1B5N19304 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2019

BMAT5B08– Differential Equations

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

Section A

Answer all the **twelve** questions
Each question carries 1 mark

1) The order and degree of the differential equation $u_{xx} + u_{yy} = 0$ is $\cdots$
2) Give an example a differential equation whose solution is $\sin x$.
3) General form of Bernoulli's equation is $\cdots$
4) Integrating factor of the differential equation $y' - y = e^{2t}$ is $\cdots$
5) Find the Wronskian of t and e^t.
6) $\mathcal{L}(\sin at) = \cdots$
7) Let a be positive real number, then $\mathcal{L}(t^a) = \cdots$
8) If the Laplace transform of $f(t)$ and $f'(t)$ exist then $\mathcal{L}(f''(t))$ is $\cdots$
9) Define the convolution of two functions $f(t)$ and $g(t)$.
10) Define a periodic function.
11) Give an example of an odd function.
12) Sketch the graph of the function $f(x) = \pi - x$ for $x \in [-\pi, \pi]$.

($12\times1 = 12$ Marks)

Section B

Answer any **TEN** questions
Each question carries 4 marks

13) Solve by Method of variation of parameters $\dfrac{dy}{dx} - y = 3e^x$.
14) Solve the differential Equation $(x+2)\dfrac{dy}{dx} = xy$.
15) Solve the differential Equation $y' = \dfrac{x^2+y^2}{x^2+xy}$.
16) Solve the initial value problem $(y-1)dx + (x-3)dy = 0, y(0) = \frac{2}{3}$.
17) Show that $y_1(t) = \sin t$ and $y_2(t) = \cos t$ forms a fundamental set of solutions of the differential equation $y'' + y = 0$.
18) Find the general solution of the differential equation $x^2y'' - 3xy' + 4y = 0$
19) If y_1 and y_2 are two solutions of a second order linear homogeneous differential equation, then show that $c_1y_1 + c_2y_2$ is again a solution of that differential equation.
20) Show that $\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$.
21) Find $\mathcal{L}(e^{at}\cos bt)$.

22) Find the inverse laplace transform of $\frac{e^{-3s}}{s^3}$.

23) Let $f \star g$ denote the convolution of f and g, then show that $f \star g = g \star f$.

24) Show that the product of two even functions is an even function.

25) Find the Fourier series of the even function $f(x) = |x|$ in $[-\pi, \pi]$ with $f(x + 2\pi) = f(x)$, for all $x \in \mathbb{R}$.

26) Find the Fourier coeffient a_n in the Fourier series expansion of the function given by $f(x) = x \sin x$, $0 < x < 2\pi$ and $f(x + 2\pi) = f(x)$.

(10×4 = 40 **Marks**)

Section C

Answer any **SIX** questions
Each question carries 7 marks

27) Solve the differential equation $t\frac{dy}{dx} + y = t^3y^6$.

28) Solve the differential equation $\frac{dy}{dx} = \frac{y - x + 1}{y - x + 5}$.

29) Solve the following initial value problem by Picard's iteration method(Do 3 steps) $2y' = x + y$, given that $y(0) = 2$. Also find $y(0.1)$

30) State and prove Abel's Theorem.

31) Solve the following system of differential Equations.
$x' = x - 2y, \quad x(0) = -1$
$y' = 3x - 4y, \quad y(0) = 2.$

32) Find the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$

33) Find the inverse Laplce transform of $\frac{s}{(s-1)^2 - 4}$.

34) Obtain the half range cosine series of $f(x) = x$, when $0 < x < 4$.

35) Solve by product method: $u_x + u_y = 0$, where u is a function of x and y.

(6×7 = 42 **Marks**)

Section D

Answer any **TWO** questions
Each question carries 13 marks

36) Solve the nonhomogeneous differential equation $y'' - 2y' + y = t + e^t$.

37) Solve the following initial value problem: $y'' + 4y = 4t$, with $y(0) = 1, y'(0) = 5$,using Laplace transform.

38) Find the Fourier series expansion for the function $f(x) = x^2$ when $-\pi < x < \pi$ with $f(x) = f(x + 2\pi) \quad \forall x \in \mathbb{R}$ hence deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

(2×13 = 26 **Marks**)

1B5N19303 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2019

BMAT5B07– Basic Mathematical Analysis

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

SECTION A

Answer all the twelve questions.
Each question carries 1 mark .

