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2B5N17198 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2017

MAT5B05 – Vector Calculus

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A (1 – 12)
Answer all twelve questions.
Each question carries 1 Mark

1. Define a bounded region in a plane.
2. What are the level curves of the function $f(x, y) = x + y$
3. Find the limit $\lim_{(x,y)\to(0,0)} \frac{4x^2-4y^2}{x-y}$
4. Find the rate of change of the function $f(x.y) = x^2 + y^2 + 2x + 4$ in the direction of the vector $\hat{\imath}$
5. Define gradiant of a function of three variables at a point.
6. Rewrite the integral $\int_0^1 \int_{y=x^2}^{y=1} dydx$ as an equivalent integral in the order $dxdy$.
7. State the tangential form of Green's Theorem in the plane.
8. Give an example of a surface which is not orientable
9. Define flux across a plane curve.
10. If C is the unit circle with centre at the origin, then what is the value of the integral $\int_C xdy - ydx$
11. Find the jacobian of the transformation $x = r\cos\theta, y = r\sin\theta$.
12. State tangential form of the Green's theorem.

(12 x 1 = 12 Marks)

Part B
Answer any Ten from the following Fourteen question (13 – 26).
Each question carries 4 Marks.

13. Find the local extreme values of $f(x, y) = xy$
14. Find $\frac{dy}{dx}$ if $xe^y + \sin xy + y - ln2 = 0$.
15. Check whether $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{xy}$ exists or not?
16. Find the line integral of $f(x, y, z) = x + y$ over the line segment$x = t, y = 1 - t, z = 0$ from (0,1,0) to (1,0,0).
17. Find a linearization of $f(x, z) = xy + 2yz - 3xz$ at the point (1,1,0).
18. Find the circulation density of the vector field $\vec{F}(x, y) = (x^2 - y)\,\hat{\imath} + (xy - y^2)\hat{\jmath}$
19. Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\,dxdy$
20. Change the integral $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)dxdy$ into polar form then evaluate the integral
21. Evaluate $\iiint_D dxdydz$, where D is the solid bounded in the first octant bounded by the cordinate planes and the planes $x = 1, y = 1$ and $z = 1$.

22. Evaluate the line integral $\int_C (x+y)ds$, where C is the straight line $x=t$, $y = 1-t, z = 0$ from $(0,1,0)$ to $(1,1,0)$

23. Find $\lim_{(x,y)\to(0,0)} f(x,y) = \frac{x^2-y^2}{x^2+y^2}$

24. Using Green's Theorem evaluate the outward flux of the field $\vec{F} = x\,\hat{\imath} + x^2\hat{\jmath}$ across the square bounded by the lines $x = \pm 1$ and $y = \pm 1$.

25. Find the direction of the function $f(x,y) = x^2 + 2xy + y^2$ in which function icreases most rapidly at $(-1,1)$.

26. Find the unit normal vector to the surface $\vec{r}(\theta) = (a\cos\emptyset\sin\theta)\hat{\imath} + (a\sin\emptyset\sin\theta)\hat{\jmath} + (a\cos\emptyset)\hat{k}, 0 \le \emptyset \le \pi, 0 \le \theta \le 2\pi$

(10 x 4 =40 Mar

Part C

Answer any Six from the following Nine questions (27 – 35).
Each question carries 7 Marks.

27. Show that the partial derivatives of the function $f(x,y) = \begin{cases} 0, & xy \neq 0 \\ 1 & xy = 0 \end{cases}$ at (0,0) exists f is not continuous at (0,0).

28. If $w = x^2 + y - z + \sin t$ and $t = x + y$, then find $\left(\frac{\partial w}{\partial t}\right)_{(x,z)}$ and $\left(\frac{\partial w}{\partial z}\right)_{(x,y)}$.

29. Using chain rule express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ in terms of u and v, if $w = \ln(x^2+y^2+z^2)$, $x = ue^v\sin u$, $y = ue^v\cos u$, z= ue^v. Also evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point $(u,v) = (-2,0)$.

