 & -1 \\ 6 & 5 \end{bmatrix}$  Find  $P(Q+R)$  and  $PQ+PR$ ,  
Hence prove  $P(Q+R) = PQ+PR$ .
- If  $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ -1 & 1 \end{bmatrix} \begin{matrix} 3 \\ 4 \\ 2 \end{matrix}$  and  $B = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 3 & 4 \\ 0 & -2 & -3 \end{bmatrix}$  Show that  $AB \neq BA$ .
- Prove that  $A \cap (A \cup B) = A \cup (A \cap B)$  by means of Venn diagram.
- Define Skewness. And Positive Skewness and Negative Skewness?
- Find the first four central moments for the value given below:  
8, 10, 12, 7, 18. Find coefficient of Skewness and measure of Kurtosis.
- Define consumer price index number?

(8 × 2 = 16 marks)

**Part III**

Answer **any** six questions.  
Each question carries 4 marks

- Solve the system of equations :  
 $9x = 3y - 4z = 35$ ,  $x + y - z = 4$ ,  $2x - 5y - 4z + 48 = 0$
- A club consist of members whose ages in A.P, the common difference being 3 months. If the youngest member of the club is 7 years old and the sum of the ages of all the members is 250 years, find the number of members in the Club?

23. Explain the components given below?

- Line Diagram
- Bar Diagram
- Pie Diagram (Pie Charts)
- Histogram

24. Find the Quartile measure of dispersion and its coefficient for the data given below.

|               |      |       |       |       |       |       |       |       |
|---------------|------|-------|-------|-------|-------|-------|-------|-------|
| Age:          | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| No of Person: | 15   | 30    | 53    | 75    | 100   | 110   | 115   | 125.  |

25. If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ -1 & 1 & 2 \end{bmatrix}$  Where  $f(x)$  is given by  $f(x) = x^2 - 5x - 6$ .

26. A man deposits a certain sum of money into bank. It amounts to Rs.12325 in 8 years and amount to Rs.13565 in 10 years. Find the sum invested.

27. What are the important functions of Statistics? Explain them.

28. Find the inverse of the matrix  $A = \begin{bmatrix} 0 & 6 & 7 \\ 0 & 2 & 1 \\ -2 & 3 & 4 \end{bmatrix}$

(6 × 4 = 24 marks)

**Part IV**

Answer **any** two questions.  
Each question carries 15 marks

29. Solve the system of equations with the help of matrices.

$$\begin{aligned} 2x - 2y + z &= 1 \\ x + 2y + 2z &= 2 \\ 2x + y - 2z &= 7 \end{aligned}$$

30. Compute the trend values by the method of least squares from the data given below. Also estimate the Number of sheep in 2009.

|               |      |      |      |      |      |      |      |      |
|---------------|------|------|------|------|------|------|------|------|
| Year:         | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
| No. of Sheep: | 56   | 55   | 51   | 47   | 42   | 38   | 35   | 32.  |

31. Explain any *four* methods of random (probability) sampling.

(15 × 2 = 30)