1. For each $n \in N$, let $A_n = \{(n+1)k : k \in N\}$. Determine $\cup \{A_n : n \in N\}$ and $\cap \{A_n : n \in N\}$.
2. State Well-Ordering property of $\mathbb{N}$.
3. Define ε - neighborhood of a point in $\mathbb{R}$.
4. State the Completeness property of $\mathbb{R}$.
5. Define a convergent sequence of real numbers. Give an example.
6. Is the set all real numbers $\mathbb{R}$ is countable ? Justify your answer.
7. What do you mean by trichotomy law of real numbers.
8. Give an example of a subset of $\mathbb{R}$,which is bounded below but not bounded above.
9. State Density theorem.
10. Find lim $\frac{2n}{n+2}$.
11. If a set is not open will it imply that the set is closed.
12. Find Arg (1 - i) .

(12x1=12 marks)

SECTION B

Answer any ten out of fourteen questions.
Each question carries 4 marks.

13. For any three sets A,B and C , prove that A\ (B∪C) = (A\ B) ∩(A\ C).
14. If A_n is a countable set for each $n \in N$ prove that their union $\bigcup_{n=1}^{\infty} A_n$ is countable.
15. If $a \in \mathbb{R}$ is such that $0 \leq a < \varepsilon$ for every $\varepsilon > 0$, then prove that $a = 0$.
16. If $a, b \in \mathbb{R}$, then prove that $|a + b| \leq |a| + |b|$.
17. If $S = \left\{\frac{1}{n} - \frac{1}{m} : n, m \in N\right\}$, Find infimum and supremum of S .
18. Determine the set of all $x \in \mathbb{R}$ such that $|2x + 3| < 7$.

19. Prove that $\lim \frac{\sin n}{n} = 0$.

20. Prove that every convergent sequence of real numbers is a cauchy sequence .

21. Show that the sequence (0, 2, 0, 2,....., o, 2, ...) does not converge to 0.

22. If $X = (x_n)$ is a convergent sequence of real numbers and if $x_n \geq 0$ for all $n \in N$, prove that $\lim \ x_n \geq 0$.

23. Prove that the interval $[0,1]$ is not countable.

24. Prove that the union of an arbitrary collection of open subsets of $\mathbb{R}$ is open .

25. Prove that Re(iz) = - Im (z)

26. Prove that Arg (z_1z_2) = Arg (z_1) + Arg (z_2)

(10x4=40 marks)

SECTION C

Answer any six out of nine questions.
Each question carries 7 marks.

27. State and prove the Principle of Mathematical Induction.

28. Prove that $\sqrt{2}$ is irrational.

29. State and prove the Bernoulli's Inequality.

30. State and prove Archimedean property.

31. Prove that a sequence in $\mathbb{R}$ can have at most one limit.

32. Prove that a convergent sequence of real numbers is bounded.

33. Let A and B be nonempty bounded subset of $\mathbb{R}$,then prove that

$$Sup(A + B) = Sup\ A + Sup\ B.$$

34. Find all values of $(-1)^{\frac{1}{4}}$.

35. Determine the locus represented by $|z - 4i| = 4$.

(6x7=42 marks)

SECTION D

Answer any two out of three questions.
Each question carries 13 marks.

36. (a) If $X = (x_n)$ is a sequence of real numbers ,then there is a subsequence of X that is monotone.

(b) Prove that a bounded sequence of real numbers has a convergent subsequence.

37. (a) Define Nested intervals .

(b) State and prove the nested interval property of $\mathbb{R}$

38. Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all of its cluster points.

(2x13=26 marks)

1B5N19302 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2019

BMAT5B06 – Abstract Algebra

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

Section A

Answer all the* twelve *questions

Each question carries 1 mark.

1. How many generators are there for the group $\mathbb{Z}_{12}$?
2. What is the inverse of $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 4 & 6 & 5 \end{pmatrix}$ in the group S_6.
3. Give an example of a non abelian group in which all proper subgroups are abelian.
4. The number of left cosets of the subgroup $6\mathbb{Z}$ in the group $2\mathbb{Z}$ is ______
5. State True or False: " Every group of order 17 is cyclic "
6. Define a transposition.
7. Number of unit elements in the ring $\mathbb{Z}_8$ is ______
8. Define normal subgroup H of a group G.
9. Give an example of a ring with exactly two units.
10. A non commutative division ring is called _______.
11. Find the remainder when -32 is divided by 5.
12. Define a field.