30. Find the equations of tangent plane and normal line of the surface $x^2 + y^2 + z - 9 =$

31. Find the absolute maximum and minimum values of $f(x.y) = 2 + 2x + 2y - x^2 -$ on the triangular plate in the first quadrant bounded by the lines $x = 0, y = 0, y = 9 -$

32. Find a quadratic approximation of $f(x,y) = \cos x \cos y$ at the origin. Estimate the er in the approximation if $|x| \le 0\cdot 1$, and $|y| \le 0\cdot 1$

33. Evaluate the integral $\int_0^{\frac{2}{3}} \int_y^{2-2y} (x+2y)e^{(y-x)^2}\,dxdy$.

34. Find the volume of a solid bounded below by the sphere $\rho = 2\cos\phi$ and above by the cone $z = \sqrt{x^2+y^2}$.

35. Find the outward flux of $\vec{F} = (y-x)\,\hat{\imath} + (x-y)\hat{\jmath} + (y-x)\hat{k}$ across the boundary of the cube bounded by the planes $x = \pm 1, y = \pm 1$ and $z = \pm 1$.

(6 x 7 = 42 Mar

Part D

Answer any two from the following Three questions (27 – 35).
Each question carries 13 Marks.

36. Find the volume of the region cut from the elliptical plane $x^2 + 4y^2 \le 4$ by the plane $z = x + 2$ and the xy-plane.

37. Maximize the function $f(x,y,z) = x^2 + 2y - y^2$ subject to the constraints $2x - y =$ and $y + z = 0$

38. Verify the Stoke's theorem for the field $\vec{F} = x^2\hat{\imath} + 2x\hat{\jmath} + z^2\hat{k}$ around the curve $C{:}4x^2$ $y^2 = 4$ in the xy-plane when viewed from above.

(2 x 13 = 26 Mar

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2B5N17199 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2017

MAT5B06 – Abstract Algebra

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A

Answer all the* twelve *questions

Each question carries 1 mark

1. Determine whether $H = \{ n^2 : n\epsilon\mathbb{Z}^+ \}$ is closed under the operation addition.
2. Define commutative binary operation. Give one example.
3. List all subgroups of $(\mathbb{Z}_6, +_6)$.
4. State true or false: "All groups of order 4 are isomorphic".
5. Give an example of an infinite group which is not cyclic.
6. D_4 has exactly _____ subgroups of order 2.
7. Define even permutation.
8. The index $(\mathbb{Z}: 3\mathbb{Z})$ is ______.
9. Define kernel of a group homomorphism $\varphi: G \to G'$.
10. What is the unity in the ring $\mathbb{Z} \times \mathbb{Z}$.
11. Characteristic of the ring $(\mathbb{R}, +, .)$ is _____.
12. Give an example of a field having countably infinite number of elements.

(12×1=12 marks)

Section B

Answer any* ten *out of fourteen questions.

Each question carries 4 marks.

13. Let S be a set and let f, g and h be functions mapping S in to S. Show that $(f \circ g) \circ h = f \circ (g \circ h)$.
14. Show that the groups $(\mathbb{R}, +)$ and $(\mathbb{R}^+, .)$ are isomorphic.
15. Define cyclic subgroup generated by an element. Give one example.
16. List all subgroups of the Klein-4 group V. Give the lattice diagram.
17. Show that every cyclic group is abelian. Is the converse is true? Justify your claim.
18. Define permutation of a set. Is $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x + 1$ a permutation? Justify your answer.
19. Find the orbit of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 1 & 5 & 2 & 7 & 6 & 9 & 8 \end{pmatrix}$.
20. Find the order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 4 & 6 & 5 & 7 \end{pmatrix}$ in S_7.
21. Show that $\mathbb{Z}_p$ has no proper nontrivial subgroups if p is a prime number.
22. Find all cosets of the subgroups $\langle 2 \rangle$ of $\mathbb{Z}_{12}$.
23. Show that a group homomorphism $\varphi: G \to G'$ is a one-to-one map iff $\text{Ker}(\varphi) = \{e\}$.
24. Define units in a ring R. Describe all units in the ring $\mathbb{Z} \times \mathbb{Z}$.
25. Define field of quotient of an integral domain D. Give one example.
26. Solve the equation $3x = 2$ in the field $\mathbb{Z}_7$.

(10× 4 = 40 marks)

Section C

Answer any six out of nine questions

Each question carries 7 Marks

27. Show that in a group G, $(ab)^{-1} = a^{-1}b^{-1}$ iff $ab = ba$ for all $a, b \in G$
28. Show that a nonempty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H \quad \forall a, b \in H$
29. Let G be a group and let a be fixed element in G. Show that $H_a = \{ x \in G : xa = ax \}$ is a subgroup of G.
30. Show that every prime order group is cyclic.
31. Show that subgroup of a cyclic group is cyclic.
32. State and Prove : Lagrange's theorem for finite groups.
33. Show that $\mathbb{M}_2(\mathbb{R})$, the collection of all 2×2 real matrices form a ring under matrix addition and matrix multiplication.
34. Define zero divisors in a ring $(R, +, .)$. Find all zero divisors in the ring $(\mathbb{Z}_8, +_8, \times_8)$
35. Show that every Field is an Integral domain.

(6×7=42 marks)

Section D

Answer any two out of three questions

Each question carries 13 Marks

36. Let G be cyclic group with generator a. Prove the following

    a) If G has infinite order , then $G \cong \mathbb{Z}$

    b) If G has finite order n, then $G \cong \mathbb{Z}_n$

37. Show that D_4 , the collection of symmetries of a square in the plane forms a group under permutation multiplication.
38. Let φ be a homomorphism of group G into G'. Then prove the following

    a) If $\in G$, then $\varphi(a^{-1}) = \varphi(a)^{-1}$

    b) If H is a subgroup of G, then $\varphi[H]$ is a subgroup of G'.

(2×13= 26 marks)

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2017

MAT5B07 – Basic Mathematical Analysis

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A

Answer all the twelve questions

Each question carries 1 mark

1. State well-ordering property of N.
2. Define absolute value of a real number.
3. For a∈ R and $\varepsilon > 0$, the ε-neighbourhood $V_\varepsilon(a)$ = ----------------.
4. A positive real number is rational if and only if its decimal representation is -------------.
5. Give examples of a set (i) having both infimum and supremum in the set (ii) having infimum but no supremum.
6. $\text{Lim}_{n\to\infty}(n^{\frac{1}{n}}) =$ ....
7. What is the set of all cluster points of the set (1, 3).
8. Give an example of a properly divergent sequence.
9. State Cauchy convergence criterion.
10. Represent $-1-i$ in polar form.
11. State De Moivre's formula.
12. Identify the region $|z—2| =3$. **(12x1 = 12 marks)**

Section B

Answer any ten questions out of fourteen questions

Each question carries 4 marks

13. State and prove triangle inequality of real numbers.
14. If a, b ϵ R, show that $a^2 + b^2 = 0$ if and only if $a = 0$ and $b = 0$.
15. Define supremum and infimum of a set. Let $S = \{ 1 - \frac{(-1^n)}{n} : n \in N \}$. Find inf S and sup S.
16. Determine the set of all x satisfying $| x - 1 | < | x |$.
17. State completeness property of real numbers.
18. Prove that $11^n - 7^n$ is divisible by 4, for all n ϵ N.
19. Verify that the set of integers Z is denumerable
20. Using the definition of limit, prove that $\lim \frac{2n-1}{3n+2} = \frac{2}{3}$.
21. Prove that a convergent sequence is absolutely convergent.
22. Discuss the convergence of the sequence $\left(\frac{\cos n}{n}\right)$.
23. Is the intersection of any collection of open sets is open? Justify your answer.
24. Give an examples of divergent sequences (x_n) and (y_n) such that (x_n/y_n) is convergent.
25. Prove that $|z_1 + z_2| \geq |z_1| - |z_2|$ for all $z_1, z_2 \in \mathbb{C}$.
26. Find all values of $(-1)^{\frac{1}{3}}$. **(10 x 4 = 40 marks)**