(12×1=12 marks)

Section B

Answer any* ten *out of fourteen questions.

Each question carries 4 marks.

13. Let S be a set and let $f, g \,\&\, h$ be functions mapping S into S. Show that $(f \circ g) \circ h = f \circ (g \circ h)$.
14. Show that $(\mathbb{Z}, +) \simeq (3\mathbb{Z}, +)$.
15. Show that $\mathbb{M}_2(\mathbb{R})$ under matrix addition is a group.
16. Show that for any $a, b \in G$, the equation $a * x = b$ has unique solution in any group G.
17. Show that every cyclic group is abelian.
18. Compute $|<\sigma>|$ for $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$.
19. Find the orbits of the permutation $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$.

20. Let H be a subgroup of a group G. Show that the relation $\sim_L$ on G defined by $a \sim_L b$ if and only if $a^{-1}b \in$ H is an equivalence relation on G.

21. Show that the group homomorphism $\varphi: G \to G'$ is one-to-one map iff $Ker(\varphi) = \{e\}$.

22. Exhibit all left cosets of the subgroup $\{\rho_0, \mu_2\}$ of the dihedral group D_4.

23. Describe the Klein-4-group V.

24. Show that cancellation law holds in a ring R if and only if R has no zero divisors.

25. Define units in a ring R. Describe all units in the ring $\mathbb{Z} \times \mathbb{Z}$.

26. If p is a prime, show that Z_p has no divisors of 0.

(10×4=40 marks)

Section C

Answer any* six *out of nine questions
Each question carries 7 Marks

27. Show that subgroup of a cyclic group is cyclic.

28. Show that every infinite cyclic group is isomorphic to the group $(\mathbb{Z}, +)$.

29. Find all subgroups of D_4. Draw its subgroup diagram.

30. Let $\varphi: G \to G'$ be a group homomorphism and let G is abelian. Show that G' is abelian.

31. Show that a non empty set H of a group G is a subgroup of G iff $ab^{-1} \in$ H for all $a, b \in$ H.

32. Express σ as product of disjoint cycles and then as product of transpositions where

$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 1 & 3 & 5 & 4 & 6 & 7 & 10 & 8 & 9 \end{pmatrix}$. Is σ an odd permutation ? Justify your answer.

33. Define ring homomorphism. Describe all ring homomorphisms of $\mathbb{Z}$ into $\mathbb{Z}$.

34. Show that every finite integral domain is a field.

35. Define zero divisors in a ring (R, +, .). Find all zero divisors in the ring $(\mathbb{Z}_{12}, +_{12}, \times_{12})$.

(6×7=42 marks)

Section D

Answer any* two *out of three questions
Each question carries 13 Marks

36. State and Prove Cayley's Theorem.

37. State and Prove Lagrange's Theorem for finite groups and hence deduce that every prime order group is cyclic.

38. (a) Show that every field is an integral domain.

    (b) Solve the equation $x^2 + 2x + 2 = 0$ in the field $\mathbb{Z}_6$.

    (c) Define field of quotients of an integral domain D. Give one example.

(2×13=26 marks)

1B5N19301 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2019

BMAT5B05 – Vector Calculus

(2017 Admission onwards)

Time: 3 hours Max. Marks: 120

Part A

Answer ALL questions (1 - 12). Each question carries 1 mark.

1. Evaluate $\lim_{(x,y)\to(1,1)} \frac{2x^2+xy-3y^2}{x-y}$.
2. Find the critical point of $x^2+xy+y^2+3x-3y+4$.
3. If a vector function $\vec{r}(t)$ has constant magnitude, prove that $\vec{r}\cdot\dfrac{d\vec{r}}{dt}=0$.
4. Find the rate of change of $f(x,y)=x^2+y^3-4x+6y-1$ in the direction of $\hat{i}$.
5. Define the potential function of a vector field $M(x,y,z)\hat{i}+N(x,y,z)\hat{j}+P(x,y,z)\hat{k}$.
6. Write the tangential form of Green's theorem on a plane.
7. State the Fubini's theorem (strong form).
8. Evaluate $\int_1^e\int_1^e\int_1^e \frac{1}{xyz}dxdydz$.
9. If $u=2x+3y$ and $v=3x+4y$, then write the relation between $dxdy$ and $dudv$.
10. **Write** the double integral of $x+y$ over the triangular region bounded by the lines $x=1$, $y=1$ and $x+y=1$.
11. Write the parametrisation of the sphere $x^2+y^2+z^2=4$.
12. State the Stoke's Theorem.