Section C

Answer any six questions out of nine questions
Each question carries 7 marks

27. State and prove Bernoulli's inequality.
28. Show that the set of real numbers [0 , 1] is not countable.
29. Let S be a nonempty subset of R and $a \in R$. If $a + S := \{ a + s : s \in S \}$, then show that $\sup (a + S) = a + \sup S$.
30. Prove that a subset of R is closed if and only if it contains all of its limit points.
31. If $X = (x_n)$ and $Y = (y_n)$ converges to x and y respectively, then show that the sequence $XY = (x_n y_n)$ converges to xy.
32. A sequence (a_n) of positive terms is defined by $a_{n+1} = \sqrt{2 + a_n}$, where $a_1 = 0$, show that the sequence converges to the positive root of $x^2 - x - 2 = 0$.
33. Define Cantor set. Show that Cantor set contains uncountable number of points.
34. State and prove ratio test for the convergence of real sequences.
35. Sketch the region $| z - 1 | + | z + 1 | < 4$. State whether it is open or closed.

(6 x 7= 42 marks)

Section D

Answer any two questions out of three questions
Each question carries 13 marks

36. (a) Define a sequence of nested intervals and give an example.
    (b) State and prove nested interval property of real numbers.

37. (a) State and prove monotone convergence theorem.
    (b) Test the convergence of the sequence (x_n) where $x_n = \sqrt{n+1} - \sqrt{n}$.

38. (a) Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2 |z_1|^2 + 2 |z_2|^2$.
    (b) Interpret the result geometrically.

(2 x 13 = 26 marks)

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Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2017

MAT5B08 – Differential Equations

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A
Answer all the twelve questions
Each question carries 1 mark

1. What is the order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} = \text{sinx}$.
2. Find the integrating factor of the differential equation x $\frac{dy}{dx}$ + y = tanx.
3. Define the Wroskian of the two functions f(t) and g(t).
4. What is the period of the function f(x) = tan(πx)
5. Write the standard form of a second order linear differential equation.
6. Write the one dimensional heat conduction equation.
7. True or False: If the function f(x) is even , then its reciprocal function is also even.
8. What is the Laplace transformation of the function f(t)=t^2+at+b?
9. Give the definition of impulse function.
10. Define the convolution of two functions f(t) and g(t).
11. Find the eigen values of the matrix A = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
12. Check whether the function f(x) = x + sin(x) is even or odd.

(12 x 1 = 12 Marks)

Section B
Answer any TEN questions
Each question carries 4 marks

13. Show that the differential equation (2xy+y–tany)dx+(x^2–$xtan^2$y+sec^2y + 2)dy=0 is exact.
14. Solve the differential equation $y' = 1 + y^2$.
15. State some differences between linear and non-linear differential equations.
16. Find the Wroskian of the functions cos(ax) and sin(ax).
17. Solve the differential equation $y'' + y'$ - 2y = 0.
18. Use the method of reduction of order to find a second a solution $y_2(x)$of the differential equation x^2 y''+2xy'-2y = 0 , x>0 if one solution is given by y_1(x) = x.
19. Define the unit step function u_c (t) and find its Laplace transform.
20. Find the Laplace transformation of the function f(t) = cos^2 (t).
21. Find the inverse Laplace transformation of $\frac{60+6s^2+s^4}{s^7}$
22. Find the convolution of the functions t and e^t
23. Find the eigen values of the matrix A = $\begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$.
24. Show that the product of two even functions is an even function.
25. Write the Euler-Fourier formulas to find the Fourier coefficients a_0 , a_n and b_n in the Fourier series expansion of a function having period 2L.
26. Show that Laplace transformation is a linear operator.

(10 x 4 = 40 Marks)

Section C

Answer any SIX questions

Each question carries SEVEN marks.