Part B

Answer ANY TEN from the FOURTEEN questions (13 - 26). Each question carries 4 marks.

13. Test the existence of $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{2x+3y}$.
14. If a vector function $\vec{r}(t)$ has constant direction, prove that $\vec{r}\times\dfrac{d\vec{r}}{dt}=\vec{0}$.
15. If $f(x,y)=x^2-3xy+4y+2$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(2,1)$.
16. Find linearisation of x^2+y^3 at $(1,1)$.
17. Find $\frac{dw}{dt}$ if $w=x^2y+y^2+x$, $x=e^t$ and $y=\cos t$ at $t=0$.
18. Locate the critical points and find the local extreme values of $f(x,y)=xy$.

19. Find the area of the ellipse $4x^2+9y^2=36$ using the method of double integral.

20. Evaluate $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\,dy\,dx$.

21. Rewrite $\int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} dz\,dy\,dx$ as an equivalent triple integral in the order $dy\,dz\,dx$.

22. Find the potential function of the conservative vector field $\mathbf{F}=2xyz\hat{i}+x^2z\hat{j}+x^2y\hat{k}$.

23. Show that the differential form in the integral $\int_{(0,0,0)}^{(2,3,-6)}(2x\,dx+2y\,dy+2z\,dz)$ is exact. Evaluate the integral.

24. Find the line integral of $2x+3y+z$ along the line segment from $(1,2,3)$ to $(3,5,4)$.

25. Write the formula for surface integral. Explain all terms used in it.

26. For a scalar field $f(x,y,z)$, prove that $Curl\ grad\ f=\vec{0}$.

Part C

Answer ANY SIX from the NINE questions (27 - 35). Each question carries 7 marks.

27. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ at $r=1$ and $s=-1$
when $w=(x+y+z)^2$, $x=r-s$, $y=\cos(r+s)$ and $z=\sin(r+s)$.

28. Find equations of tangent plane and normal at $(4,2,3)$ on the surface $x^2+y^3-z^3+3=0$.

29. Find the average height of the paraboloid $z=x^2+y^2$ over the square $0\le x\le 2$, $0\le y\le 2$.

30. Write an equivalent polar integral of $\int_0^2 \int_{-\sqrt{1-y^2}}^{0} \frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}\,dx\,dy$ and evaluate it.

31. Find the volume of the region bounded by the paraboloids $z=8-x^2-y^2$ and $z=x^2+y^2$.

32. Use $u=x-y$ and $v=2x+y$ evaluate the integral $\iint_R(2x^2-xy-y^2)\,dx\,dy$ where R is the region in t he first quadrant bounded by the lines $y=-2x+4$, $y=-2x+7$, $y=x-2$ and $y=x+1$.

33. Find the flow of the velocity field $(x+y)\hat{i}-(x^2+y^2)\hat{j}$ along the path from $(1,0)$ to $(-1,0)$ in the xy plane.

34. Find the surface area of the cap cut from the hemisphere $x^2+y^2+z^2=2\ z\ge 0$, by the cylinder $x^2+y^2=1$.

35. Parametrise the sphere $x^2+y^2+z^2=4$ and hence find its surface area.

Part D

Answer ANY TWO from the THREE questions (36 - 38). Each question carries 13 marks.

36. Using the method of Lagrange multipliers maximise $f(x,y,z)=x^2+y^2+z^2$ subject to the constraints $2y+4z-5=0$ and $4x^2+4y^2-z^2=0$.

37. a) Test for exactness of the differential form $2xe^z\,dx+3y^2e^z\,dy+(x^2+y^3)e^z\,dz$. 4 Marks

    b) Evaluate the line integral of $2xe^z\hat{i}+3y^2e^z\hat{j}+(x^2+y^3)e^z\hat{k}$ along the arc of an ellipse having eccentricity 0.5 joining the points $(0,0,0)$ to $(1,1,1)$. 9 Marks
    Hint: You can use a suitable theorem for an easy evaluation of the line integral.

38. Verify the Divergence theorem for the field $\vec{F}=x\hat{i}+y\hat{j}+z\hat{k}$ over the sphere $x^2+y^2+z^2=9$.