27. Show that the differential equation $(1+4xy+2y^2)dx + (1+4xy+2x^2)dy = 0$ is exact and hence solve it.
28. Use Euler's method with h=0.1 to find approximate values of the solution of the differential equation $\frac{dy}{dt} = \frac{3t^2}{3y^2-4}$ at t=1.2 , 1.4 , 1.6 and 1.8
29. Solve by the method of variation of parameters $y''+y = \sec(x)$.
30. Solve the non-homogenous equation $y''-3y'+2y = e^x$ by the method of undetermined coefficients.
31. Solve the initial value problem $y''+0.2y'+4.01y=0$, $y(0)=0$, $y'(0)=2$.
32. Show that the convolution of two functions is commutative.
33. Solve the integral equation $y(t) = 2t - 4\int_0^t y(u)(t-u)du$.
34. Find the Fourier Series for $f(x)=|x|$ in $[-\pi , \pi]$.
35. Solve by the method of separation of variables $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

(6 x 7 = 42 Marks)

Section D

Answer any TWO questions

Each question carries 13 marks.

36. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.
37. Using Laplace transformation solve the differential equation $y''+y = t$, $y(0)=1$, $y'(0)=-2$.
38. Find the Fourier series expansion of $f(x) = \begin{Bmatrix} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{Bmatrix}$, $f(x+2\pi) = f(x)$.

    Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = \frac{\pi}{4}$.

(2 x 13 =26 Marks)

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1B5N17202 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics (Open Course) Degree Examination, November 2017

MAT5D03 – Mathematics for Social Science

(2015 Admission onwards)

Max. Time: 2 hours Max. Marks: 40

Section A

Answer allthe six questions

Each question carries 1 mark

1. Solve the equation $5x - 39 = 5(x - 8) + 1$.
2. Check whether the equation $x = 4$ is a function or not? Why?
3. Evaluate $\lim_{x\to 4} \frac{x-4}{x^2-16}$.
4. Differentiate $f(x) = \sqrt{16 - x^2}$ using generalized power rule.
5. Test the concaveness for the function $y = -4x^3 + 5x^2 + 14x - 15$
6. Write any two properties of exponents.

(6 x 1 = 6 marks)

Section B

Answer any* five *out of seven questions

Each question carries 2 marks.

7. Find the equation of the line passes through the point (3, 10) and has slope −4.
8. Identify the domain and range of $y = \frac{7}{x(x-4)}$
9. Test whether the function $y = 2x^2 - 48x + 27$ is increasing or decreasing at $x = 3$.
10. Solve $4\ lnx - 10 = 0$, for x.
11. Determine the area under the curve $y = 20 - 4x$ over the interval 0 to 5.
12. Find the total differential for the function$z = x^3 + 7xy + 5y^4$.
13. Use L' H$\hat{o}pital's$ rule to evaluate $\lim_{x\to\infty} \frac{x-7}{e^x}$

(5 x 2 = 10 marks)

Section C

Answer any **three** *out of five questions*

Each question carries **4** *marks.*

14. Optimize the function $y = 3x^3 - 36x^2 + 135x - 17$.

15. If $f(x) = 6x - 5$ and $g(x) = 8x - 3$, then

    find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$ and $(f \div g)(x)$

16. Evaluate the area between the curves $y = 7 - x^2$ and $y = 3$ from $x = -2$ to $x = 2$.

17. Find the anti derivative of $\int \left(25\, x^{\frac{1}{4}} + 16x^{\frac{1}{3}}\right) dx$, given $F(0) = 19$

18. Find the equilibrium price p_0 and quantity q_0 given Supply : $q = 200p - 1400$

    and Demand : $q = -50p + 1850$.

(3 x 4 = 12 marks)

Section D

Answer any two *out of three questions*

Each question carries **6** *marks.*

19. Use Lagrange multipliers maximize the function $f(x, y, z) = 3x^2yz$ subject to the constraint $x + y + z = 32$.

20. Express profit π as an explicit function of x for given $R(x) = 280x - 2x^2$, $C(x) = 60x + 5,600$ and determine the maximum level of profit by finding the vertex of $\pi(x)$. Also find the x – intercept of the graph.

21. Use integration by parts to evaluate $\int 9x(x + 5)^2 dx$.

(2 x 6 = 12 marks)